8.1

1. A penny, a dime, and a loonie are in one bag. A nickel, a quarter, and a toonie are in another bag. Tessa removes 1 coin from each bag. Use a graphic organizer to list the total amounts she could have removed.

Use an organized list.
1¢ + 5¢ = 6¢   10¢ + 5¢ = 15¢ $1 + 5¢ = $1.05
1¢ + 25¢ = 26¢   10¢ + 25¢ = 35¢ $1 + 25¢ = $1.25
1¢ + $2 = $2.01   10¢ + $2 = $2.10 $1 + $2 = $3
So, there are 9 total amounts that Tessa could have removed.

2. The Braille code consists of patterns of raised dots arranged in a 3 by 2 array. The pattern for the letter Z is shown. How many different patterns are possible? Explain your reasoning.

Each position in the 3 by 2 cell may or may not have a raised dot.
Use the fundamental counting principle.
The number of possible patterns is: 2 · 2 · 2 · 2 · 2 · 2 = 64

8.2

3. A hiking group consists of 12 students and 2 leaders. A leader must be at the front and back of the line. How many ways can the group hike in a line?

The number of ways the students can hike in a line is:
12! = 479 001 600
The leaders A and B must be at the front and back of the line.
There are 2 ways for the leaders to line up: AB and BA
So, the number of ways the group can hike in a line is:
479 001 600 · 2 = 958 003 200

4. A code consists of 4 letters from the English alphabet and 3 letters from the Greek alphabet. There are 8 English letters and 6 Greek letters to choose from and repetition is not allowed. How many 7-letter codes are possible?

Order matters, so use permutations.

Determine the number of ways to arrange 4 English letters chosen from 8 letters:
$8 \cdot 7 \cdot 6 \cdot 5 = 1680$

Determine the number of ways to arrange 3 Greek letters chosen from 6 letters:
$6 \cdot 5 \cdot 4 = 120$

Use the fundamental counting principle.
$1680 \cdot 120 = 201\,600$
So, there are 201 600 possible 7-letter codes.

8.3

5. How many 12-letter permutations of GOBBLEDEGOOK can be created?

There are 12 letters: 2 are Gs, 2 are Es, 2 are Bs, and 3 are Os.
Number of permutations:
$\dfrac{12!}{2!2!2!3!} = 9\,979\,200$
So, 9 979 200 permutations can be created.

6. How many ways are there to arrange all the words in this tongue twister?
CAN YOU CAN A CAN AS A CANNER CAN CAN A CAN?

There are 12 words: 6 are CAN, and 3 are A.
Number of permutations:
$\dfrac{12!}{6!3!} = 110\,880$
So, there are 110 880 ways to arrange all the words.

7. How many 9-letter permutations of EQUATIONS can be created if the vowels must appear together in the order A, E, I, O, and U? Explain.

There are 5 vowels and 4 consonants.
Since the vowels have to be together, consider them as 1 object.
So, there are 5 objects: the group of vowels and 4 different consonants.
The number of permutations of 5 objects is: $5! = 120$
The vowels must be in the given order, so the number of permutations of the vowels is 1.
So, all the letters can be arranged in 120 ways.

8. A student volunteers at a food bank. He fills hampers with these items:
5 cans of soup chosen from 7 different soups
3 bags of pasta chosen from 4 different types of pasta
4 bags of vegetables chosen from 8 different types of vegetables
3 boxes of cereal chosen from 6 different types of cereal
How many ways can the student fill a hamper?

Use combinations. The number of ways of choosing:
5 cans of soup from 7 different soups is: $_7C_5 = 21$
3 bags of pasta from 4 different types of pasta is: $_4C_3 = 4$
4 bags of vegetables from 8 different types of vegetables is: $_8C_4 = 70$
3 boxes of cereal from 6 different types of cereal is $_6C_3 = 20$
Use the fundamental counting principle.
$21 \cdot 4 \cdot 70 \cdot 20 = 117\,600$
The student can fill the hamper in 117 600 ways.

9. Solve each equation for n or r.

a) $_nC_3 = 84$

$$_nC_3 = \frac{n!}{(n-3)!3!}$$

$$84 = \frac{n!}{(n-3)!6}$$

$$6 \cdot 84 = n(n-1)(n-2)$$

$$504 = n(n-1)(n-2)$$

Use a calculator.
$$\sqrt[3]{504} \doteq 7.96$$
So, try 3 consecutive
numbers with 8 as the
middle number:
$7 \cdot 8 \cdot 9 = 504$
So, $n = 9$

b) $_7C_r = 35$

$$_7C_r = \frac{7!}{(7-r)!r!}$$

$$35 = \frac{5040}{(7-r)!r!}$$

$$(7-r)!r! = \frac{5040}{35}$$

$$(7-r)!r! = 144$$

Use guess and test.
Since $4! = 24$,
$(7-4)!4! = 6 \cdot 24$
$$= 144$$
So, $r = 4$
Or, since $_7C_4 = {_7C_3}$, $r = 3$

10. a) These are the first 7 numbers in row 14 of Pascal's triangle:
1, 13, 78, 286, 715, 1287, 1716
Complete the row. What strategy did you use?

Row 14 of Pascal's triangle has 14 terms.
The triangle is symmetrical: the numbers in each row read the same
from left to right and from right to left. So, the remaining numbers in
row 14 are the given numbers in reverse order: 1716, 1287, 715, 286,
78, 13, 1

b) Use your results from part a to write the numbers in row 15 of the triangle.

1 1 + 13 = 14   13 + 78 = 91   78 + 286 = 364
   286 + 715 = 1001   715 + 1287 = 2002   1287 + 1716 = 3003
   1716 + 1716 = 3432   1716 + 1287 = 3003   1287 + 715 = 2002
   715 + 286 = 1001   286 + 78 = 364   78 + 13 = 91   13 + 1 = 14   1
So, the numbers in row 15 are:
1, 14, 91, 364, 1001, 2002, 3003, 3432, 3003, 2002, 1001, 364, 91, 14, 1

11. Determine the value of each number in Pascal's triangle.

a) the 4th number in row 13

Use $_nC_r$. Substitute: $n = 12, r = 3$
$_{12}C_3 = 220$

b) the 5th number in row 21

Use $_nC_r$. Substitute: $n = 20, r = 4$
$_{20}C_4 = 4845$

8.6

12. The 4th term in the expansion of $(x + 1)^8$ is $56x^5$. Which other term in the expansion has a coefficient of 56? Explain.

Because both coefficients in the binomial $(x + 1)^8$ are 1, the coefficients in the expansion are the 9 terms in row 9 of Pascal's triangle. These terms are the same when read from left to right and right to left. So, in row 9, the 5th term is the middle term, and the 6th term is the same as the 4th term, 56.

13. Expand using the binomial theorem.

a) $(4x - 1)^5$

$(x + y)^n = {_nC_0}x^n + {_nC_1}x^{n-1}y + {_nC_2}x^{n-2}y^2 + \ldots + {_nC_n}y^n$
Substitute: $n = 5, x = 4x, y = -1$
$(4x - 1)^5 = {_5C_0}(4x)^5 + {_5C_1}(4x)^4(-1) + {_5C_2}(4x)^3(-1)^2 + {_5C_3}(4x)^2(-1)^3$
$\qquad + {_5C_4}(4x)(-1)^4 + {_5C_5}(-1)^5$
$\qquad = 1(1024x^5) + 5(256x^4)(-1) + 10(64x^3)(1) + 10(16x^2)(-1)$
$\qquad + 5(4x)(1) - 1$
$\qquad = 1024x^5 - 1280x^4 + 640x^3 - 160x^2 + 20x - 1$

b) $(5x^2 + 2y)^4$

$(x + y)^n = {_nC_0}x^n + {_nC_1}x^{n-1}y + {_nC_2}x^{n-2}y^2 + \ldots + {_nC_n}y^n$

Substitute: $n = 4$, $x = 5x^2$, $y = 2y$

$(5x^2 + 2y)^4 = {_4C_0}(5x^2)^4 + {_4C_1}(5x^2)^3(2y) + {_4C_2}(5x^2)^2(2y)^2$

$\qquad\qquad + {_4C_3}(5x^2)(2y)^3 + {_4C_4}(2y)^4$

$\qquad = 1(625x^8) + 4(125x^6)(2y) + 6(25x^4)(4y^2) + 4(5x^2)(8y^3) + 16y^4$

$\qquad = 625x^8 + 1000x^6y + 600x^4y^2 + 160x^2y^3 + 16y^4$

14. Determine the indicated term in each expansion.

a) the 3rd term in $(-7x + 1)^6$　　　　**b)** the 7th term in $(6x + 2y)^7$

The kth term is: ${_nC_{k-1}}x^{n-(k-1)}y^{k-1}$

Substitute: $k = 3$, $n = 6$,

$x = -7x$, $y = 1$

${_6C_2}(-7x)^4(1)^2 = 15(2401x^4)(1)$

$\qquad\qquad = 36\,015x^4$

The 3rd term is $36\,015x^4$.

The kth term is: ${_nC_{k-1}}x^{n-(k-1)}y^{k-1}$

Substitute: $k = 7$, $n = 7$, $x = 6x$,

$y = 2y$

${_7C_6}(6x)^1(2y)^6 = 7(6x)(64y^6)$

$\qquad\qquad = 2688xy^6$

The 7th term is $2688xy^6$.

15. Use the binomial theorem to evaluate $(1.2)^6$. Use a calculator to verify your result.

Write $(1.2)^6$ as $(1 + 0.2)^6$.

$(x + y)^n = {_nC_0}x^n + {_nC_1}x^{n-1}y + {_nC_2}x^{n-2}y^2 + \ldots + {_nC_n}y^n$

Substitute: $n = 6$, $x = 1$, $y = 0.2$

$(1 + 0.2)^6 = {_6C_0}(1)^6 + {_6C_1}(1)^5(0.2) + {_6C_2}(1)^4(0.2)^2 + {_6C_3}(1)^3(0.2)^3$

$\qquad\qquad + {_6C_4}(1)^2(0.2)^4 + {_6C_5}(1)(0.2)^5 + {_6C_6}(0.2)^6$

$\qquad = 1(1) + 6(1)(0.2) + 15(1)(0.2)^2 + 20(1)(0.2)^3 + 15(1)(0.2)^4$

$\qquad\quad + 6(1)(0.2)^5 + 1(0.2)^6$

$\qquad = 1 + 1.2 + 0.6 + 0.16 + 0.024 + 0.001\,92 + 0.000\,064$

$\qquad = 2.985\,984$