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CUMULATIVE REVIEW Chapters 1-8, pages 761–772

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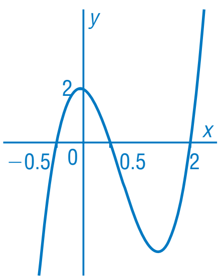
1. Sketch a graph of this polynomial function:
$g(x) = 4x^3 - 8x^2 - x + 2$

Factor the polynomial.
Use mental math to substitute $x = 1$, then $x = -1$ in $g(x)$ to determine that neither $x - 1$ nor $x + 1$ is a factor.
Try $x = 2$: $g(2) = 32 - 32 - 2 + 2$
$= 0$
So, $x - 2$ is a factor.
Divide to determine the other factor.

$$\begin{array}{r|rrrr} 2 & 4 & -8 & -1 & 2 \\ & & 8 & 0 & -2 \\ \hline & 4 & 0 & -1 & 0 \end{array}$$

So, $4x^3 - 8x^2 - x + 2 = (x - 2)(4x^2 - 1)$
Factor the second binomial: $4x^2 - 1 = (2x - 1)(2x + 1)$
So, $4x^3 - 8x^2 - x + 2 = (x - 2)(2x - 1)(2x + 1)$
So, the x-intercepts of the graph are: 2, 0.5, −0.5
The equation has degree 3, so it is an odd-degree polynomial function.
The leading coefficient is positive, so as $x \to -\infty$, the graph falls and as $x \to \infty$, the graph rises.
The constant term is 2, so the y-intercept is 2.
Plot points at the intercepts, then join them with a smooth curve.

$g(x) = 4x^3 - 8x^2 - x + 2$



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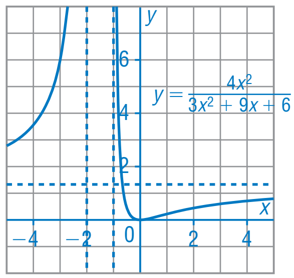
2. Sketch the graph of $y = \frac{4x^2}{3x^2 + 9x + 6}$, then state its domain and range.

The function is undefined when:
$3x^2 + 9x + 6 = 0$
$3(x + 2)(x + 1) = 0$
$x = -2$ or $x = -1$
These are the equations of the vertical asymptotes.
The leading coefficients are 4 and 3, so there is a horizontal asymptote with equation $y = \frac{4}{3}$.



Determine the behaviour of the graph close to the asymptotes, and the coordinates of a point between the vertical asymptotes.
Approximate the *y*-values.

x	−2.01	−1.99	−1.01	−0.99	−100	100	−1.5
y	533	−533	−137	129	1.37	1.29	−12

When $x = 0$, $y = 0$, so the graph passes through the origin.
Draw broken lines for the asymptotes.
Plot points to show the behaviour of the graph near the asymptotes.
Join the points to form smooth curves.
The domain is: $x \neq -2, x \neq -1$
Using a graphing calculator, the range is: $y \geq 0$ or $y \leq -\frac{32}{3}$

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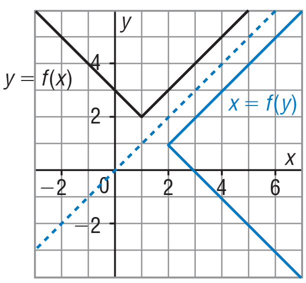
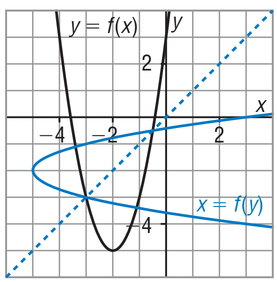
3. Restrict the domain of each function below so its inverse is a function.

a)



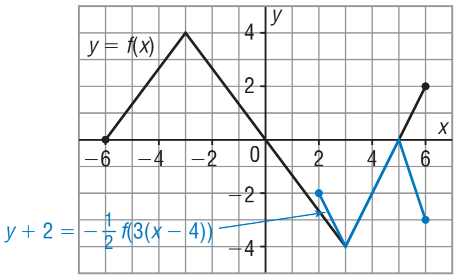
Sample response:
Sketch the graph of the inverse.
The inverse is a function if the domain of $y = f(x)$ is restricted to either $x \leq 1$ or $x \geq 1$.

b)



Sample response:
Sketch the graph of the inverse.
The inverse is a function if the domain of $y = f(x)$ is restricted to either $x \leq -2$ or $x \geq -2$.

4. Here is the graph of $y = f(x)$. On the same grid, sketch the graph of $y + 2 = -\frac{1}{2}f(3(x - 4))$.



Compare: $y - k = af(b(x - h))$ to $y + 2 = -\frac{1}{2}f(3(x - 4))$:

$k = -2$, $a = -\frac{1}{2}$, $b = 3$, and $h = 4$

(x, y) on $y = f(x)$ corresponds to

$\left(\frac{x}{3} + 4, -\frac{1}{2}y - 2\right)$ on $y + 2 = -\frac{1}{3}f(3(x - 4))$

(x, y)	$\left(\frac{x}{3} + 4, -\frac{1}{2}y - 2\right)$
$(-6, 0)$	$(2, -2)$
$(-3, 4)$	$(3, -4)$
$(3, -4)$	$(5, 0)$
$(6, 2)$	$(6, -3)$

Join the points with line segments.

4

5. For $y = 2x^2 + 4x + 5$, determine two sets of possible functions f and g so that $y = f(g(x))$.

Sample response: Complete the square.

$y = 2(x^2 + 2x + 1 - 1) + 5$

$y = 2(x + 1)^2 + 3$

One solution is: $g(x) = x + 1$ and $f(x) = 2x^2 + 3$

Another solution is: $g(x) = (x + 1)^2$ and $f(x) = 2x + 3$

6. A principal of \$800 is invested in a savings account that pays 7% annual interest, compounded semi-annually. To the nearest quarter of a year, how long will it take until this investment is worth \$1000?

Use the formula for the growth of money with compound interest.

$A = A_0\left(1 + \frac{i}{n}\right)^{nt}$, **where** t **is the time in years since the principal was invested**

Substitute: $A = 1000$, $A_0 = 800$, $i = 0.07$, $n = 2$

$1000 = 800\left(1 + \frac{0.07}{2}\right)^{2t}$ **Divide each side by 800.**

$1.25 = 1.035^{2t}$ **Take the common logarithm of each side.**

$\log 1.25 = \log (1.035)^{2t}$ **Apply the power law.**

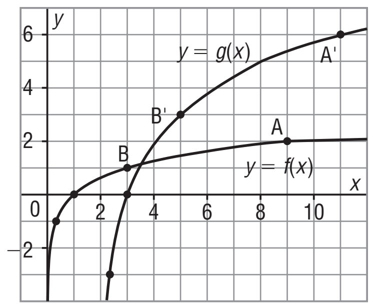
$\log 1.25 = 2t \log 1.035$

$t = \frac{\log 1.25}{2 \log 1.035}$

$t = 3.2432\ldots$

It will take approximately 3.25 years until the investment is worth \$1000.

7. a) The graphs of a logarithmic function and its transformation image are shown. Corresponding points are indicated. Identify the transformations.



The horizontal distance between: A and B is 6; A′ and B′ is 6
The vertical distance between: A and B is 1; A′ and B′ is 3
The vertical distance triples, so there is a vertical stretch by a factor of 3.
After this stretch, the coordinates of A(9, 2) will be (9, 6).
The translation that moves (9, 6) to A′(11, 6) is 2 units right.

b) Given that $f(x) = \log_3 x$, what is $g(x)$? Justify your answer.

Write the equation of the image graph in the form:
$y - k = a \log_3 d(x - h)$
a **represents the vertical stretch, so** $a = 3$.
h **represents the horizontal translation, so** $h = 2$.
There is no horizontal stretch or compression, and no vertical translation, so $k = 0$ **and** $d = 1$.
So, an equation of the image graph is: $g(x) = 3 \log_3(x - 2)$

8. For each angle below:

i) Sketch the angle in standard position.

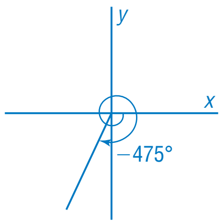
ii) Write an expression for the measures of all angles that are coterminal with the angle in standard position.

iii) Determine the values of the six trigonometric ratios of the angle. Give exact answers where possible, or to the nearest thousandth.

a) $-475°$ **b)** $\frac{15\pi}{4}$

a)

i)



ii) The measures of all coterminal angles can be represented by:
$-475° + k360°, k \in \mathbb{Z}$

iii) Use a calculator.

$\sin(-475°) \doteq -0.906$

$\csc(-475°) = \frac{1}{\sin(-475°)} \doteq -1.103$

$\cos(-475°) \doteq -0.423$

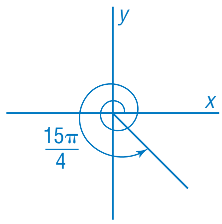
$\sec(-475°) = \frac{1}{\cos(-475°)} \doteq -2.366$

$\tan(-475°) \doteq 2.145$

$\cot(-475°) = \frac{1}{\tan(-475°)} \doteq 0.466$

b)

i)



ii) The measures of all coterminal angles can be represented by:
$\frac{15\pi}{4} + 2\pi k, k \in \mathbb{Z}$

iii) $\frac{15\pi}{4} - 2\pi = \frac{7\pi}{4}$

The terminal arm of the angle lies in Quadrant 4 where the cosine and secant ratios are positive, and the other 4 trigonometric ratios are negative.

$\sin\frac{15\pi}{4} = \sin\frac{7\pi}{4} = -\frac{1}{\sqrt{2}}$

$\csc\frac{15\pi}{4} = -\sqrt{2}$

$\cos\frac{15\pi}{4} = \cos\frac{7\pi}{4} = \frac{1}{\sqrt{2}}$

$\sec\frac{15\pi}{4} = \sqrt{2}$

$\tan\frac{15\pi}{4} = \tan\frac{7\pi}{4} = -1$

$\cot\frac{15\pi}{4} = -1$

9. Given $\sin\theta = \frac{2}{\sqrt{13}}$, determine the exact values of the other 5 trigonometric ratios in the domain $90° \leq \theta \leq 270°$.

Since $\sin\theta$ is positive, the terminal arm of angle θ lies in Quadrant 1 or 2.
Since $\sin\theta = \frac{2}{\sqrt{13}}$, point P($x$, 2) on a circle, with radius $\sqrt{13}$, is a terminal point of angle θ in standard position. Use mental math and the Pythagorean Theorem to determine that $x = \pm\sqrt{13 - 4}$, or $x = \pm 3$.
From the given domain, the terminal arm of angle θ lies in Quadrant 2, so $x = -3$. Then:

$\csc\theta = \frac{\sqrt{13}}{2}$ $\cos\theta = -\frac{3}{\sqrt{13}}$ $\sec\theta = -\frac{\sqrt{13}}{3}$

$\tan\theta = -\frac{2}{3}$ $\cot\theta = -\frac{3}{2}$

10. A circle has radius 5 cm. In radians and degrees, what is the measure of the central angle subtended by an arc with length 10 cm? Give the answers to the nearest tenth where necessary.

Use: $a = \theta r$ Substitute: $a = 10$, $r = 5$

$10 = 5\theta$

$\theta = 2$

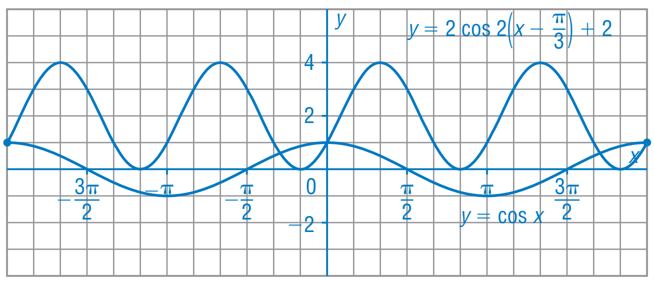
The measure of the central angle is 2 radians.
Convert 2 radians to degrees.

$\theta = 2\left(\frac{180°}{\pi}\right)$

$\theta = 114.5915...°$

The measure of the central angle is approximately 114.6°.

11. a) Sketch the graph of $y = \cos x$ for the domain $-2\pi \leq x \leq 2\pi$.



b) On the grid in part a, sketch the graph of $y = 2\cos 2\left(x - \frac{\pi}{3}\right) + 2$. Describe your strategy.

The graph of $y = \cos x$ is: stretched vertically by a factor of 2, compressed horizontally by a factor of $\frac{1}{2}$, then translated $\frac{\pi}{3}$ units right and 2 units up.
I chose points on the graph of $y = \cos x$, applied the transformations to each point, then joined the image points.

c) For the function in part b, list the: amplitude, period, phase shift, zeros, domain, and range.

The amplitude is 2; the period is π; the phase shift is $\frac{\pi}{3}$; the zeros are: $-\frac{7\pi}{6}, -\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$; the domain is $x \in \mathbb{R}$; the range is $0 \leq y \leq 4$

12. The pedals of an exercise bicycle are mounted on a bracket that is 30 cm above the floor. Each pedal is 18 cm from the centre of the bracket. A person pedals at 60 rotations per minute.

a) Determine a sinusoidal function that models the height of a pedal above the floor, *h* metres, as a function of time, *t* seconds.

The timing begins when the pedal is at the lowest point, which is:
30 cm − 18 cm = 12 cm
So, at $t = 0, h = 12$
At 60 rotations per minute, the time for one rotation is 1 s.
So, after 0.5 s, the pedal is at its highest point, which is:
12 cm + 36 cm = 48 cm
So, at $t = 0.5, h = 48$
The graph begins at (0, 12), which is a minimum point.
The first maximum point is at (0.5, 48).
The position of the first maximum is known, so use a cosine function:
$h(t) = a\cos b(t - c) + d$
The amplitude is $30 - 12 = 18$, so $a = 18$.
The period is 1, so $b = \frac{2\pi}{1}$, or 2π.
The phase shift is $c = 0.5$.
The vertical translation is $d = 30$.
So, an equation is: $h(t) = 18\cos 2\pi(t - 0.5) + 30$

b) During the first rotation, when is the pedal 40 cm above the floor? Give the answers to the nearest hundredth of a second.

Use technology.
Graph $Y = 18\cos 2\pi(X - 0.5) + 30$ and $Y = 40$, for $0 < X < 2$, then determine the approximate X-coordinates of the first two points of intersection: $X \doteq 0.34$ and $X \doteq 0.66$
The pedal is 40 cm above the floor after approximately 0.34 s and 0.66 s.

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13. Use graphing technology to solve each equation for $0 \leq x < 2\pi$, then write the general solution. Give the roots to the nearest hundredth.

a) $3 \sin 2x + 1 = 2 \cos 2x$

Graph $y = 3 \sin 2x + 1$ and $y = 2 \cos 2x$.
Determine the approximate x-coordinates of the points of intersection.
To the nearest hundredth, the roots are: $x = 0.15$, $x = 2.01$, $x = 3.30$, and $x = 5.15$
The period is π, so the general solution is approximately:
$x = 0.15 + \pi k, k \in \mathbb{Z}$ or
$x = 2.01 + \pi k, k \in \mathbb{Z}$

b) $3 \cos^2 x = 1 - 2 \cos x$

Graph the related function
$y = 3 \cos^2 x - 1 + 2 \cos x$
Determine the approximate zeros.
To the nearest hundredth, the roots are: $x = 1.23$, $x = 3.14$, and $x = 5.05$
The period is 2π, so the general solution is approximately:
$x = 1.23 + 2\pi k, k \in \mathbb{Z}$,
$x = 3.14 + 2\pi k, k \in \mathbb{Z}$, or
$x = 5.05 + 2\pi k, k \in \mathbb{Z}$

14. Use algebra to solve each equation for the domain $-180° \leq x \leq 180°$. Give the roots to the nearest degree.

a) $5 - 3 \tan x = 2 \tan x + 1$

$-5 \tan x = -4$
$\tan x = \frac{4}{5}$, or 0.8
Since $\tan x$ is positive, the terminal arm of angle x lies in Quadrant 1 or 3.
In Quadrant 1, $x = \tan^{-1}(0.8)$
So, $x = 38.6598...°$
In Quadrant 3,
$x = -180° + 38.6598...°$
$x = -141.3401...°$
The roots are: $x \doteq 39°$ and $x \doteq -141°$

b) $4 \sin^2 x + 5 \sin x = -1$

$4 \sin^2 x + 5 \sin x + 1 = 0$
$(4 \sin x + 1)(\sin x + 1) = 0$
Either $4 \sin x + 1 = 0$
$\sin x = -\frac{1}{4}$, or -0.25
Since $\sin x$ is negative, the terminal arm of angle x lies in Quadrant 3 or 4.
The reference angle is:
$\sin^{-1}(0.25) = 14.4775...°$
In Quadrant 3,
$x = -180° + 14.4775...°$
$x = -165.5224...°$
In Quadrant 4,
$x = -14.4775...°$
Or $\sin x + 1 = 0$
$\sin x = -1$
The terminal arm of angle x lies on the negative y-axis, so $x = -90°$.
The roots are: $x \doteq -14°$, $x \doteq -166°$, and $x = -90°$

15. For each expression below:

i) Determine any non-permissible values of θ.

ii) Write the expression in simplest form.

a) $(\sec\theta)(\sec\theta - \cos\theta)$ **b)** $\dfrac{\cos\theta + \sin\theta\tan\theta}{\tan\theta}$

a) i) $\sec\theta = \dfrac{1}{\cos\theta}$, so $\cos\theta \neq 0$,

$\theta \neq \dfrac{\pi}{2} + \pi k, k \in \mathbb{Z}$

ii) $(\sec\theta)(\sec\theta - \cos\theta)$

$= \sec^2\theta - \sec\theta\cos\theta$

$= \sec^2\theta - 1$

$= \tan^2\theta$

b) i) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, so $\cos\theta \neq 0$,

$\theta \neq \dfrac{\pi}{2} + \pi k, k \in \mathbb{Z}$ and

$\sin\theta \neq 0, \theta \neq \pi k, k \in \mathbb{Z}$

ii) $\dfrac{\cos\theta + \sin\theta\tan\theta}{\tan\theta}$

$= \dfrac{\cos\theta + \sin\theta\left(\dfrac{\sin\theta}{\cos\theta}\right)}{\dfrac{\sin\theta}{\cos\theta}}$

$= \dfrac{\cos^2\theta + \sin^2\theta}{\sin\theta}$

$= \dfrac{1}{\sin\theta}$

$= \csc\theta$

16. Prove this identity: $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$

L.S. $= \tan^2 x - \sin^2 x$

$= \dfrac{\sin^2 x}{\cos^2 x} - \sin^2 x$ Use a common denominator.

$= \dfrac{\sin^2 x - (\sin^2 x)(\cos^2 x)}{\cos^2 x}$ Factor.

$= \dfrac{(\sin^2 x)(1 - \cos^2 x)}{\cos^2 x}$

$= \dfrac{(\sin^2 x)(\sin^2 x)}{\cos^2 x}$

$= \dfrac{\sin^2 x}{\cos^2 x} \cdot \sin^2 x$

$= \tan^2 x \sin^2 x$

$=$ R.S.

The left side is equal to the right side, so the identity is proved.

17. Solve this equation over the domain $-90° < x < 270°$:

$\dfrac{\tan 4x - \tan 3x}{1 + \tan 4x \tan 3x} = \sqrt{3}$

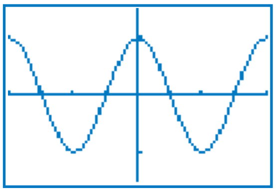
$\tan(4x - 3x) = \sqrt{3}$

$\tan x = \sqrt{3}$

$x = 60°$ or $x = 240°$

18. Use graphing technology to verify the identity $\cos 2\theta = 2\cos^2\theta - 1$. Sketch the graph.

The graphs of $y = \cos 2\theta$ and $y = 2\cos^2\theta - 1$ coincide, so the identity is verified.



19. Prove this identity: $\tan 2\theta \cos 2\theta = \dfrac{2\tan\theta}{1 + \tan^2\theta}$

$$\text{L.S.} = \tan 2\theta \cos 2\theta = \left(\frac{\sin 2\theta}{\cos 2\theta}\right)(\cos 2\theta) = \sin 2\theta$$

$$\text{R.S.} = \frac{2\tan\theta}{1 + \tan^2\theta} = \frac{2\dfrac{\sin\theta}{\cos\theta}}{\sec^2\theta} = \frac{2\dfrac{\sin\theta}{\cos\theta}}{\dfrac{1}{\cos^2\theta}} = \frac{2\sin\theta\cos\theta}{1} = \sin 2\theta$$

The left and right sides simplify to the same expression, so the identity is proved.

8

20. a) License plates in British Columbia have the form ABC 123 or 123 ABC. All 26 letters (A to Z) and 10 digits (0 to 9) may be used more than once. How many plates are possible?

Use the fundamental counting principle.
For each letter, there are 26 choices.
For each digit, there are 10 choices.
The number of possible plates with the form ABC 123 is:
$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 17\,576\,000$
The number of possible plates with the form 123 ABC is:
$10 \cdot 10 \cdot 10 \cdot 26 \cdot 26 \cdot 26 = 17\,576\,000$
So, the total number of plates possible is:
$17\,576\,000 + 17\,576\,000 = 35\,152\,000$

b) Suppose a repetition of a letter or digit is not permitted. How many plates are possible?

When a letter or digit cannot be used more than once:
The number of possible plates with the form ABC 123 is:
$26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 = 11\ 232\ 000$
The number of possible plates with the form 123 ABC is:
$10 \cdot 9 \cdot 8 \cdot 26 \cdot 25 \cdot 24 = 11\ 232\ 000$
So, the total number of plates possible is:
$11\ 232\ 000 + 11\ 232\ 000 = 22\ 464\ 000$

c) Explain why the answers in parts a and b are different.

When repetition is not allowed, the total number of arrangements decreases. For example, when repetition is allowed, there are 4 possible arrangements for the letters A and B: AA, BB, AB, BA
When repetition is not allowed, there are 2 possible arrangements: AB, BA

21. What is the number of permutations of all the letters in the name of each capital city?

a) WINNIPEG

There are 8 letters.
2 are Is and 2 are Ns.
Number of permutations:

$$\frac{8!}{2!2!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot {}^{2}\not{4} \cdot 3}{\not{2}} = 10\ 080$$

b) REGINA

There are 6 letters.
All letters are different.
Number of permutations:
$6! = 720$

22. The pre-requisite grade 12 courses for a Bachelor of Arts program at the University of Alberta are English Language Arts 30-1; and 4 courses from a list of 10 Grade 12 courses. How many ways can a student meet the pre-requisite courses?

Number of ways of choosing from the list of Grade 12 courses:

$$_{10}C_4 = \frac{10!}{(10 - 4)!4!} = \frac{10!}{6!4!} = 210$$

There is only one way to choose English Language Arts 30-1.
So, the number of ways a student can meet the pre-requisites is:
$210 \cdot 1 = 210$
A student can meet the pre-requisite courses in 210 ways.

23. Determine the indicated term in each expansion.

a) the first term in the expansion of $(-3x - 0.2y)^9$

The first term in the expansion of $(x + y)^n$ is x^n.
Substitute: $x = -3x$ and $n = 9$
$(-3x)^9 = -19\,683x^9$
So, the first term is $-19\,683x^9$.

b) the third term in the expansion of $(0.5x + 2y)^7$

Use the expression for the kth term in the expansion of $(x + y)^n$:
${}_nC_{k-1}x^{n-(k-1)}y^{k-1}$
Substitute: $n = 7$, $k = 3$, $x = 0.5x$, $y = 2y$

$$\begin{aligned}{}_7C_{3-1}(0.5x)^{7-(3-1)}(2y)^{3-1} &= {}_7C_2(0.5x)^5(2y)^2\\ &= 21(0.031\,25x^5)(4y^2)\\ &= 2.625x^5y^2\end{aligned}$$

So, the third term in the expansion is $2.625x^5y^2$.

24. Expand using the binomial theorem: $(2x - 3y^2)^6$

$(x + y)^n = {}_nC_0x^n + {}_nC_1x^{n-1}y + {}_nC_2x^{n-2}y^2 + {}_nC_3x^{n-3}y^3 + \ldots + {}_nC_{n-1}xy^{n-1} + {}_nC_ny^n$
Substitute: $x = 2x$, $y = -3y$, $n = 6$

$$\begin{aligned}(2x - 3y^2)^6 &= {}_6C_0(2x)^6 + {}_6C_1(2x)^{6-1}(-3y^2) + {}_6C_2(2x)^{6-2}(-3y^2)^2 + {}_6C_3(2x)^{6-3}(-3y^2)^3 +\\ &\quad {}_6C_4(2x)^{6-4}(-3y^2)^4 + {}_6C_5(2x)^{6-5}(-3y^2)^5 + {}_6C_6(2x)^{6-6}(-3y^2)^6\\ &= 1(2x)^6 + 6(2x)^5(-3y^2) + 15(2x)^4(-3y^2)^2 + 20(2x)^3(-3y^2)^3 + 15(2x)^2(-3y^2)^4 +\\ &\quad 6(2x)^1(-3y^2)^5 + 1(2x)^0(-3y^2)^6\\ &= 64x^6 - 576x^5y^2 + 2160x^4y^4 - 4320x^3y^6 + 4860x^2y^8 - 2916xy^{10} + 729y^{12}\end{aligned}$$