Lesson 8.3 Exercises, pages 712–715

A

3. Evaluate.

a) $\frac{4!}{2!2!}$

$= \frac{\cancel{4}^{2} \cdot 3}{\cancel{2} \cdot 1}$

$= 6$

b) $\frac{9!}{3!6!}$

$= \frac{\cancel{9}^{3} \cdot \cancel{8}^{4} \cdot 7}{\cancel{3} \cdot \cancel{2} \cdot 1}$

$= 84$

c) $\frac{8!}{2!3!4!}$

$= \frac{\cancel{8}^{4} \cdot 7 \cdot \cancel{6} \cdot 5}{\cancel{2} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}$

$= 140$

d) $\frac{10!}{2!2!5!}$

$= \frac{10 \cdot 9 \cdot \cancel{8}^{2} \cdot 7 \cdot 6}{\cancel{2} \cdot \cancel{2}}$

$= 7560$

4. Which word in each pair has the greater number of permutations of all its letters?

a) BID or BIB

Number of permutations:

BID: $3! = 6$

BIB: $\frac{3!}{2!1!} = 3$

BID has the greater number of permutations.

b) DEED or DIED

Number of permutations:

DEED: $\frac{4!}{2!2!} = \frac{\cancel{4}^{2} \cdot 3}{\cancel{2}}$

$= 6$

DIED: $\frac{4!}{2!1!1!} = 4 \cdot 3$

$= 12$

DIED has the greater number of permutations.

c) KAYAK or KOALA

Number of permutations:

KAYAK: $\frac{5!}{2!2!1!} = \frac{5 \cdot \cancel{4}^{2} \cdot 3}{\cancel{2}}$

$= 30$

KOALA: $\frac{5!}{2!1!1!1!} = 5 \cdot 4 \cdot 3$

$= 60$

KOALA has the greater number of permutations.

d) RUDDER or REDDER

Number of permutations:

RUDDER: $\frac{6!}{2!2!1!1!} = \frac{6 \cdot 5 \cdot \cancel{4}^{2} \cdot 3}{\cancel{2}}$

$= 180$

REDDER: $\frac{6!}{2!2!2!} = \frac{6 \cdot 5 \cdot \cancel{4} \cdot 3}{\cancel{2} \cdot \cancel{2}}$

$= 90$

RUDDER has the greater number of permutations.

B

5. How many permutations are there of the 4 digits in each number?

a) 1234

All digits are different.
Number of permutations:

$4! = 24$

b) 1123

Two of the 4 digits are identical.
Number of permutations:

$\frac{4!}{2!} = 4 \cdot 3$

$= 12$

c) 1113

3 of the 4 digits are identical.
Number of permutations:
$\frac{4!}{3!} = 4$

d) 1111

All digits are identical.
Number of permutations is 1.

6. a) How many permutations are there of all the letters in each of these Aboriginal words?

i) ISKWEW

There are 6 letters.
2 are Ws.
Number of permutations:
$$\frac{6!}{2!} = 6 \cdot 5 \cdot 4 \cdot 3$$
$$= 360$$

ii) TSILIKST

There are 8 letters.
2 are Ts, 2 are Ss, and 2 are Is.
Number of permutations:
$$\frac{8!}{2!2!2!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4} \cdot 3}{\cancel{2} \cdot \cancel{2}}$$
$$= 5040$$

iii) SUMSHASAT

There are 9 letters.
3 are Ss and 2 are As.
Number of permutations:
$$\frac{9!}{3!2!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \overset{2}{\cancel{4}}}{\cancel{2}}$$
$$= 30\ 240$$

iv) KINNIKINNICK

There are 12 letters.
3 are Ks, 4 are Is, and 4 are Ns.
Number of permutations:
$$\frac{12!}{3!4!4!} = 138\ 600$$

b) How do identical letters change the number of permutations?

When there are identical letters in a word, the number of permutations is less than the number of permutations when all the letters are different.

7. The number of permutations of all the letters in the word BRICK is 120. How can you use this information to determine the number of permutations of all the letters in the word BROOK?

All the letters in BRICK are different.
The number of 5-letter permutations of BRICK is 120.
BROOK also has 5 letters, but there are 2 Os.
So, the number of permutations is: $\frac{120}{2!} = 60$

8. The number of permutations of all the digits in a 5-digit number is one. What do you know about the number? Justify your answer.

Since the number of permutations is 1, all digits must be identical; for example, 22 222.

9. How many 9-digit numbers can be created from the digits 5, 5, 6, 6, 6, 7, 7, 7, 7?

There are 9 digits of which 2 are 5s, 3 are 6s, and 4 are 7s.
So, the number of 9-digit numbers that can be created is:
$\frac{9!}{2!3!4!} = \frac{9 \cdot \overset{4}{\cancel{8}} \cdot 7 \cdot \cancel{6} \cdot 5}{\cancel{2} \cdot \cancel{6}}$, or 1260
1260 nine-digit numbers can be created.

10. Create a 5-digit number so that the number of permutations of all the digits is:

a) the greatest possible

The greatest number of permutations occurs when all digits are different; for example, 24 789.
Number of permutations is:
$5! = 120$

b) the least possible

The least number of permutations occurs when all digits are identical; for example, 66 666.
Number of permutations is 1.

c) 10

When all the digits are different, the number of permutations is 120. So, some digits must be identical. Since the number of permutations is 10, and $120 \div 10 = 12$, look for factorials whose product is 12: $2! \cdot 3! = 12$. So, 2 digits are identical and 3 digits are identical; for example, 22 333.

d) 5

When all the digits are different, the number of permutations is 120. So, some digits must be identical. Since the number of permutations is 5, and $120 \div 5 = 24$, look for a factorial whose value is 24: $4! = 24$. So, 4 digits are identical; for example, 13 111.

11. Identify a common word that satisfies each requirement.

a) Contains 3 letters; numbers of permutations of all letters is 6.

$3! = 6$; since the word contains 3 letters, all letters must be different; for example, DOG

b) Contains 4 letters; numbers of permutations of all letters is 12.

$4! = 24$ and $24 \div 12 = 2$
Since $2! = 2$, two letters are identical; for example, FOOD

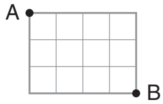
c) Contains 4 letters; numbers of permutations of all letters is 24.

$4! = 24$; since the word contains 4 letters, all letters must be different; for example, WORD

d) Contains 5 letters; numbers of permutations of all letters is 30.

$5! = 120$ and $120 \div 30 = 4$
Since $2! \cdot 2! = 4$, there are 2 pairs of identical letters; for example, SEEDS

12. a) How many ways are there to get from A to B travelling along grid lines and moving only to the right or down?



Total number of grid squares travelled: 7
Squares travelled right: 4; squares travelled down: 3
So, the number of ways to get from A to B is: $\frac{7!}{4!3!} = \frac{7 \cdot 6 \cdot 5}{6}$
$= 35$

b) Why does order matter in this problem?

Order matters because travelling 4 squares right and 3 squares down is different from travelling 3 squares right, 1 square down, 1 square right, and 2 squares down. The total number of squares travelled in each direction is the same but the paths are different.

C

13. How many ways can all the letters in the word ABACUS be arranged so that the vowels are always together?

There are 3 vowels and 3 consonants.
Since the vowels have to be together, consider them as 1 object.
So, there are 4 objects: the vowels and 3 different consonants.
The number of permutations of 4 objects is: $4! = 24$
Two of the 3 vowels are identical so the number of permutations of the vowels: $\frac{3!}{2!} = 3$
So, the number of ways is: $24 \cdot 3 = 72$ ways

14. A 26-term series is written using only the integers +8 and −5. How many such series can be written with a sum of 0? Explain your reasoning.

Let m represent the number of terms of $+8$.
Let n represent the number of terms of -5.
Solve this linear system to determine how many 8s and -5s can be combined to have a sum of 0: $8m - 5n = 0$
$m + n = 26$

$$\begin{aligned} 8m - 5n &= 0 \\ +5m + 5n &= 130 \\ \hline 13m &= 130 \\ m &= 10 \end{aligned}$$

$$\begin{aligned} 10 + n &= 26 \\ n &= 16 \end{aligned}$$

A series with ten 8s and sixteen -5s will have a sum of 0.
The series is a collection of terms where 10 terms are of one kind and 16 terms are of another kind. So, the number of series that can be written is:
$\frac{26!}{10!16!} = 5\,311\,735$