## Checkpoint: Assess Your Understanding, pages 602–604

### 7.1

**1. Multiple Choice** How many roots does the equation $\sin 6x = \frac{1}{3}$ have over the domain $0 \leq x < 2\pi$?

**A.** 2 **B.** 4 **C.** 6 **D.** 12

**2.** Use graphing technology to solve each equation over the given domain. Give the roots to the nearest hundredth.

**a)** $1 + 2 \sin x = 1 - 3 \cos x$; $0 \leq x \leq 2\pi$

**Graph the corresponding function: $y = 2 \sin x + 3 \cos x$**
**Determine the approximate zeros in the given domain.**
**The roots are approximately: $x = 2.16$ and $x = 5.30$**
**Substitute each root into the given equation to verify.**

**b)** $2 = \cos x + 2 \cos^2 x$; $-2\pi \leq x \leq 2\pi$

**Graph the corresponding function: $y = \cos x + 2 \cos^2 x - 2$**
**Determine the approximate zeros in the given domain.**
**The roots are approximately: $x = \pm 0.67$ and $x = \pm 5.61$**
**Substitute each root into the given equation to verify.**

3. Use graphing technology to determine the general solution of each equation over the set of real numbers. Give the answers to the nearest hundredth.

a) $4 \tan x - 5 = 0$

Graph the corresponding function: $y = 4 \tan x - 5$
The period of the function is $\pi$.
Determine the zero in the domain $0 \leq x < \pi$.
The root is approximately: $x = 0.90$
The general solution is approximately: $x = 0.90 + \pi k, k \in \mathbb{Z}$

b) $6 \cos^2 x + \cos x = 1$

Graph the corresponding function: $y = 6 \cos^2 x + \cos x - 1$
The period of the function is $2\pi$.
Determine the zeros in the domain $0 \leq x < 2\pi$.
The roots are approximately: $x = 1.23$, $x = 2.09$, $x = 4.19$, $x = 5.05$
The general solution is approximately: $x = 1.23 + 2\pi k, k \in \mathbb{Z}$ or $x = 2.09 + 2\pi k, k \in \mathbb{Z}$ or $x = 4.19 + 2\pi k, k \in \mathbb{Z}$ or $x = 5.05 + 2\pi k, k \in \mathbb{Z}$

## 7.2

4. **Multiple Choice** Which number is a root of the equation $3 \sin x + 1 = 5 \sin x - 1$ over the domain $0 \leq x < 2\pi$?

**A.** 0  **B.** $\pi$  **C.** $\frac{\pi}{2}$  **D.** $\frac{3\pi}{2}$

5. Use algebra to solve the equation $\sqrt{2} \cos 2x + 1 = 0$ over the domain $-\pi < x < \pi$, then write the general solution of the equation.

$$\sqrt{2} \cos 2x = -1$$
$$\cos 2x = -\frac{1}{\sqrt{2}}$$

The terminal arm of angle $2x$ lies in Quadrant 2 or 3.

The reference angle for angle $2x$ is: $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

In Quadrant 2, $2x = \frac{3\pi}{4}$, $x = \frac{3\pi}{8}$  In Quadrant 3, $2x = -\frac{3\pi}{4}$, $x = -\frac{3\pi}{8}$

The period of $\cos 2x$ is $\pi$, so other roots are:

$x = \frac{3\pi}{8} - \pi$, $x = -\frac{5\pi}{8}$  and  $x = -\frac{3\pi}{8} + \pi$, $x = \frac{5\pi}{8}$

The roots are: $x = \pm\frac{3\pi}{8}$ and $x = \pm\frac{5\pi}{8}$

The general solution is: $x = \frac{3\pi}{8} + \pi k, k \in \mathbb{Z}$ or $x = \frac{5\pi}{8} + \pi k, k \in \mathbb{Z}$

**6.** Verify that $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ are two roots of the equation $4\cos^2 x - 3 = 0$.

**Substitute each given value in the equation.**

**For $x = \frac{\pi}{6}$:**

$$\begin{aligned}\text{L.S.} &= 4\cos^2\left(\frac{\pi}{6}\right) - 3\\ &= 4\left(\frac{\sqrt{3}}{2}\right)^2 - 3\\ &= 0\\ &= \text{R.S.}\end{aligned}$$

**For $x = \frac{5\pi}{6}$:**

$$\begin{aligned}\text{L.S.} &= 4\cos^2\left(\frac{5\pi}{6}\right) - 3\\ &= 4\left(-\frac{\sqrt{3}}{2}\right)^2 - 3\\ &= 0\\ &= \text{R.S.}\end{aligned}$$

**For each value of $x$, the left side is equal to the right side, so the roots are verified.**

**7.** Use algebra to solve the equation $10\sin^2 x + 11\sin x = -3$ over the domain $90° \leq x \leq 360°$. Give the roots to the nearest degree.

$$10\sin^2 x + 11\sin x + 3 = 0$$
$$(2\sin x + 1)(5\sin x + 3) = 0$$

**Either $2\sin x + 1 = 0$**

$\sin x = -0.5$

**The reference angle is: $\sin^{-1}(0.5) = 30°$**

**The terminal arm of angle $x$ lies in Quadrant 3 or 4.**

**In Quadrant 3, $x = 180° + 30°$, or $210°$**

**In Quadrant 4, $x = 360° - 30°$, or $330°$**

**or $5\sin x + 3 = 0$**

$\sin x = -0.6$

**The reference angle is: $\sin^{-1}(0.6) \doteq 37°$**

**The terminal arm of angle $x$ lies in Quadrant 3 or 4.**

**In Quadrant 3, $x \doteq 180° + 37°$, or $217°$**

**In Quadrant 4, $x \doteq 360° - 37°$, or $323°$**

**The roots are: $x = 210°$, $x \doteq 217°$, $x \doteq 323°$, $x = 330°$**