Grade 12
Pre-Calculus Mathematics
Achievement Test

Marking Guide

January 2013

Manitoba

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Websites are subject to change without notice.

Disponible en français.

Available in alternate formats upon request.

Table of Contents

General Marking Instructions

Please make no marks in the student test booklets. If the booklets have marks in them, the marks need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the *Answer/Scoring Sheet* are identical
- **students and markers only use a pencil to complete the *Answer/Scoring Sheets***
- the totals of each of the four parts are written at the bottom
- each student's final result is recorded, by booklet number, on the corresponding *Answer/Scoring Sheet*
- the *Answer/Scoring Sheet* is complete
- a photocopy has been made for school records

Once marking is completed, please forward the *Answer/Scoring Sheets* to Manitoba Education in the envelope provided (for more information see the administration manual).

Marking the Test Questions

The test is composed of short-answer questions, long-answer questions, and multiple-choice questions. Short-answer questions are worth 1 or 2 marks each, long-answer questions are worth 3 to 5 marks each, and multiple-choice questions are worth 1 mark each. An answer key for the multiple-choice questions can be found at the beginning of the section "Booklet 2 Questions."

Each question is designed to elicit a well-defined response according to the associated specific learning outcome(s) and relevant mathematical processes. Their purpose is to determine whether a student meets the standards for the course as they relate to the knowledge and skills associated with the question.

To receive full marks, a student's response must be complete and correct. Where alternative answering methods are possible, the *Marking Guide* attempts to address the most common solutions. For general guidelines regarding the scoring of students' responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an *Answer/Scoring Sheet* is marked with "0" and/or "NR" only (e.g., student was present but did not attempt any questions) please document this on the *Irregular Test Booklet Report*.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

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Information for Markers

The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the *Answer/Scoring Sheet* that represents the marks awarded based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called “Communication Errors” (see Appendix A) and will be tracked on the *Answer/Scoring Sheet* in a separate section. There is a ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e. committing a second error for any type will not further affect a student’s mark), with a maximum deduction of 5 marks from the total test mark.

The student’s final mark is determined by subtracting the communication errors from the preliminary mark.

Example:

A student has a preliminary mark of 72. The student committed two E1 errors (½ mark deduction), four E7 errors (½ mark deduction), and one E8 error (½ mark deduction). Although seven communication errors were committed in total, there is a deduction of only 1½ marks.

COMMUNICATION ERRORS / ERREURS DE COMMUNICATION

Shade in the circles below for a maximum total deduction of 5 marks (0.5 mark deduction per error).
Noircir les cercles ci-dessous pour une déduction maximale totale de 5 points (déduction de 0,5 point par erreur).

E1 ●	**E2** ○	**E3** ○	**E4** ○	**E5** ○
E6 ○	**E7** ●	**E8** ●	**E9** ○	**E10** ○

Mark assigned to the student / Note accordée à l'élève

Booklet 1 / Cahier 1	+	Multiple Choice / Choix multiple	+	Booklet 2 / Cahier 2	-	Communication Errors / Erreurs de communication	=	Total
25 / 31	+	7 / 9	+	40 / 49	-	1½ (maximum deduction of 5 marks / déduction maximale de 5 points)	=	70½ / **89**

Scoring Guidelines

Booklet 1 Questions

Question 1 T5

Gina correctly started to answer the following question. Complete her solution.

Question: Solve the following equation for all real values of θ.
Express your answer in radians correct to 3 decimal places.

$$3\sin^2\theta - 14\sin\theta - 5 = 0$$

Gina's solution: $3\sin^2\theta - 14\sin\theta - 5 = 0$

$$(3\sin\theta + 1)(\sin\theta - 5) = 0$$

Solution

Method 1

$$3\sin^2\theta - 14\sin\theta - 5 = 0$$
$$(3\sin\theta + 1)(\sin\theta - 5) = 0$$

$\sin\theta = -\frac{1}{3}$ $\qquad$ $\sin\theta = 5$ no solution — ½ mark for $\sin\theta = 5$; ½ mark for no solution

$\theta_r = 0.339\ 837$

$\theta = 3.481\ 429,\ 5.943\ 348$ — 1 mark (½ mark for each value of θ)

$\theta = 3.481 + 2k\pi,\ k \in \mathrm{I}$
$\theta = 5.943 + 2k\pi,\ k \in \mathrm{I}$ — 1 mark for general solution

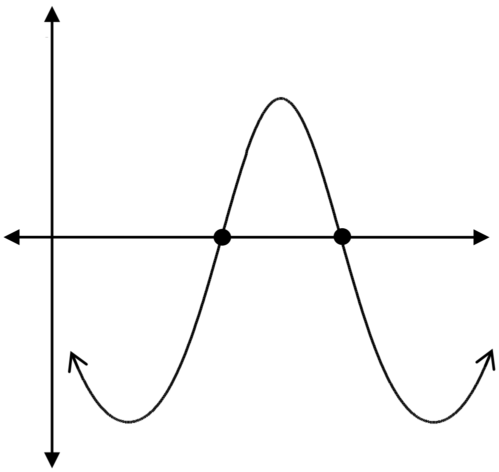
3 marks

Note(s):

- give maximum of 2 marks for solution in degrees: $\theta = 199.471^\circ + 360^\circ k,\ k \in \mathrm{I}$
  $\theta = 340.529^\circ + 360^\circ k,\ k \in \mathrm{I}$

Method 2

$y = 3\sin^2\theta - 14\sin\theta - 5$ ½ mark for equation



or ½ mark for justification

Find all zeros over the reals.

$\theta = 3.481 + 2k\pi,\ k \in \mathrm{I}$ 1 mark for solutions

$\theta = 5.943 + 2k\pi,\ k \in \mathrm{I}$ 1 mark for general solution

3 marks

Find and simplify the 6th term in the binomial expansion of $\left(3x^4 - \frac{1}{x^3}\right)^9$.

Solution

$t_6 = {}_9C_5\left(3x^4\right)^4\left(-\frac{1}{x^3}\right)^5$

2 marks (1 mark for ${}_9C_5$, ½ mark for each consistent factor)

$= (126)\left(81x^{16}\right)\left(-\frac{1}{x^{15}}\right)$

$= -10\ 206x$

1 mark for simplification (½ mark for evaluating coefficient, ½ mark for simplifying variable)

3 marks

Question 3 R10

The number of times a website is visited can be modeled by the function:

$$A = 800(e)^{rt}$$

where A = the total number of visitors at time t

t = the time in days $(t \geq 0)$

r = the rate of growth

After 5 days, 40 000 people have visited the site.
Determine the number of visitors expected after 9 days.

Express your answer as a whole number.

Solution

Method 1

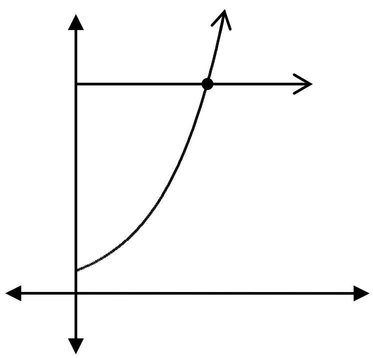
$40\ 000 = 800e^{r5}$	½ mark for substitution
$\frac{40\ 000}{800} = e^{5r}$	
$\ln 50 = \ln e^{5r}$	½ mark for applying logs
$\ln 50 = 5r$	1 mark for log theorem
$\frac{\ln 50}{5} = r$	
$r = 0.782\ 404\ 601$	
$A = 800e^{(0.782\,404\ 601)(9)}$	½ mark for substitution
$A = 914\ 610.103$	½ mark for calculations with base e
$A = 914\ 610$	**3 marks**

Method 2

Use calculator to find the value of r.

$y = 40\ 000$ — ½ mark for equations

$y = 800e^{5x}$



or — ½ mark for justification

Find the point of intersection of these two functions.

$x = 0.782\ 404\ 601$ — 1 mark for finding the value of x at the intersection point

Substitute x for r into formula.

$\therefore A = 800e^{(0.782\ 404\ 601)(9)}$

$A = 914\ 610.103$ — 1 mark for value of A

$A = 914\ 610$

3 marks

Question 4 R10

Solve algebraically:

$$10^{3x} = 7^{x+5}$$

Express your answer correct to 3 decimal places.

Solution

Method 1

$$10^{3x} = 7^{x+5}$$

$$\log 10^{3x} = \log 7^{x+5}$$ ½ mark for applying logs

$$3x(\log 10) = (x+5)\log 7$$ 1 mark for power rule

$$3x\log 10 = x\log 7 + 5\log 7$$

$$3x\log 10 - x\log 7 = 5\log 7$$ ½ mark for collecting terms with x

$$x = \frac{5\log 7}{3\log 10 - \log 7}$$ ½ mark for solving for x

$$x = 1.960\ 873$$ ½ mark for evaluating quotient of logarithms

$$x = 1.961$$

Method 2

$$10^{3x} = 7^{x+5}$$

$$\log 10^{3x} = \log 7^{x+5}$$

$$3x(\log 10) = (x+5)\log 7$$

$$3x = x\log 7 + 5\log 7$$

$$3x - x\log 7 = 5\log 7$$

$$x = \frac{5\log 7}{3 - \log 7}$$

$$x = 1.960\ 873$$

$$x = 1.961$$

3 marks

Question 5 P2

A word contains two Ms, two Es, two Ns, and no other repeated letters.

Suppose one of the Ns is replaced by an M.
Will this replacement result in greater or fewer permutations?

Justify your reasoning.

Solution

Method 1

$$2\,\text{Ms},\ 2\,\text{Es},\ 2\,\text{Ns:}\ \frac{(\text{total number of letters})!}{2!2!2!}$$

1 mark for dividing the total number of letters by 2!2!2!

$$\frac{(\text{total number of letters})!}{8}$$

$$3\,\text{Ms},\ 2\,\text{Es},\ 1\,\text{N:}\ \frac{(\text{total number of letters})!}{3!2!1!}$$

1 mark for dividing the total number of letters by 3!2!1!

2 marks

$$\frac{(\text{total number of letters})!}{12}$$

If one of the Ns is changed to an M, there would be fewer permutations.

Method 2

If one of the Ns is changed to an M, there would be fewer permutations because you would be dividing the total number of letters by a larger number.

2 marks for justification

2 marks

Question 6 P3

There is a group of 16 boys and 12 girls. How many ways can a committee of 3 people be formed if there must be at least 2 girls on the committee?

Express your answer as a whole number.

Solution

Method 1

Case 1: 2 girls, 1 boy	${}_{12}C_2 \bullet {}_{16}C_1 = 1056$	1 mark for case 1 (½ mark for each factor shown in a product)
Case 2: 3 girls	${}_{12}C_3 = 220$	1 mark for case 2
	$1056 + 220 = 1276$	1 mark for addition of cases

3 marks

Method 2

Case 1: 2 boys, 1 girl	${}_{16}C_2 \bullet {}_{12}C_1 = 1440$	1 mark for case 1 (½ mark for each factor shown in a product)
Case 2: 3 boys	${}_{16}C_3 = 560$	1 mark for case 2
All possibilities	${}_{28}C_3 = 3276$	½ mark for all possible cases
	$3276 - 1440 - 560 = 1276$	½ mark for subtraction from total

3 marks

Question 7 T1

A student is using the formula $s = \theta r$ to find an arc length of a circle. Given a central angle measure of 35° and a radius of 6 cm, the student's solution is as follows:

$$s = (35)(6)$$
$$s = 210 \text{ cm}$$

Explain why this solution is incorrect.

Write the correct solution.

Solution

When using the formula $s = \theta r$, the angle must be in radians. ½ mark for explanation

$$35^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{35\pi}{180} \quad \textbf{or} \quad \frac{7\pi}{36}$$

1 mark for conversion to radians

$$\therefore s = \theta r$$
$$= \frac{7\pi}{36}(6)$$
$$= \frac{42\pi}{36} \text{ cm} \quad \textbf{or} \quad \frac{7\pi}{6} \text{ cm} \quad \textbf{or} \quad 3.665 \text{ cm}$$

½ mark for simplification

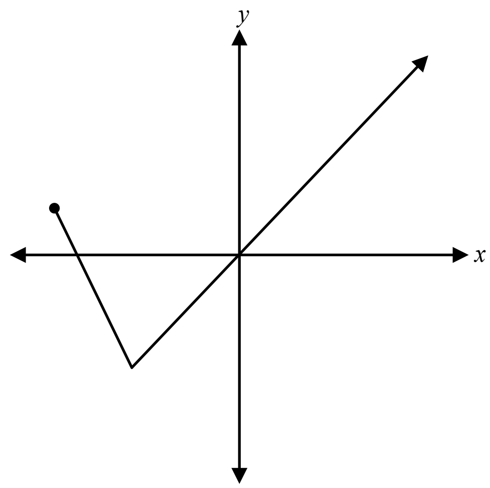
2 marks

Note(s):

- award 1 mark for correct final answer with no work

Question 8 R1

Given the graph of $f(x)$ below, explain how you would sketch the graph of $y = |f(x)|$.



Solution

The negative *y* values are reflected over the *x*-axis.

1 mark for explanation

1 mark

Claire correctly solves the following equation:

$$\log_2(6-x)+\log_2(3-x)=2$$

She finds two possible values of x: $x=2$ and $x=7$.

Identify which one of these values is unacceptable and explain why.

Solution

If x is greater than 3, you have a negative argument $\therefore x=2$ but $x \neq 7$.

or

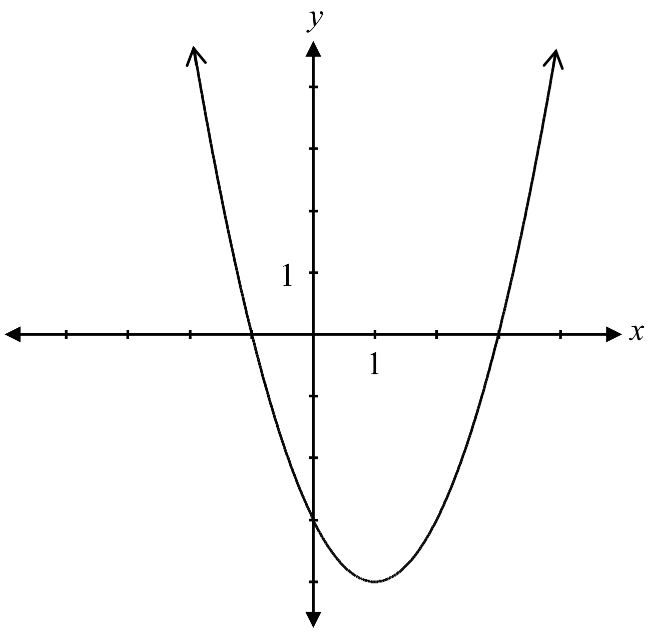
The domain is restricted to values of $x<3 \therefore x=2$.

1 mark for explanation

1 mark

Question 10 R13

Given the graph of the function $f(x)$ below, state the domain of $y = \sqrt{f(x)}$.



Solution

The domain is $\{x \in \mathbb{R} \mid x \leq -1 \text{ or } x \geq 3\}$.

or

The domain is $(-\infty, -1] \cup [3, \infty)$.

1 mark for domain

1 mark

A school offers 4 different Science courses, 3 different Mathematics courses, and 2 different English courses.

Julie must select 1 Science course, 1 Mathematics course, and 1 English course. She thinks this creates 9 options for her timetable.

Show why Julie is incorrect.

Solution

Julie should have multiplied the number of options.

1 mark for justification

or

1 mark

$4 \times 3 \times 2 = 24$ different options

Question 12 P4

Explain how Pascal's triangle can be used to determine the coefficients in the binomial expansion of $(x+y)^n$.

Solution

The $(n+1)^{\text{st}}$ row of Pascal's triangle corresponds to the coefficients of the terms in the expansion of the binomial $(x+y)^n$.

1 mark for explanation

1 mark

Prove the identity below for all permissible values of x:

$$\frac{1 + \cos 2x}{\sin 2x} = \cot x$$

Solution

Method 1

$\text{LHS} = \dfrac{1 + 2\cos^2 x - 1}{2\sin x \cos x}$	1 mark for identity ½ mark for identity
$= \dfrac{2\cos^2 x}{2\sin x \cos x}$	
$= \dfrac{\cos x}{\sin x}$	1 mark for simplification
$= \cot x$	½ mark for identity
$= \text{RHS}$	**3 marks**

Method 2

$\text{LHS} = \dfrac{1 + 1 - 2\sin^2 x}{2\sin x \cos x}$	½ mark for identity ½ mark for identity
$= \dfrac{2 - 2\sin^2 x}{2\sin x \cos x}$	
$= \dfrac{1 - \sin^2 x}{\sin x \cos x}$	½ mark for simplification
$= \dfrac{\cos^2 x}{\sin x \cos x}$	½ mark for identity
$= \dfrac{\cos x}{\sin x}$	½ mark for simplification
$= \cot x$	½ mark for identity
$= \text{RHS}$	**3 marks**

Solution

Method 3

$$\text{LHS} = \frac{1+\cos^2 x - \sin^2 x}{2\sin x\cos x}$$

½ mark for identity
½ mark for identity

$$= \frac{2\cos^2 x}{2\sin x\cos x}$$

½ mark for identity $\left(1-\sin^2 x = \cos^2 x\right)$

$$= \frac{\cos x}{\sin x}$$

1 mark for simplification

$$= \cot x$$

½ mark for identity

$$= \text{RHS}$$

3 marks

Question 14 T3

Your classmate, Leo, was absent for one of his math lessons.

Explain to Leo how to determine the cosecant ratio for an angle in standard position given that $P(-3, -4)$ is a point on the terminal arm of the angle.

Solution

Method 1

- Locate the point $(-3, -4)$ on the coordinate plane.
- Draw a right-angle triangle by connecting the point $(-3, -4)$ to the x-axis and to the origin.
- The angle created with the x-axis and the line connecting the point with the origin (the terminal arm) is θ. ½ mark
- Determine the length of the terminal arm by using the Pythagorean Theorem. ½ mark
- Once you have the lengths of all three sides of the triangle, find $\sin\theta$. ½ mark
- Invert the $\sin\theta$ ratio to find $\csc\theta$. ½ mark

2 marks

Method 2

- Determine the value of r using the Pythagorean Theorem. ½ mark
- Determine the value of $\sin\theta \left(\sin\theta = \dfrac{y}{r}\right)$. ½ mark
- Identify the correct quadrant. ½ mark
- Invert the $\sin\theta$ ratio to find $\csc\theta$. ½ mark

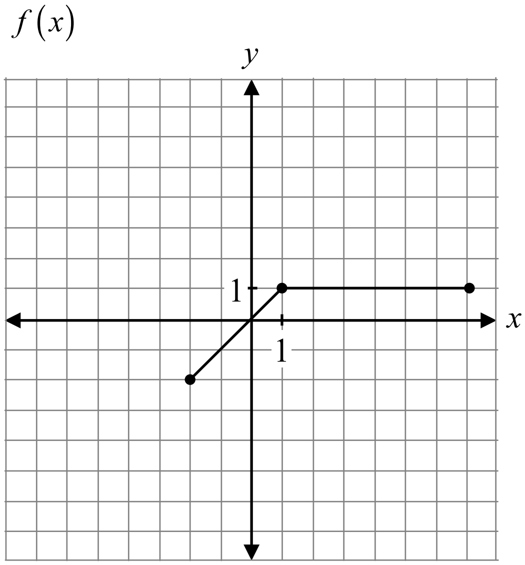
2 marks

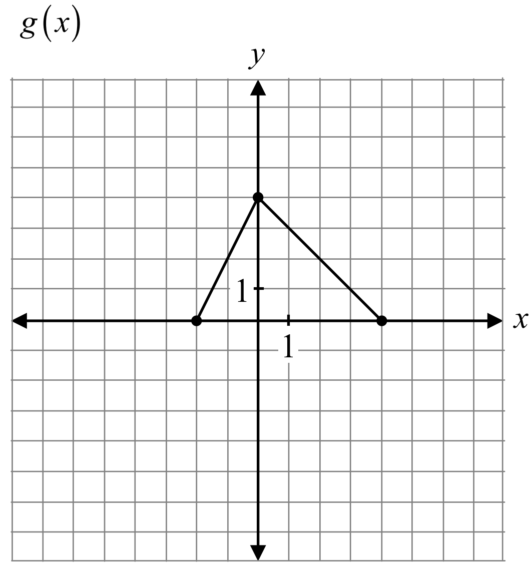
Note(s):

- award a maximum of 1 mark for correct work shown without an explanation in words

Question 15 R1

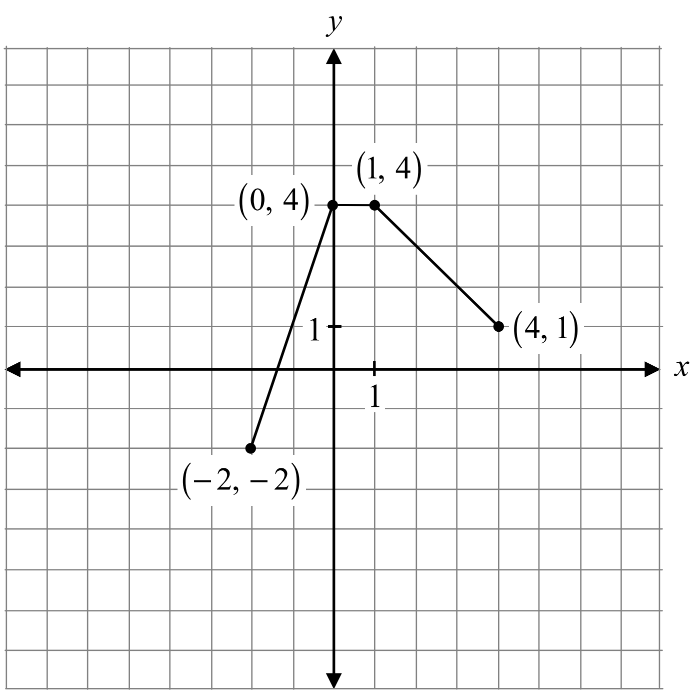
Given the following graphs:





Sketch the graph of $f(x) + g(x)$.

Solution



2 marks (½ mark for each point where the graph changes direction)
$[(-2, -2); (0, 4); (1, 4); (4, 1)]$

2 marks

Note(s):

- deduct a maximum of 1 mark for incorrect domain

Booklet 2 Questions

Answer Key for Multiple-Choice Questions

Question	Answer	Learning Outcome
16	D	R3
17	D	T1
18	A	R7
19	C	T6
20	C	R14
21	D	P4
22	A	R6
23	B	P2
24	D	R9

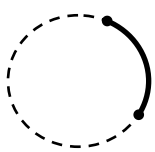
Question 16 R3

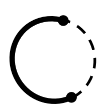
If $(2, 3)$ is a point on the graph of $y = f(x)$, what point must be on the graph of $y = 3f\left(\frac{1}{4}x\right)$?

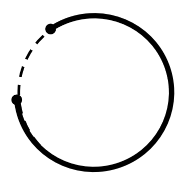
a) $\left(\frac{1}{2}, 1\right)$ b) $\left(\frac{1}{2}, 9\right)$ c) $(8, 1)$ d) $(8, 9)$

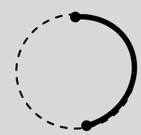
Question 17 T1

Consider the arc drawn on each circle. Which arc measure is closest to 3 radians?

a) 

b) 

c) 

d) 

Question 18 R7

If $\log_2 x = 4$, then $\log_2 (2x)$ is equal to:

a) 5 b) 8 c) 16 d) 32

Question 19 T6

Simplify the following expression:

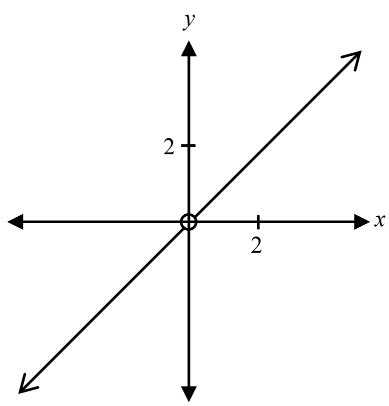
$$\cos^2 x\left(1+\cot^2 x\right)$$

a) $\sin^2 x$ b) $\cos^2 x$ c) $\cot^2 x$ d) $\sec^2 x$

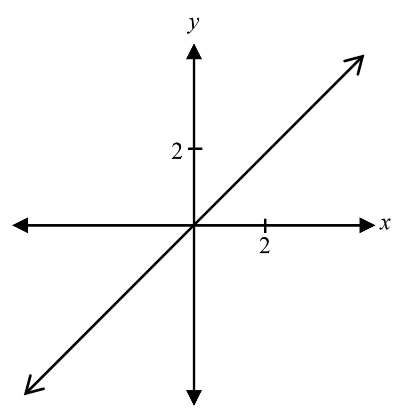
Question 20 R14

Identify the graph of the function $y = \frac{x}{x}$.

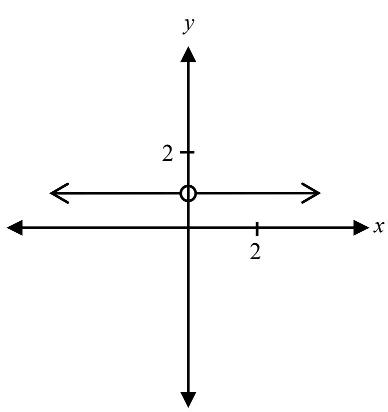
a)



b)



c)



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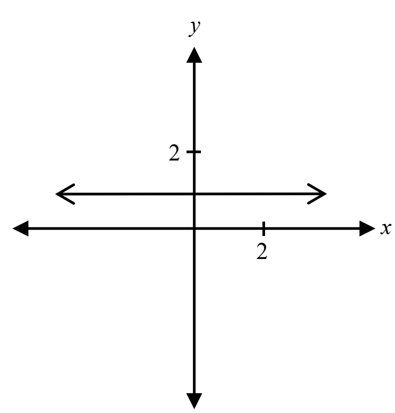


Question 21 P4

How many terms are in the expansion of $\left(3y^2 - 4z\right)^7$?

a) 2 b) 6 c) 7 d) 8

Question 22 R6

Determine one possible restriction for the domain of $y = \left(x+3\right)^2 - 4$ so that its inverse is a function.

a) $x \le -3$ b) $x \le 0$ c) $x \le 3$ d) $x \le 4$

Question 23 P2

Find the total possible number of arrangements for 7 adults and 3 children seated in a row if the 3 children must sit together.

a) $10!$ b) $8!3!$ c) $7!3!$ d) $7!$

Question 24 R9

Identify the value of the x-intercept of the function $y = \ln\left(x-2\right)$.

a) -1 b) 0 c) 2 d) 3

Question 25 R7

Given $\log_b a = 3$, give one example of possible values for a and b that make this equation true.

Solution

Answers will vary but $b^3 = a$.

Some possible solutions are: $a = 8$ $b = 2$

or

$a = 27$ $b = 3$

or

$a = 64$ $b = 4$

1 mark

Question 26 R5

The range of the graph of $y = f(x)$ is $[-3, 2]$.

Explain why there is no effect on the range of the graph that is a result of the transformation $y = f(-x)$.

Solution

$y = f(-x)$ is a reflection over the y-axis.
The domain is affected, but the range remains the same. 1 mark for explanation

1 mark

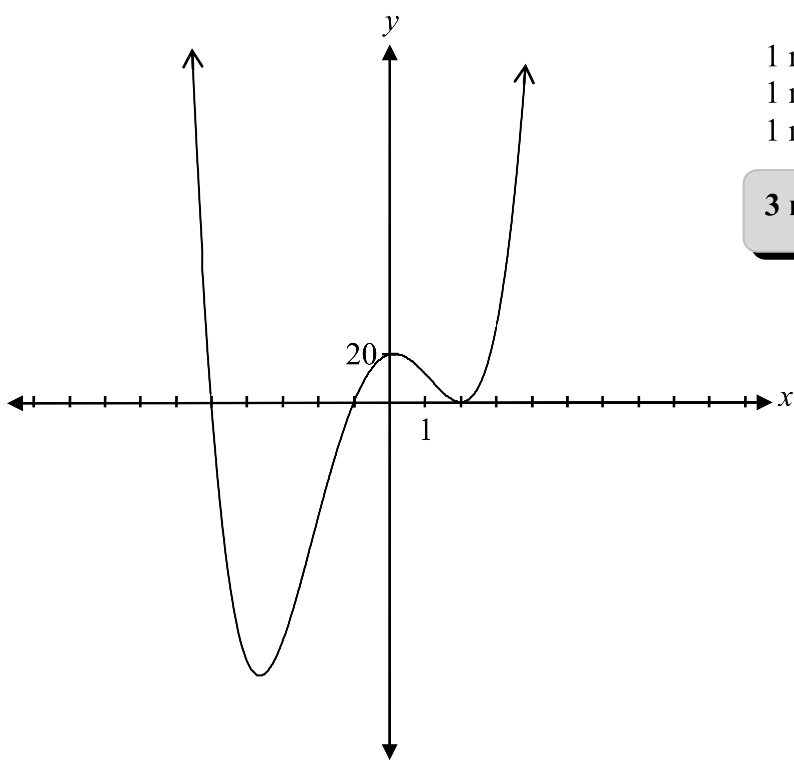
Sketch the graph of $y = (x+1)(x-2)^2(x+5)$.

Identify the x-intercepts and y-intercept.

Solution

x-intercepts: -5, -1, and 2

y-intercept: 20



1 mark for x-intercepts
1 mark for y-intercept
1 mark for multiplicity of a zero at $x = 2$

3 marks

Note(s):

- if no graph is shown, give ½ mark for x-intercepts $(-5, -1, \text{and } 2)$ and ½ mark for $y-$intercept (20)
- relative maximum and minimum are not required

Question 28 T4

The graph of the function $y = \sin x$ has been transformed to create a new graph.

The range of this new graph is $[-4, 4]$ and the zeros are $x = k\frac{\pi}{2}$, where k is an integer.

Write the equation that corresponds to this new graph.

Solution

Amplitude $= 4$	1 mark for correct amplitude
Period $= \pi$	½ mark for correct period
$\therefore b = \frac{2\pi}{\pi} = 2$	½ mark for consistent value of b

2 marks

$y = 4\sin(2x)$

or

$y = -4\sin(2x)$

Note(s):

- deduct ½ mark for a translation that results in an incorrect range and/or zeros

Question 29 R1

Given the functions $f(x) = x^2 - 1$ and $g(x) = x + 1$, state the domain of $\frac{g(x)}{f(x)}$.

Solution

Domain: $x \in \mathbb{R}$ where $x \neq 1$ and $x \neq -1$ — 1 mark (½ mark for $x \neq 1$, ½ mark for $x \neq -1$)

1 mark

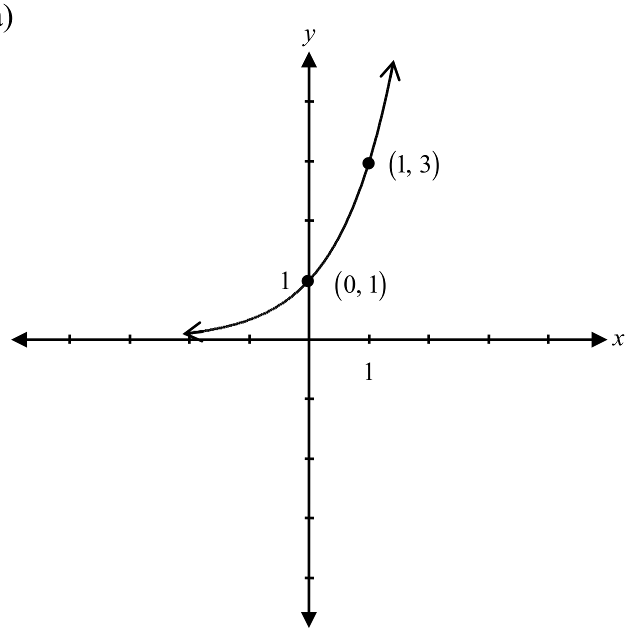
Question 30 R9

a) Sketch the graph of $y = 3^x$.

b) Explain how the graph of $y = 3^x$ can be used to sketch the graph of $y = \log_3 x$.

Solutions

a)



½ mark for increasing exponential function
½ mark for y-intercept at $(0, 1)$
½ mark for consistent point on exponential function
½ mark for asymptotic behaviour

2 marks

b)

To graph $y = \log_3 x$, you can reflect the graph of $y = 3^x$ over the line $y = x$.

or

You can switch the x and y coordinates of $y = 3^x$ to get the graph of $y = \log_3 x$.

1 mark for explanation

1 mark

Note(s):

- in (b), give ½ mark for having only stated they are inverse functions of each other

A box in the shape of a rectangular prism has side lengths x, $x+2$, and $x+10$.

Write a function, $V(x)$, to express the volume of the box in terms of x.

Find all possible values of x, given that the volume of the box is 96 cm^3.

State the dimensions of the box.

Solution

$V(x) = (x)(x+2)(x+10)$ — ½ mark for expressing volume in terms of x

$$96 = (x)(x+2)(x+10)$$
$$96 = x^3 + 12x^2 + 20x$$
$$0 = x^3 + 12x^2 + 20x - 96$$

½ mark for equating to zero

when $x = 2$:

$$0 = (2)^3 + 12(2)^2 + 20(2) - 96$$
$$0 = 8 + 48 + 40 - 96$$
$$0 = 0$$

$\therefore x = 2$ is a possible value. — 1 mark for identifying one possible value of x

2	1	12	20	−96
		2	28	96
	1	14	48	0

1 mark for division

$$(x-2)\left(x^2 + 14x + 48\right) = 0$$
$$(x-2)(x+8)(x+6) = 0$$

$x = -8, -6, 2$ — 1 mark for identifying all possible values of x

$x \neq -8$ and $x \neq -6$ because dimensions cannot be negative values.

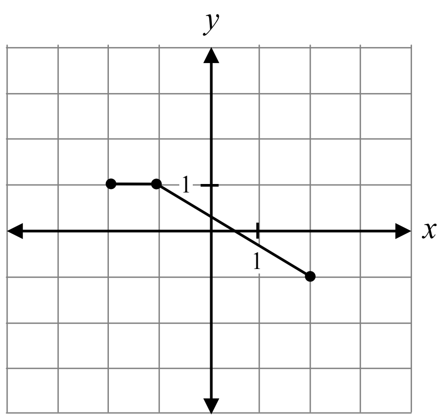
$\therefore x = 2$ is the only solution. — ½ mark for rejecting extraneous roots

The dimensions of the box are 2 cm × 4 cm × 12 cm. — ½ mark for stating the dimensions of the box

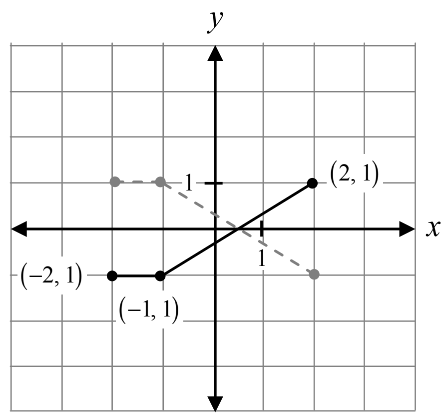
5 marks

Question 32 R5

Given the graph of $f(x)$ below, sketch the graph of $y = -f(x)$.



Solution



1 mark

Question 33 T2

Determine the coordinates of a point $(x,\ y)$ on the unit circle if you are given $\theta = 30^\circ$ where θ is in standard position.

Solution

$\mathrm{P}(\theta) = (\cos\theta,\ \sin\theta)$

If $\theta = 30^\circ$, then the coordinates of $\mathrm{P}(\theta)$ would be $\left(\cos 30^\circ,\ \sin 30^\circ\right)$.

1 mark for answer stated as coordinate point

1 mark

or

$$\mathrm{P}\left(30^\circ\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

Question 34 T4

Given the following sinusoidal equation:

$$P(t) = 3000\sin\left[\frac{\pi}{10}(t-2010)\right] + 10\ 000$$

Determine the maximum value of $P(t)$ and a value of t at which this maximum occurs.

Solution

Method 1

Maximum value $= 10\ 000 + 3000$	
$= 13\ 000$	1 mark for maximum value
$13\ 000 = 3000\sin\left[\frac{\pi}{10}(t-2010)\right] + 10\ 000$	
$3000 = 3000\sin\left[\frac{\pi}{10}(t-2010)\right]$	
$1 = \sin\left[\frac{\pi}{10}(t-2010)\right]$	½ mark for simplifying
$\frac{\pi}{2} = \frac{\pi}{10}(t-2010)$	1 mark for exact value
$5 = t - 2010$	
$t = 2015$	½ mark for solving for t

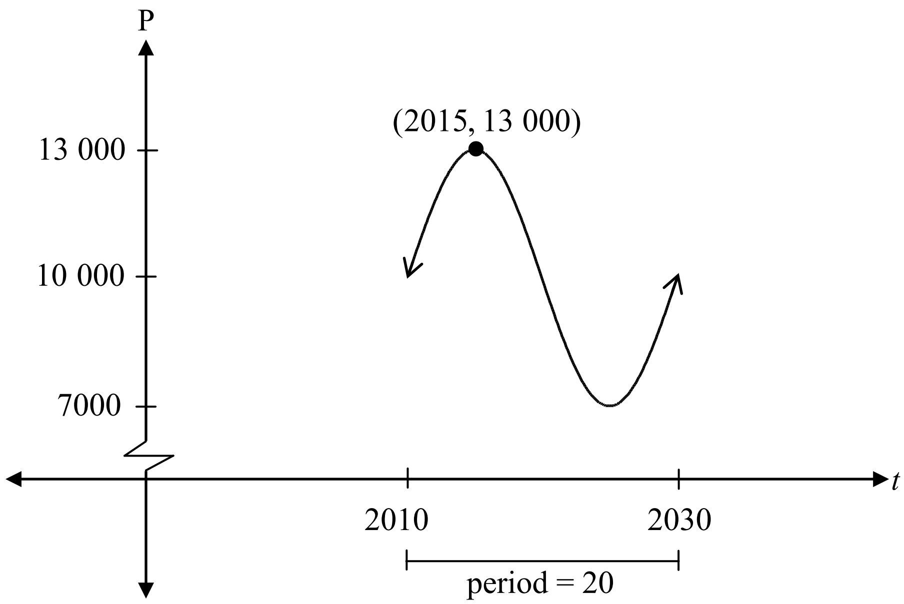
3 marks

Note(s):

- the period of the function is 20 $\therefore$ other acceptable answers are: $t = 2015 \pm 20$

Solution

Method 2



1 mark for period

$P(t) = 13\ 000$

$t = 2015$

$\therefore$ the maximum value is 13 000 when $t = 2015$.

1 mark for maximum value

1 mark for solving for t

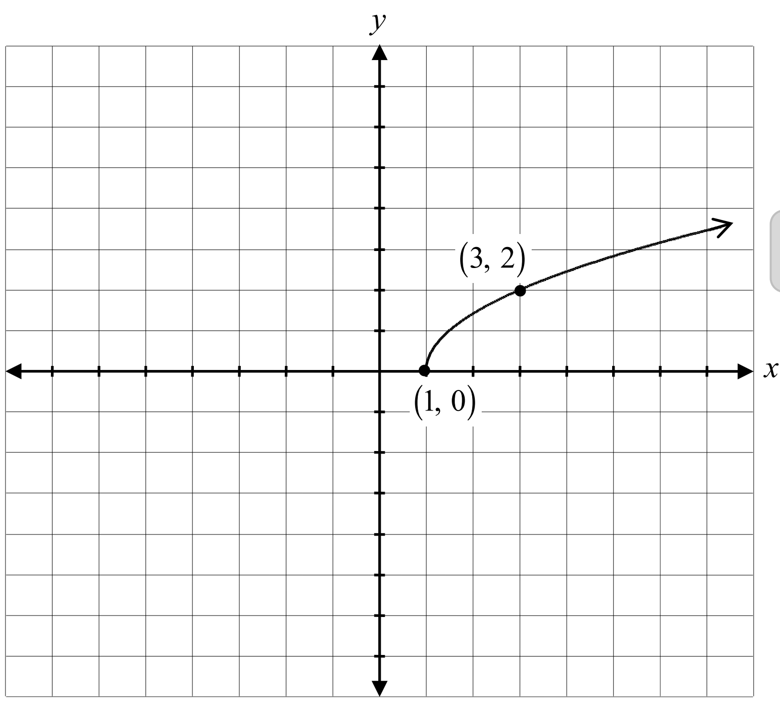
3 marks

Sketch the graph of $y = \sqrt{2x - 2}$.

Solution

Method 1

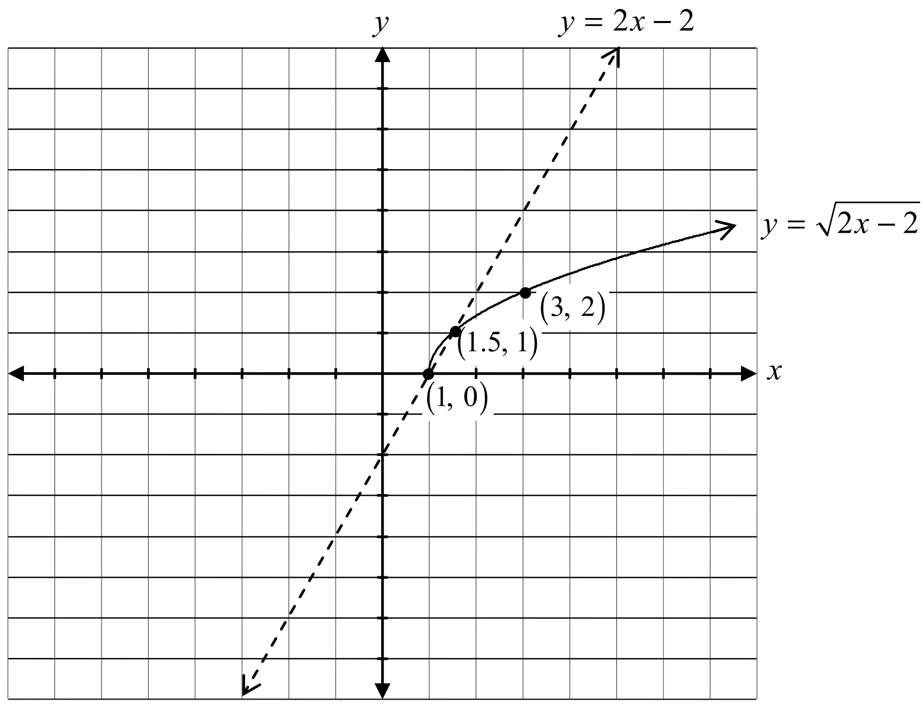
$$y = \sqrt{2x-2}$$
$$= \sqrt{2(x-1)}$$



1 mark for domain: $[1, \infty)$
1 mark for shape (graph of a radical function)
1 mark for horizontal compression

3 marks

Method 2



1 mark for domain of $y = \sqrt{2x-2}$: $[1, \infty)$

1 mark for invariant points where $y = 0$ and $y = 1$ (½ mark for each point)

½ mark for graph of $y = \sqrt{2x-2}$ drawn above the graph of $y = 2x - 2$ between the invariant points

½ mark for graph of $y = \sqrt{2x-2}$ drawn below the graph of $y = 2x - 2$ after the invariant point where $y = 1$

3 marks

Question 36 R6

Given $f(x) = 2x - 6$, write the equation of $f^{-1}(x)$.

Solution

Switch the x and y values.

$x = 2y - 6$ 1 mark for switching x and y values

$x + 6 = 2y$

$\frac{x+6}{2} = y$ ½ mark for solving for y

$\therefore f^{-1}(x) = \frac{x+6}{2}$ ½ mark for writing equation of $f^{-1}(x)$

2 marks

Question 37 R8

Frank tried to expand a logarithmic expression using the laws of logarithms. He made one error.

Frank's solution: $\log_a \frac{(x+2)}{zw} = \log_a x + \log_a 2 - \log_a z - \log_a w$

Write the correct solution.

Solution

Correct solution: $\log_a \frac{(x+2)}{zw} = \log_a (x+2) - \log_a z - \log_a w$ 1 mark for correct solution

1 mark

Determine all non-permissible values of θ over the interval $[0, 2\pi]$.

$$\frac{\sin\theta}{1+\cos\theta}+\csc\theta+\cot\theta$$

Explain your reasoning.

Solution

To determine non-permissible values, the denominator needs to equal zero.

1 mark for explanation

The denominators of this expression are "$1+\cos\theta$" and "$\sin\theta$" $\left(\text{since } \csc\theta=\frac{1}{\sin\theta} \text{ and } \cot\theta=\frac{\cos\theta}{\sin\theta}\right)$.

1 mark (½ mark for identifying each restriction)

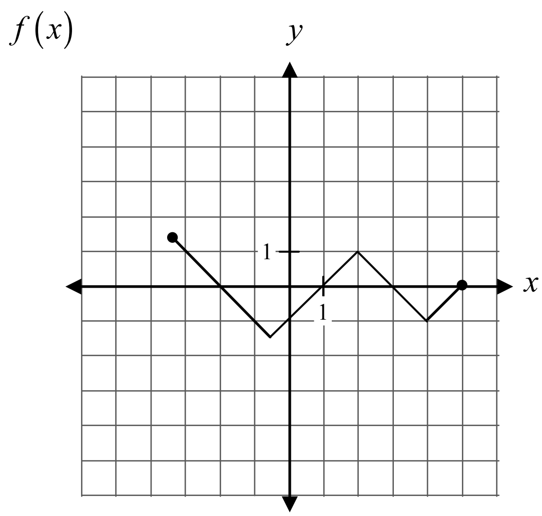
$$\therefore 1+\cos\theta=0 \qquad \sin\theta=0$$
$$\cos\theta=-1 \qquad \theta=0,\ \pi,\ 2\pi$$
$$\theta=\pi$$

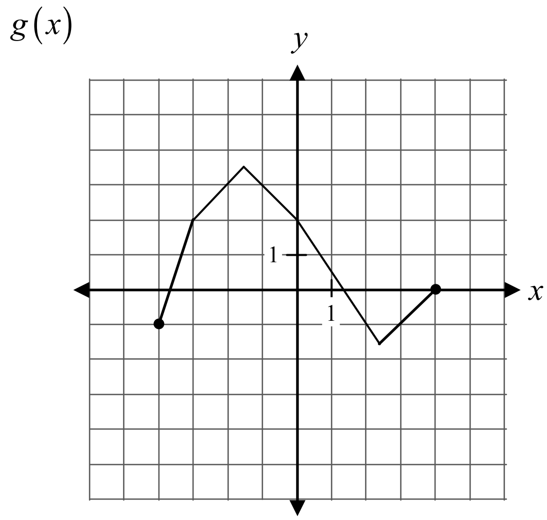
1 mark for all non-permissible values of θ (½ mark for each equation)

$\therefore$ the non-permissible values of θ over $[0, 2\pi]$ are $0,\ \pi,$ and 2π.

3 marks

Given the following graphs:





a) Determine the value of $[f \bullet g](0)$.

b) Determine the value of $g(f(4))$.

c) Determine a value for k where $f(k)=1$.

Solutions

a) $f(0) = -1$

$g(0) = 2$

$[f \bullet g](0) = (-1)(2)$
$= -2$

1 mark for the value of $[f \bullet g](0)$

1 mark

b) $f(4) = -1$

$g(-1) = 3$

½ mark for $f(4)$
½ mark for $g(f(4))$ consistent with $f(4)$ value

1 mark

c) $k = 2$ **or** $k = -3$

1 mark for a value for k

1 mark

Question 40 R1

Given that $h(x) = 2x^2 + 5x - 3$ and that $h(x) = f(x) \bullet g(x)$, determine $f(x)$ and $g(x)$.

Solution

$f(x) = 2x - 1$
$g(x) = x + 3$

1 mark for two correct factors of $h(x)$

1 mark

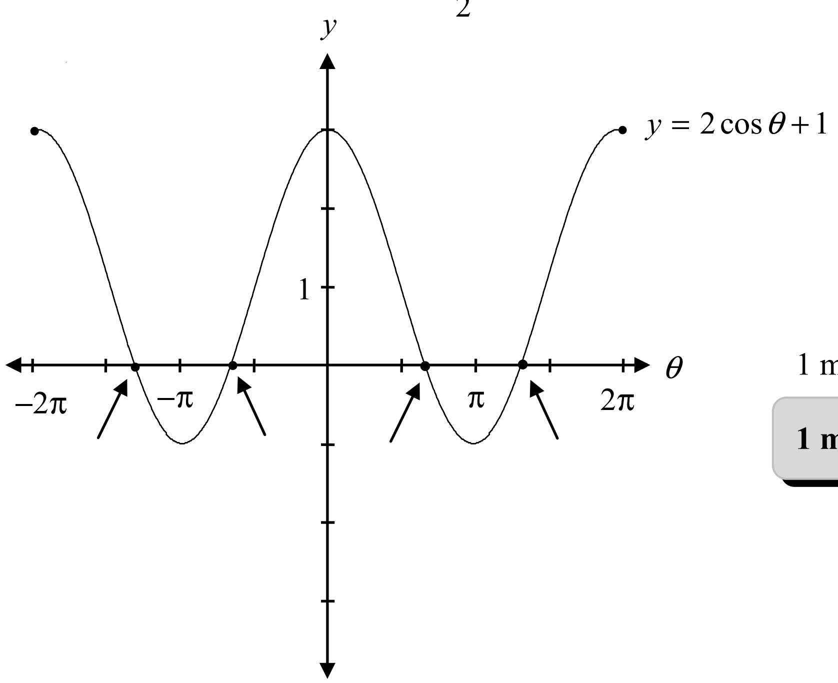
Other answers are possible.

Question 41 T5

The graph of $y = 2\cos\theta + 1$ below can be used to solve the equation $\cos\theta = -\frac{1}{2}$ over the interval $[-2\pi, 2\pi]$. Indicate on the graph where to find the solutions to the equation $\cos\theta = -\frac{1}{2}$.

Solution

The solution to the equation $\cos\theta = -\frac{1}{2}$ is found where the graph of $y = 2\cos\theta + 1$ crosses the *x*-axis.



1 mark for indicating solutions on graph

1 mark

Note(s):

- give ½ mark for indicating 1, 2, or 3 of the solutions

Question 42 R1

The function $f(x)$ is transformed.

A new function, $y = \frac{1}{f(x)}$, is created that does not have any vertical asymptotes.

What can you conclude about the original function $f(x)$?

Solution

If $f(x)$ does not have any x-intercepts, then the transformation would not have any vertical asymptotes.

or

$f(x)$ cannot equal zero.

or

$f \neq 0$

1 mark for conclusion

1 mark

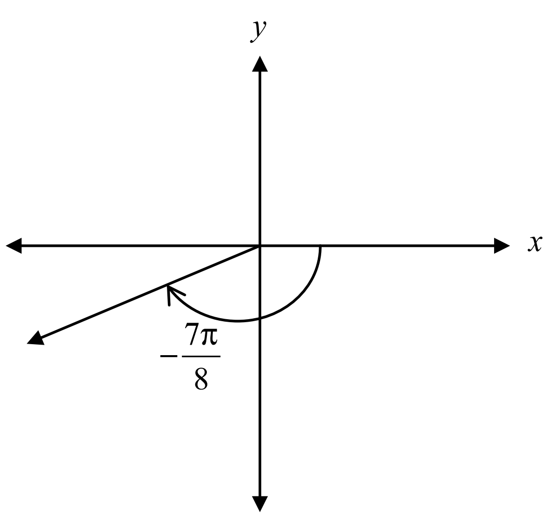
Note(s):

- award ½ mark for a correct example without a given conclusion

Question 43 T1

Draw the angle $-\frac{7\pi}{8}$ in standard position.

Solution



1 mark for angle drawn in Quadrant III

1 mark

Question 44 T3, T6

Determine the exact value of:

$$4\cos\left(\frac{11\pi}{12}\right)$$

Solution

Method 1

$$\begin{aligned} 4\cos\left(\frac{11\pi}{12}\right) &= 4\cos\left(\frac{\pi}{4}+\frac{2\pi}{3}\right) \\ &= 4\left[\cos\frac{\pi}{4}\cos\frac{2\pi}{3}-\sin\frac{\pi}{4}\sin\frac{2\pi}{3}\right] \\ &= 4\left[\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right)-\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\right] \\ &= 4\left[\frac{-\sqrt{2}-\sqrt{6}}{4}\right] \\ &= -\sqrt{2}-\sqrt{6} \end{aligned}$$

1 mark for combination

2 marks (½ mark for each exact value)

3 marks

Solution

Method 2

$\frac{11\pi}{12}$ has a reference angle of $\frac{\pi}{12}$. ½ mark for correct reference angle

$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$$

$$= \cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}$$

½ mark for substitution into correct identity

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

1 mark for exact values

$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

Since $\frac{11\pi}{12}$ is in Quadrant II, cosine is negative.

$$\therefore 4\cos\left(\frac{11\pi}{12}\right) = 4\left(\frac{-\sqrt{6} - \sqrt{2}}{4}\right)$$

$$= -\sqrt{6} - \sqrt{2}$$

1 mark for the concept that $\cos\left(\frac{11\pi}{12}\right)$ is < 0

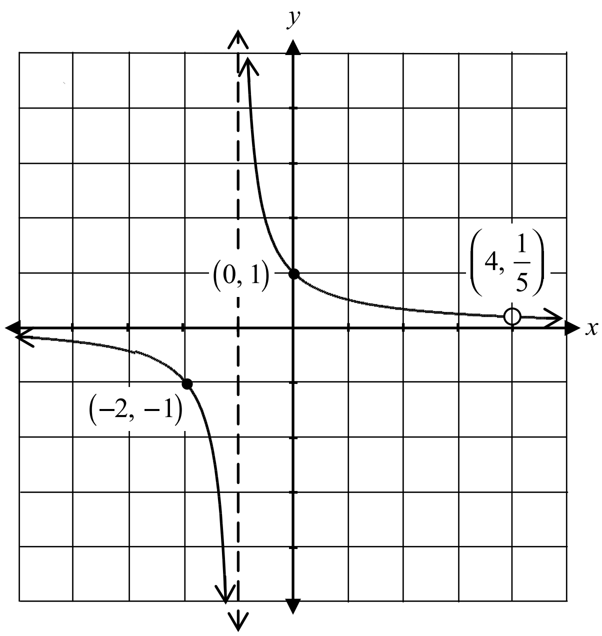
3 marks

Note(s):

- $\frac{-2 - 2\sqrt{3}}{\sqrt{2}}$ is an acceptable way to write the solution
- in Method 1, another possible combination is $\left(\frac{\pi}{6} + \frac{3\pi}{4}\right)$
- deduct ½ mark if answer is not expressed as a single fraction
- in Method 2, the reference angle need not be explicitly stated in order to obtain full marks

Sketch the graph of $f(x) = \dfrac{x-4}{x^2-3x-4}$.

Solution



$$f(x) = \frac{x-4}{x^2-3x-4}$$
$$= \frac{x-4}{(x-4)(x+1)}$$
$$= \frac{1}{x+1} \text{ with a point of discontinuity at } x = 4$$

point of discontinuity: $f(4) = \dfrac{1}{5}$

$\therefore$ there is a point of discontinuity at $\left(4, \dfrac{1}{5}\right)$

y-intercept: $f(0) = \dfrac{0-4}{(0)^2 - 3(0) - 4}$

$$= \frac{-4}{-4}$$
$$= 1$$

1 mark for vertical asymptote at $x = -1$
½ mark for graph left of vertical asymptote
½ mark for graph right of vertical asymptote
1 mark for point of discontinuity at $x = 4$

3 marks

Question 46 R7

Estimate the value of $\log_5 35$.

Justify your answer.

Solution

$5^2 = 25$ $5^3 = 125$

½ mark for justification

The value of $\log_5 35$ is more than 2 but less than 2.5.

½ mark for estimated answer

1 mark

Question 47 R11

If $p(x) = x^5 - 12x + 1$, determine the remainder when $p(x)$ is divided by $(x + 2)$.

Solution

Method 1

$p(x) = x^5 - 12x + 1$

$p(-2) = (-2)^5 - 12(-2) + 1$

$= -32 + 24 + 1$

$= -7$

1 mark for substitution

1 mark

Method 2

$$-2 \,|\, 1 \quad 0 \quad 0 \quad 0 \quad -12 \quad 1$$

1 mark for correct set-up of synthetic division

$$\begin{array}{r|rrrrrr} -2 & 1 & 0 & 0 & 0 & -12 & 1 \\ & \downarrow & -2 & 4 & -8 & 16 & -8 \\ \hline & 1 & -2 & 4 & -8 & 4 & -7 \end{array}$$

1 mark

The remainder is -7.

Describe the effects on the graph of $y = f(x)$ when asked for the graph of $y = f(x-3)+5$.

Solution

Shift right 3 and up 5

½ mark for horizontal translation
½ mark for vertical translation

or

$(x+3,\ y+5)$

1 mark

Question 49 T3

Find the exact value of the following expression:

$$\sin\left(\frac{11\pi}{3}\right) \bullet \sec\left(\frac{4\pi}{3}\right) \bullet \tan\left(-\frac{5\pi}{6}\right)$$

Solution

$$= \left(-\frac{\sqrt{3}}{2}\right)(-2)\left(\frac{1}{\sqrt{3}}\right)$$

$$= 1$$

1 mark for $\sin\left(\frac{11\pi}{3}\right)$ (½ mark for quadrant, ½ mark for value)

1 mark for $\sec\left(\frac{4\pi}{3}\right)$ (½ mark for quadrant, ½ mark for value)

1 mark for $\tan\left(-\frac{5\pi}{6}\right)$ (½ mark for quadrant, ½ mark for value)

3 marks

Appendices

Appendix A

MARKING GUIDELINES

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply.

- arithmetic error
- procedural error
- terminology error
- lack of clarity in explanation
- incorrect shape of graph (only when marks are not allocated for shape)

Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the *Answer/Scoring Sheet*.

E1	▪ answer given as a complex fraction ▪ final answer not stated ▪ answer stated in degrees instead of radians or vice versa
E2	▪ changing an equation to an expression ▪ equating the two sides when proving an identity
E3	▪ variable omitted in an equation or identity ▪ variables introduced without being defined
E4	▪ "$\sin x^2$" written instead of "$\sin^2 x$" ▪ missing brackets but still implied
E5	▪ missing units of measure ▪ incorrect units of measure
E6	▪ rounding error ▪ rounding too early
E7	▪ transcription error ▪ notation error
E8	▪ answer included outside the given domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order
E9	▪ incorrect or missing endpoints or arrowheads ▪ scale values on axes not indicated ▪ coordinate points labelled incorrectly
E10	▪ asymptotes drawn as solid lines ▪ graph crosses or curls away from asymptotes

Appendix B

IRREGULARITIES IN PROVINCIAL TESTS

A GUIDE FOR LOCAL MARKING

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the Department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all “NR”) or only incorrect responses (“0”)

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the Department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report*.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Booklet Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the Department along with the test materials.

Irregular Test Booklet Report

Test: ______________________________

Date marked: ______________________________

Booklet No.: ______________________________

Problem(s) noted: ______________________________

Question(s) affected: ______________________________

Action taken or rationale for assigning marks: ______________________________

Follow-up: ______________________________

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Decision: ______________________________

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Marker's Signature: ______________________________

Principal's Signature: ______________________________

For Department Use Only—After Marking Complete

Consultant: ______________________________

Date: ______________________________

Appendix C

Table of Questions by Unit and Learning Outcome

Unit A: Transformations of Functions		
Question	**Learning Outcome**	**Mark**
8	R1	1
15	R1	2
16	R3	1
22	R6	1
26	R5	1
29	R1	1
32	R5	1
36	R6	2
39	R1	3
40	R1	1
42	R1	1
48	R2	1
Unit B: Trigonometric Functions		
Question	**Learning Outcome**	**Mark**
7	T1	2
14	T3	2
17	T1	1
28	T4	2
33	T2	1
34	T4	3
43	T1	1
44	T3, T6	2
49	T3	3
Unit C: Binomial Theorem		
Question	**Learning Outcome**	**Mark**
2	P4	3
5	P2	2
6	P3	3
11	P1	1
12	P4	1
21	P4	1
23	P2	1
Unit D: Polynomial Functions		
Question	**Learning Outcome**	**Mark**
27	R12	3
31	R11, R12	5
47	R11	1

Unit E: Trigonometric Equations and Identities		
Question	**Learning Outcome**	**Mark**
1	T5	3
13	T6	3
19	T6	1
38	T6	3
41	T5	1
44	T3, T6	1
Unit F: Exponents and Logarithms		
Question	**Learning Outcome**	**Mark**
3	R10	3
4	R10	3
9	R10	1
18	R7	1
24	R9	1
25	R7	1
30	R9	3
37	R8	1
46	R7	1
Unit G: Radicals and Rationals		
Question	**Learning Outcome**	**Mark**
10	R13	1
20	R14	1
35	R13	3
45	R14	3