Grade 12
Pre-Calculus Mathematics
Achievement Test

Marking Guide

January 2014

Manitoba

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Available in alternate formats upon request.

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General Marking Instructions

Please make no marks in the student test booklets. If the booklets have marks in them, the marks need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the *Answer/Scoring Sheet* are identical
- **students and markers use only a pencil to complete the *Answer/Scoring Sheets***
- the totals of each of the four parts are written at the bottom
- each student's final result is recorded, by booklet number, on the corresponding *Answer/Scoring Sheet*
- the *Answer/Scoring Sheet* is complete
- a photocopy has been made for school records

Once marking is completed, please forward the *Answer/Scoring Sheets* to Manitoba Education in the envelope provided (for more information see the administration manual).

Marking the Test Questions

The test is composed of short-answer questions, long-answer questions, and multiple-choice questions. Short-answer questions are worth 1 or 2 marks each, long-answer questions are worth 3 to 5 marks each, and multiple-choice questions are worth 1 mark each. An answer key for the multiple-choice questions can be found at the beginning of the section "Booklet 2 Questions."

To receive full marks, a student's response must be complete and correct. Where alternative answering methods are possible, the *Marking Guide* attempts to address the most common solutions. For general guidelines regarding the scoring of students' responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an *Answer/Scoring Sheet* is marked with "0" and/or "NR" only (e.g., student was present but did not attempt any questions) please document this on the *Irregular Test Booklet Report*.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

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Communication Errors

The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the *Answer/Scoring Sheet* that represents the marks given based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called "Communication Errors" (see Appendix A) and will be tracked on the *Answer/Scoring Sheet* in a separate section. There is a ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type will not further affect a student's mark), with a maximum deduction of 5 marks from the total test mark.

The total mark deduction for communication errors for any student response is not to exceed the marks given for that response. When multiple communication errors are made in a given response, any deductions are to be indicated in the order in which the errors occur in the response, without exceeding the given marks.

The student's final mark is determined by subtracting the communication errors from the preliminary mark.

Example: A student has a preliminary mark of 72. The student committed two E1 errors (½ mark deduction), four E7 errors (½ mark deduction), and one E8 error (½ mark deduction). Although seven communication errors were committed in total, there is a deduction of only 1½ marks.

COMMUNICATION ERRORS / ERREURS DE COMMUNICATION

Shade in the circles below for a maximum total deduction of 5 marks (0.5 mark deduction per error).
Noircir les cercles ci-dessous pour une déduction maximale totale de 5 points (déduction de 0,5 point par erreur).

E1 ●	E2 ○	E3 ○	E4 ○	E5 ○
E6 ○	E7 ●	E8 ●	E9 ○	E10 ○

Mark assigned to the student / Note accordée à l'élève

Booklet 1 / Cahier 1	+	Multiple Choice / Choix multiple	+	Booklet 2 / Cahier 2	-	Communication Errors / Erreurs de communication	=	Total
25	+	7	+	40	-	1½	=	70½
36		9		45		maximum deduction of 5 marks / déduction maximale de 5 points		**90**

Scoring Guidelines

Booklet 1 Questions

Find the coterminal angle to $\frac{27\pi}{5}$ over the interval $\left[-360^\circ, 0^\circ\right)$.

Solution

Method 1

$$\frac{27\pi}{5}\left(\frac{180^\circ}{\pi}\right) = 27\left(36^\circ\right)$$

$$= 972^\circ$$

1 mark for conversion to degrees

$$972^\circ - \left(360^\circ\right)(3) = -108^\circ$$

½ mark for coterminal angle
½ mark for correct domain

2 marks

Method 2

$-\frac{3\pi}{5}$ is a coterminal angle to $\frac{27\pi}{5}$.

½ mark for coterminal angle
½ mark for correct domain

$$-\frac{3\pi}{5} \bullet \frac{180^\circ}{\pi} = -108^\circ$$

1 mark for conversion to degrees

2 marks

Exemplar 1

$\frac{10\pi}{5} = 2\pi$

$\frac{27\pi}{5} - \frac{10\pi}{5} = \frac{17\pi}{5} - \frac{10\pi}{5} = \frac{7\pi}{5} - \frac{10\pi}{5} = \frac{-3\pi}{5}$

1 out of 2

+ ½ mark for coterminal angle
+ ½ mark for correct domain
E7 (notation error in line 1)

Exemplar 2

$\frac{27\pi}{5\pi} = \frac{4860}{5} = 972 = 252^\circ$

$-(360)$

(360)

1½ out of 2

+ 1 mark for conversion to degrees
+ ½ mark for coterminal angle
E7 (notation errors in line 1: $\frac{27}{5} = \frac{4860}{5}$ and $972^\circ = 252^\circ$)

Question 2 T5

Solve the following equation over the interval $0 \le \theta < 2\pi$.

$$(\tan\theta - 3)(\tan\theta + 1) = 0$$

Solution

$$(\tan\theta - 3)(\tan\theta + 1) = 0$$

1 mark (½ mark for each branch)

$\tan\theta = 3$ $\quad\quad$ $\tan\theta = -1$

$\theta_r = 1.249\ 046$ $\quad\quad$ $\theta_r = \frac{\pi}{4}$

2 marks (½ mark for each value of θ)

3 marks

$\theta = 1.249\ 046$ $\quad\quad$ $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

$\theta = 4.390\ 639$

or

$\theta = 1.249,\ 4.391$ $\quad\quad$ $\theta = 2.356\ 194$

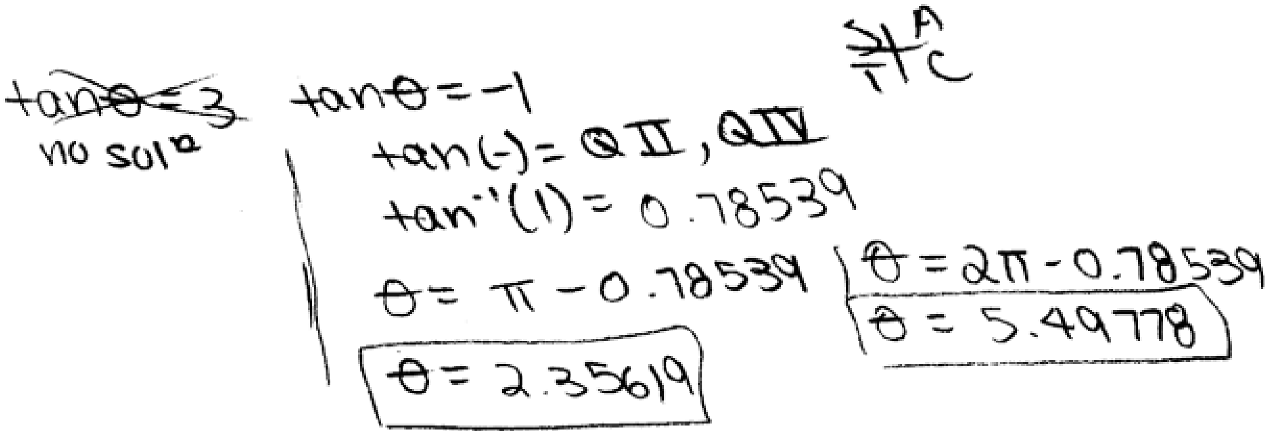
$\theta = 5.497\ 787$

$$\therefore \theta = 1.249, \frac{3\pi}{4}, 4.391, \frac{7\pi}{4}$$

or

$$\theta = 1.249,\ 2.356,\ 4.391,\ 5.498$$

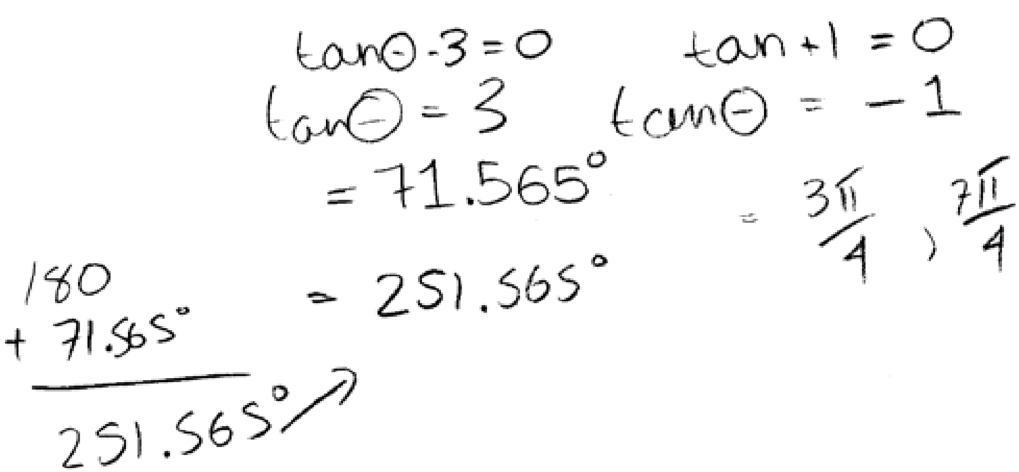
Exemplar 1



2 out of 3

+ 1 mark (½ mark for each branch)
+ 1 mark (½ mark for correct answer)
E7 (notation error in line 2)

Exemplar 2



3 out of 3

award full marks
E3 (variable omitted in line 1)
E5 (answer stated in degrees instead of radians in lines 3 and 4)
E7 (notation error in line 3)

An earthquake in Vancouver had a magnitude of 6.3 on the Richter scale. An earthquake in Japan had a magnitude of 8.9 on the Richter scale.

How many times more intense was the Japan earthquake than the Vancouver earthquake?

You may use the formula below:

$$M = \log\left(\frac{A}{A_0}\right)$$

where M is the magnitude of the earthquake on the Richter scale
A is the intensity of the earthquake
A_0 is the intensity of a standard earthquake

Express your answer as a whole number.

Solution

Method 1

Vancouver: substitute $M = 6.3$

$$6.3 = \log\left(\frac{A}{A_0}\right)$$

$$10^{6.3} = \frac{A}{A_0}$$ ½ mark for exponential form

$$A = 10^{6.3} A_0$$

Japan: substitute $M = 8.9$

$$8.9 = \log\left(\frac{A}{A_0}\right)$$

$$10^{8.9} = \frac{A}{A_0}$$ ½ mark for exponential form

$$A = 10^{8.9} A_0$$

To compare the two earthquakes divide their intensities.

$$\frac{\text{the intensity of Japan}}{\text{the intensity of Vancouver}} = \frac{10^{8.9} A_0}{10^{6.3} A_0}$$ 1 mark for comparison

$$= 398.107$$

$$= 398$$

2 marks

Method 2

$$\frac{I_J}{I_V} = \frac{10^{8.9}}{10^{6.3}}$$
$$= 398$$

1 mark for comparison
½ mark for exponential form
½ mark for exponential form

2 marks

Exemplar

$$8.9 - 6.3 = \log\left(\frac{A}{A_o}\right)$$

$$2.6 = \log\left(\frac{A}{A_o}\right)$$

$$\boxed{10^{2.6} = \frac{A}{A_o}}$$

$$398.1071706 = \frac{A}{A_o}$$

The earthquake in Japan was 398 times more intense than the earthquake in Vancouver.

1½ out of 2

award full marks
– ½ mark for procedural error

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Question 4 P4

Find and simplify the last term in the expansion of $(2y - 3x)^7$.

Solution

$t_8 = {}_7C_7(2y)^0(-3x)^7$	½ mark for ${}_7C_7$
$= -2187x^7$	½ mark for $(2y)^0$
	1 mark for $(-3x)^7$ which is equal to $-2187x^7$

2 marks

Note(s):

- no deduction if ${}_7C_7$ and $(2y)^0$ are not shown

Exemplar 1

$${}_7C_7(-3x)^0(2)^7$$
$$= \frac{7!}{7!}(1)(128)$$
$$= 128$$

½ out of 2

+ ½ mark for ${}_7C_7$

Exemplar 2

$$t_{k+1} = {}_nC_k\, a^{n-k} b^k$$
$$t_{6+1} = {}_7C_6\, (2y)^{7-6}(-3x)^6$$
$$7(2y)(+729x^6)$$
$$\boxed{10206x^6y}$$

1 out of 2

award full marks
– 1 mark for concept error (incorrect term)

Question 5 R8

Given $\log_a 9 = 1.129$ and $\log_a 4 = 0.712$, find the value of $\log_a 12$.

Solution

Method 1

$$\log_a 9 = 1.129$$
$$\log_a 3^2 = 1.129$$
$$2\log_a 3 = 1.129$$
$$\log_a 3 = 0.5645$$

1 mark for power rule

$$\log_a 12 = \log_a (4 \bullet 3)$$
$$= \log_a 4 + \log_a 3$$
$$= 0.712 + 0.5645$$
$$= 1.2765$$
$$= 1.277$$

1 mark for writing 12 as a product

1 mark for product rule

3 marks

Method 2

$$\log_a 12 = \log_a \left(\sqrt{9} \bullet 4\right)$$
$$= \frac{1}{2}\log_a 9 + \log_a 4$$
$$= \frac{1}{2}(1.129) + 0.712$$
$$= 1.2765$$
$$= 1.277$$

1 mark for writing 12 as a product

1 mark for power rule

1 mark for product rule

3 marks

Method 3

$$\log_a 9 = 1.129$$
$$a^{1.129} = 9$$
$$a = 9^{\frac{1}{1.129}}$$
$$a = 7$$

1 mark for exponential form

1 mark for solving for a

$$\log_7 12 = \frac{\log 12}{\log 7}$$
$$= 1.276\ 989$$
$$= 1.277$$

1 mark for value of $\log_a 12$

3 marks

Exemplar 1

$a^{1.129} = 9$

$a = 6.98976$

$a^{0.712} = 4$

$a = 7.0079$

$(\log_a 4)(\log_a 4)(\log_a 4) = \log_a 12$

$(0.712)(3) = 2.136$

$\boxed{\log_a 12 = 2.13}$

2 out of 3

Method 3
+ 1 mark for exponential form
+ 1 mark for solving for *a*

Exemplar 2

$\sqrt{9} \times 4 = 12$

$\sqrt{\log_a 9} + \log_a 4 = \log_a 12$

$\sqrt{1.129} + 0.712 = 1.7745$

2 out of 3

Method 2
+ 1 mark for writing 12 as a product
+ 1 mark for product rule

Question 6 P2

How many different ways can 4 girls and 4 boys be arranged in a row if the girls and the boys must alternate?

Solution

Method 1

$$\underline{8} \cdot \underline{4} \cdot \underbrace{\underline{3} \cdot \underline{3} \cdot \underline{2} \cdot \underline{2} \cdot \underline{1} \cdot \underline{1}}_{\text{alternate the rest}} = 1152 \text{ ways}$$

$\underline{8}$: choose one person; $\underline{4}$: choose the other gender

1 mark for beginning with either gender
1 mark for alternating genders

2 marks

Method 2

Case 1: $\underset{B}{\underline{4}} \cdot \underset{G}{\underline{4}} \cdot \underset{B}{\underline{3}} \cdot \underset{G}{\underline{3}} \cdot \underset{B}{\underline{2}} \cdot \underset{G}{\underline{2}} \cdot \underset{B}{\underline{1}} \cdot \underset{G}{\underline{1}} = 576$

1 mark for arrangement of alternating genders

Case 2: $\underset{G}{\underline{4}} \cdot \underset{B}{\underline{4}} \cdot \underset{G}{\underline{3}} \cdot \underset{B}{\underline{3}} \cdot \underset{G}{\underline{2}} \cdot \underset{B}{\underline{2}} \cdot \underset{G}{\underline{1}} \cdot \underset{B}{\underline{1}} = 576$

½ mark for two cases

Total number of ways: $576 + 576 = 1152$ ways

½ mark for addition of cases

2 marks

Exemplar

4 4 3 3 2 2 1 1 = 576

576 options!

1 out of 2

Method 2

+ 1 mark for arrangement of alternating genders

Question 7 T3, T5, T6

Solve the following equation over the interval $[0,\ 2\pi]$.

$$2\cos 2\theta - 1 = 0$$

Solution

Method 1

$$2\cos 2\theta - 1 = 0$$

$$2\left(2\cos^2\theta - 1\right) - 1 = 0$$ 1 mark for identity

$$4\cos^2\theta - 3 = 0$$

$$\cos^2\theta = \frac{3}{4}$$

$$\cos\theta = \pm\frac{\sqrt{3}}{2}$$ 1 mark for solving for $\cos\theta$

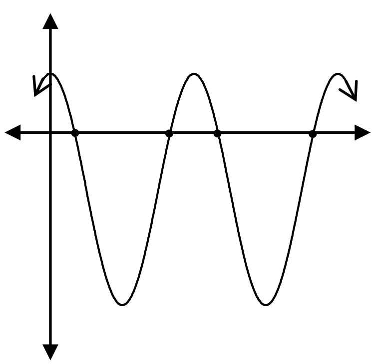
$$\theta_r = \frac{\pi}{6}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$ 2 marks for solutions (½ mark for each consistent solution)

4 marks

Method 2

$y = 2\cos 2\theta - 1$ 1 mark for equation



1 mark for the graph with zeroes indicated

Find all zeros over the interval $[0,\ 2\pi]$.

$\theta = 0.524,\ 2.618,\ 3.665,\ 5.760$ 2 marks for solutions

4 marks

Question 7 T3, T5, T6

Solution

Method 3

$$2\cos 2\theta - 1 = 0$$

$$\cos 2\theta = \frac{1}{2}$$

½ mark for solving for $\cos 2\theta$

$$\text{let } x = 2\theta$$

$$\therefore \cos x = \frac{1}{2}$$

$$x_r = \frac{\pi}{3}$$

½ mark for reference angle

$$x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$$

1 mark for correct angles

$$\therefore 2\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

1 mark for coterminal angles
1 mark for all solutions over the interval $[0, 2\pi]$.

4 marks

Note(s):

- in Method 1, deduct a maximum of 1 mark if missing $\cos\theta = -\frac{\sqrt{3}}{2}$

Exemplar 1

$$2(2\cos^2\theta - 1) - 1 = 0$$

$$4\cos^2\theta - 1 - 1 = 0$$

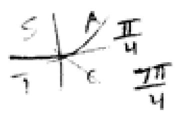
$$4\cos^2\theta - 2 = 0$$
$$+2 \quad +2$$

$$\frac{4\cos^2\theta}{4} = \frac{2}{4}$$

$$\sqrt{\cos^2\theta} = \sqrt{\frac{1}{2}}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\cos\theta = \frac{\sqrt{2}}{2}$$



$$\cos\theta = \frac{\pi}{4}, \frac{7\pi}{4}$$

2½ out of 4

award full marks

– 1 mark for missing $\cos\theta = -\frac{1}{\sqrt{2}}$

– ½ mark for arithmetic error in line 2

E7 (notation error in line 8)

Exemplar 2

$4\cos^2\theta - 2 - 1$

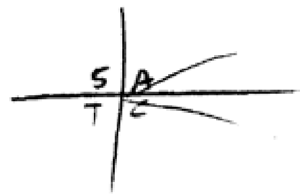
$4\cos^2\theta - 3$

$\sqrt{4\cos^2\theta} = \sqrt{3}$

$\frac{2\cos\theta}{2} = \frac{\sqrt{3}}{2}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\boxed{\theta = \frac{\pi}{6}, \frac{11\pi}{6}}$



3 out of 4

award full marks

− 1 mark for missing $\cos\theta = -\frac{\sqrt{3}}{2}$

E2 (changing an equation to an expression in lines 1 and 2)

Question 8 R4

Alex incorrectly explains to Rashid that the graph of $y = 2f(x) + 5$ means you first move the graph of $y = f(x)$ up 5 units and then multiply the y values by 2.

Explain to Rashid the correct way to transform the graph.

Solution

Alex explains the transformations correctly, but not in the correct order.
First multiply the y-values by 2, then move the graph up 5 units.

1 mark for explanation

1 mark

Exemplar 1

First you have to do the compression/stretches, wich would be multiply by 2.
Then you can do the translation of 5 units up.

1 out of 1

+ 1 mark for explanation

Exemplar 2

- You need to do stretches befur shifts

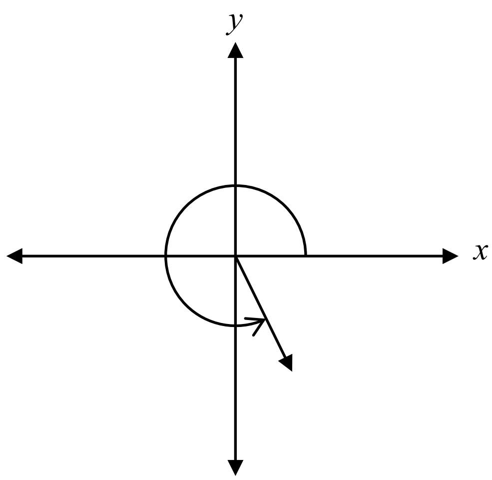
½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Question 9 T1

Sketch the angle of 5 radians in standard position.

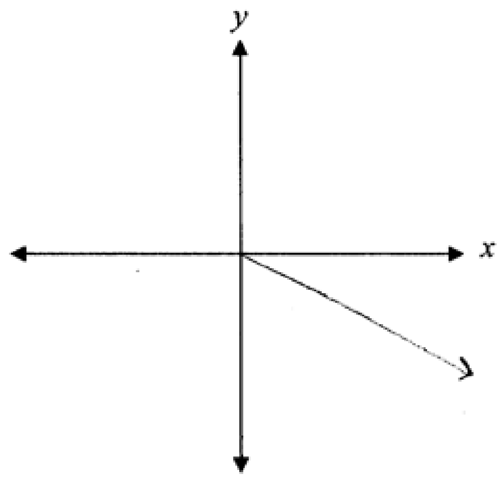
Solution



1 mark for angle drawn in Quadrant IV

1 mark

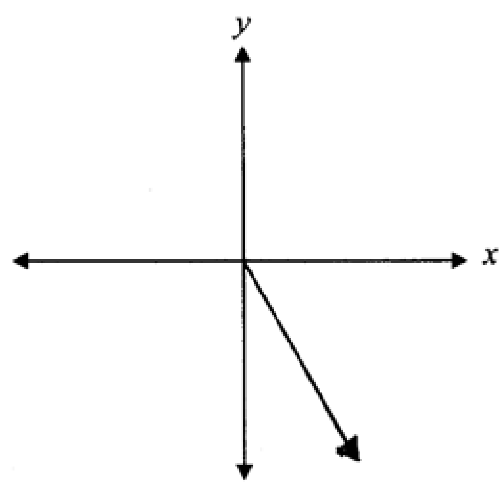
Exemplar 1



1 out of 1

award full marks
E1 (final answer not stated)

Exemplar 2



$$D = \frac{R \cdot 180^\circ}{\pi}$$

$$D = \frac{5 \cdot 180^\circ}{3,14}$$

$$D = \frac{900^\circ}{3,14}$$

$D \approx 300^\circ$ (approx.)

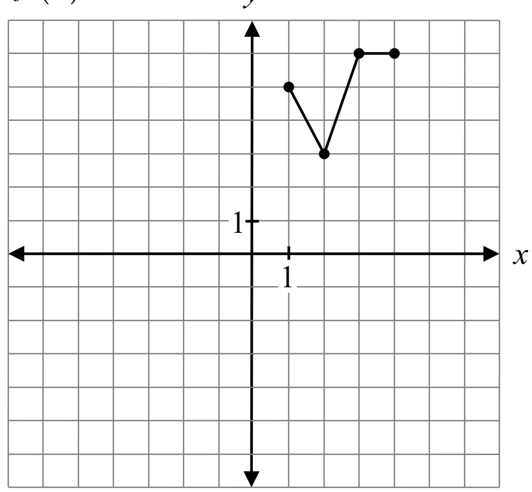
$$\begin{array}{r} \overset{4}{1}80 \\ \times \;\; 5 \\ \hline 900 \end{array}$$

1 out of 1

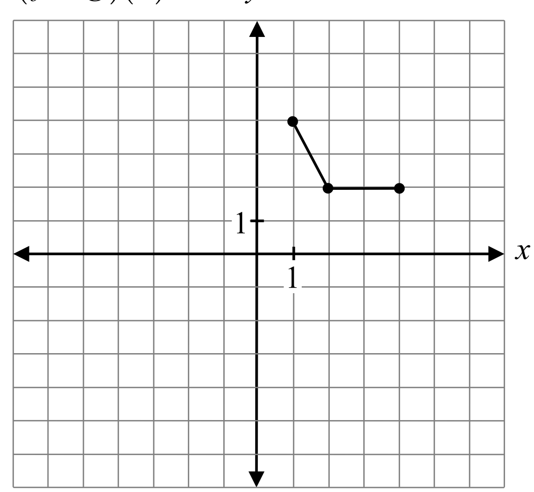
award full marks
E1 (final answer not stated)

Given the graphs of $f(x)$ and $(f-g)(x)$, sketch the graph of $g(x)$.

$f(x)$

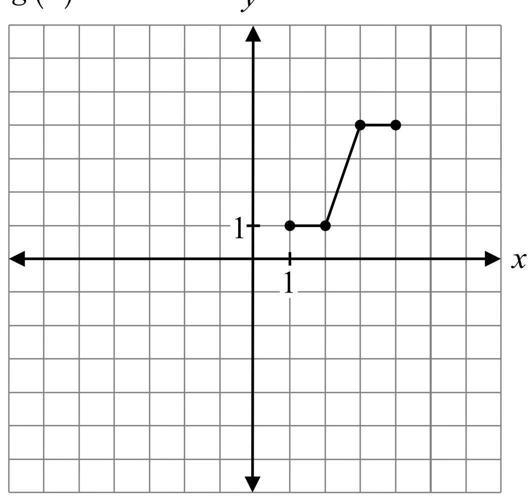


$(f-g)(x)$



Solution

$g(x)$

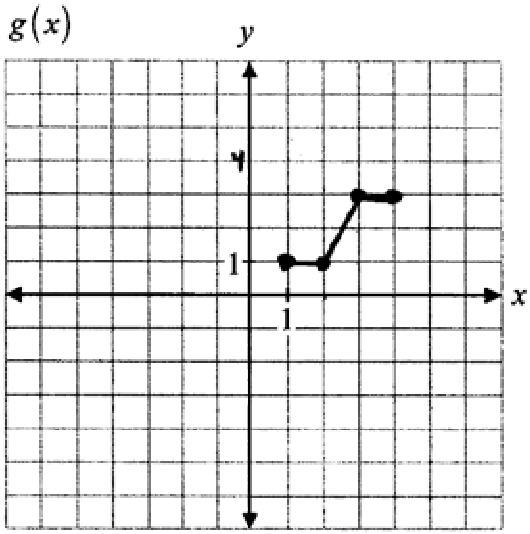


1 mark for subtraction of $f(x)-(f-g)(x)$

1 mark for shape representing the operation given

2 marks

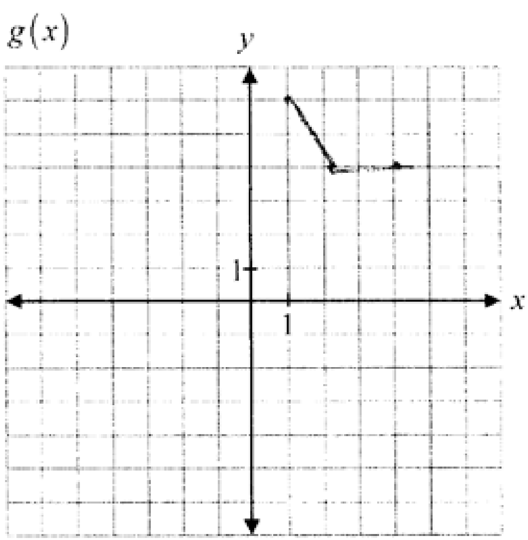
Exemplar 1



1½ out of 2

award full marks
– ½ mark for arithmetic error

Exemplar 2



0 out of 2

A particular math class has a large number of students. From this class, you are to create a committee of 4 students that has at least 1 girl.

Without actually solving the problem, explain the strategy you would use to find the total number of ways to select this committee.

Solution

Method 1

Find all ways to form the committee and subtract the number of ways with no girls on the committee.

1 mark for identifying cases
1 mark for operation on these cases

2 marks

Method 2

Find all scenarios with girls on the committee: 1 girl on the committee, 2 girls on the committee, 3 girls on the committee, and a committee with all girls. Then add all scenarios together to get the total number of ways to create a committee with at least 1 girl.

1 mark for identifying cases
1 mark for operation on these cases

2 marks

Note(s):

- in Method 2, deduct ½ mark if one of the cases is missing

Exemplar 1

There are 4 cases: 3 boys and 1 girl, 2 boys and 2 girls and 1 boy and 3 girls. I would find the number of options for each case and add them together.

1½ out of 2

+ 1½ marks (see note on previous page)

Exemplar 2

To know all the possibilities, first I would use 1 girl and 3 boys on the commitee.
The next time, I would put 2 girls and 2 boys on the commitee to find different ways to choose the commitee. Then, I would put 3 girls and 1 boy on the commitee, followed by 4 girls and 0 boys on the commitee.
After multiplying each option (ex. 2 girls, 2 boys) I would add them up to find the solution.

Diagram

G=girls
B=boys

$\frac{1}{G} \cdot \frac{3}{B} \cdot \frac{2}{B} \cdot \frac{1}{B} =$

$\frac{2}{G} \cdot \frac{1}{G} \cdot \frac{2}{B} \cdot \frac{1}{B} =$

$\frac{3}{G} \cdot \frac{2}{G} \cdot \frac{1}{G} \cdot \frac{1}{B} =$

$\frac{4}{G} \cdot \frac{3}{G} \cdot \frac{2}{G} \cdot \frac{1}{G} = +$ ______ total number of ways

1 out of 2

award full marks
– 1 mark for concept error (permutations/combinations)

Question 12 a) T6 b) T5, T6

a) Prove the identity below for all permissible values of θ.

$$\frac{1+2\cos^2\theta}{\cos^2\theta} = \tan^2\theta + 3$$

b) Determine all the non-permissible values for θ.

Solution

Method 1

a)

LHS	RHS
$\frac{1}{\cos^2\theta}+\frac{2\cos^2\theta}{\cos^2\theta}$	$\tan^2\theta+3$
$\sec^2\theta+2$	
$\tan^2\theta+1+2$	
$\tan^2\theta+3$	

$\therefore$ LHS = RHS

1 mark for appropriate algebraic strategy
1 mark for appropriate identity substitutions

2 marks

b) $\cos^2\theta = 0$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

$\therefore$ the non-permissible values of θ

are $\frac{\pi}{2}+k\pi,\ k \in \mathrm{I}$

or

$90^\circ + 180k,\ k \in \mathrm{I}$.

½ mark for $\cos^2\theta = 0$

½ mark for any non-permissible value of θ

1 mark for all non-permissible values of θ

2 marks

Solution

Method 2

a)

LHS	RHS
$\dfrac{1+2\cos^2\theta}{\cos^2\theta}$	$\tan^2\theta+3$
	$\sec^2\theta-1+3$
	$\sec^2\theta+2$
	$\dfrac{1}{\cos^2\theta}+2$
	$\dfrac{1}{\cos^2\theta}+\dfrac{2\cos^2\theta}{\cos^2\theta}$
	$\dfrac{1+2\cos^2\theta}{\cos^2\theta}$

$\therefore$ LHS = RHS

1 mark for appropriate identity substitutions
1 mark for appropriate algebraic strategy

2 marks

b) $\cos^2\theta=0$

$\cos\theta=0$

$\theta=\dfrac{\pi}{2},\ \dfrac{3\pi}{2},\ \ldots$

$\therefore$ the non-permissible values of θ

are $\dfrac{\pi}{2}+k\pi,\ k\in \mathrm{I}$

or

$90^\circ+180k,\ k\in \mathrm{I}.$

½ mark for $\cos^2\theta=0$

½ mark for any non-permissible value of θ

1 mark for all non-permissible values of θ

2 marks

Exemplar 1

a)

Left-Hand Side	Right-Hand Side
$\frac{1+2(1-\sin^2\theta)}{\cos^2\theta}$	$=\frac{\sin^2\theta}{\cos^2\theta}+3$
$\frac{1+2\sin^2\theta}{1-\sin^2\theta}$	$=\frac{\sin^2\theta}{1-\sin^2\theta}+3$
$\frac{3}{1}$	$=\frac{3}{1}$
$=3$	$=3$
$=RS$	$=LS$

1 out of 2

+ 1 mark for appropriate identity substitutions

b)

$\cos\theta \neq 0$

$\frac{\pi}{2} \neq 0$

$\frac{3\pi}{2} \neq 0$

1 out of 2

+ ½ mark for $\cos^2\theta = 0$
+ ½ mark for non-permissible values
E7 (notation error)

Exemplar 2

a)

Left-Hand Side	Right-Hand Side
$= \dfrac{1 + 2\cos^2\theta}{\cos^2\theta}$	$\dfrac{\sin^2\theta}{\cos^2\theta} + 3$
$= \dfrac{1 + 2(1 - \sin^2\theta)}{\cos^2\theta}$	
$= \dfrac{1 + 2 - 2\sin^2\theta}{\cos^2\theta}$	
$= \dfrac{3 - 2\sin^2\theta}{\cos^2\theta}$	

1 out of 2

+ 1 mark for appropriate identity substitutions

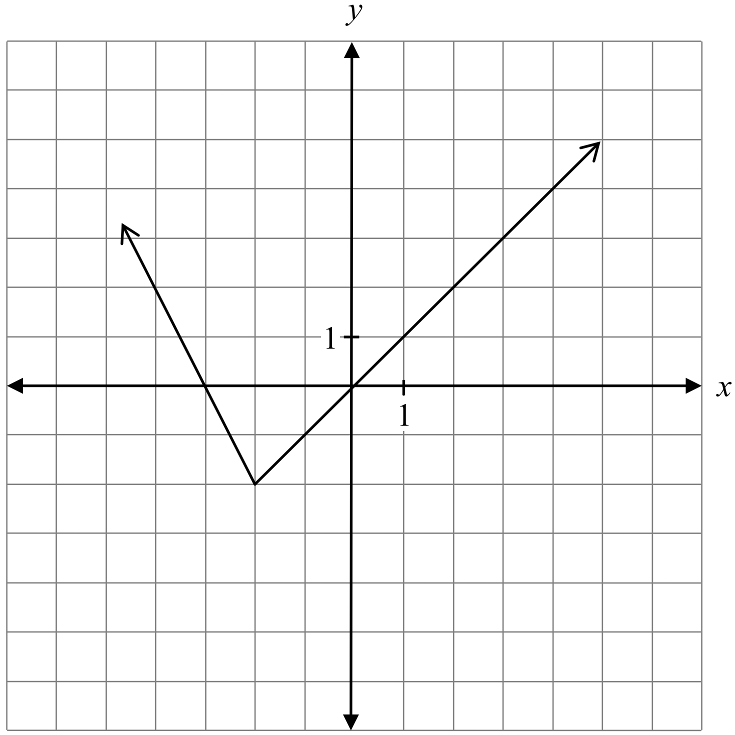
b)

$\theta \neq \frac{\pi}{2}, \frac{\pi}{2} + 2\pi k, k \in I$

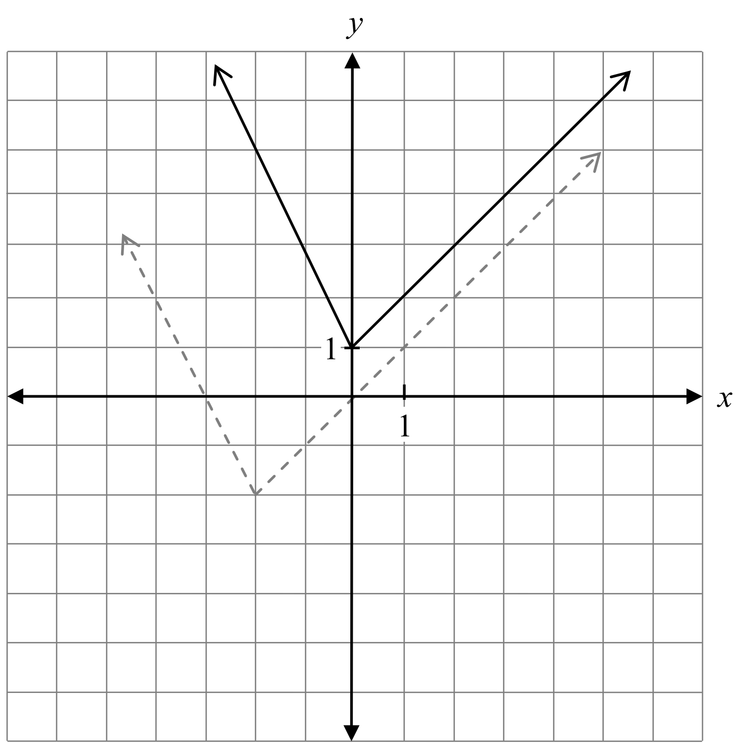
1 out of 2

+ ½ mark for $\cos^2\theta = 0$
+ ½ mark for non-permissible value

Given the graph of $f(x)$ below, sketch the graph of $g(x) = f(x-2)+3$.



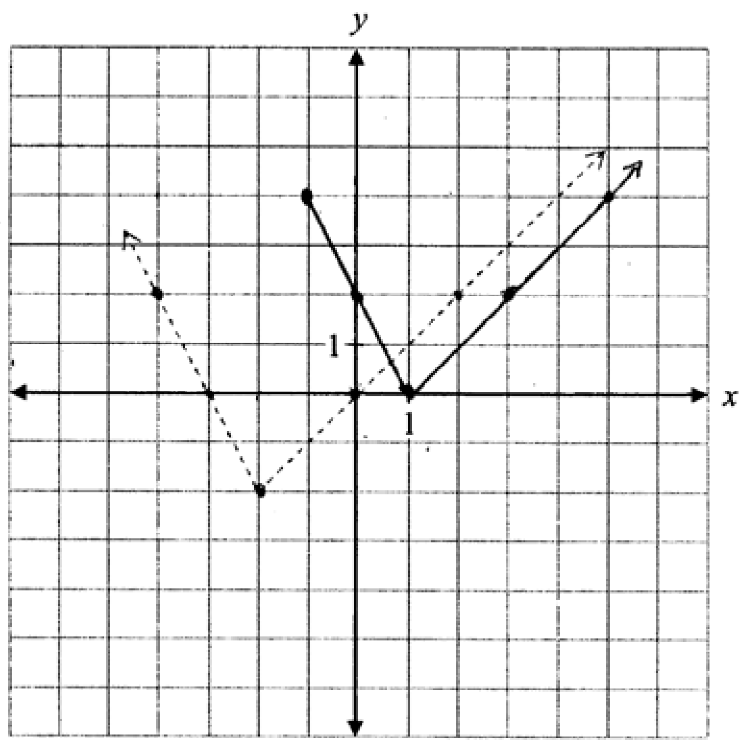
Solution



1 mark for horizontal shift
1 mark for vertical shift

2 marks

Exemplar



1 out of 2

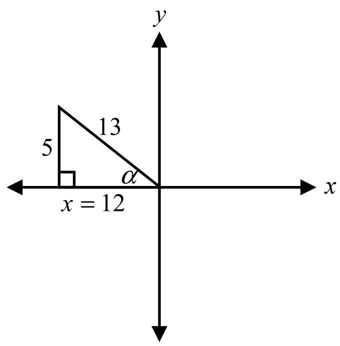
award full marks
– 1 mark for concept error ($x \leftrightarrow y$)
E9 (missing arrowhead)

Given that $\sin\alpha = \frac{5}{13}$, where α is in Quadrant II, and $\cos\beta = \frac{2}{5}$, where β is in Quadrant IV, find the exact value of:

a) $\cos(\alpha+\beta)$ b) $\sin 2\alpha$

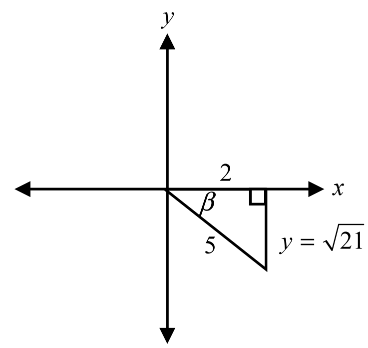
Solution

a)



$$x^2 + y^2 = r^2$$
$$x^2 + 25 = 169$$
$$x^2 = 144$$
$$x = \pm 12$$
$$x = -12$$

$$x^2 + y^2 = r^2$$
$$4 + y^2 = 25$$
$$y^2 = 21$$
$$y = \pm\sqrt{21}$$
$$y = -\sqrt{21}$$



½ mark for value of x
½ mark for value of y

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$= \left(-\frac{12}{13}\right)\left(\frac{2}{5}\right) - \left(\frac{5}{13}\right)\left(-\frac{\sqrt{21}}{5}\right)$$
$$= -\frac{24}{65} + \frac{5\sqrt{21}}{65}$$
$$= \frac{5\sqrt{21} - 24}{65}$$

½ mark for $\cos\alpha$
½ mark for $\sin\beta$

1 mark for substitution into correct identity

3 marks

b) $$\sin 2\alpha = 2\sin\alpha\cos\alpha$$
$$= 2\left(\frac{5}{13}\right)\left(-\frac{12}{13}\right)$$
$$= -\frac{120}{169}$$

1 mark for substitution into correct identity

1 mark

Note(s):

- accept any of the following values for x: $x = \pm 12$, $x = -12$ or $x = 12$
- accept any of the following values for y: $y = \pm\sqrt{21}$, $y = -\sqrt{21}$ or $y = \sqrt{21}$

Exemplar 1

a)

$4 + b^2 = 25$

$b^2 = 21$

$\cos\left(-\frac{12}{13}\right)\cos\left(\frac{2}{5}\right) - \sin\left(\frac{5}{13}\right)\sin\left(-\frac{\sqrt{21}}{5}\right)$

$\left(\frac{-24}{65}\right) - \frac{-5\sqrt{21}}{65}$

$\frac{-24 + 5\sqrt{21}}{65}$

3 out of 3

award full marks

E7 (notation error in line 1)

b)

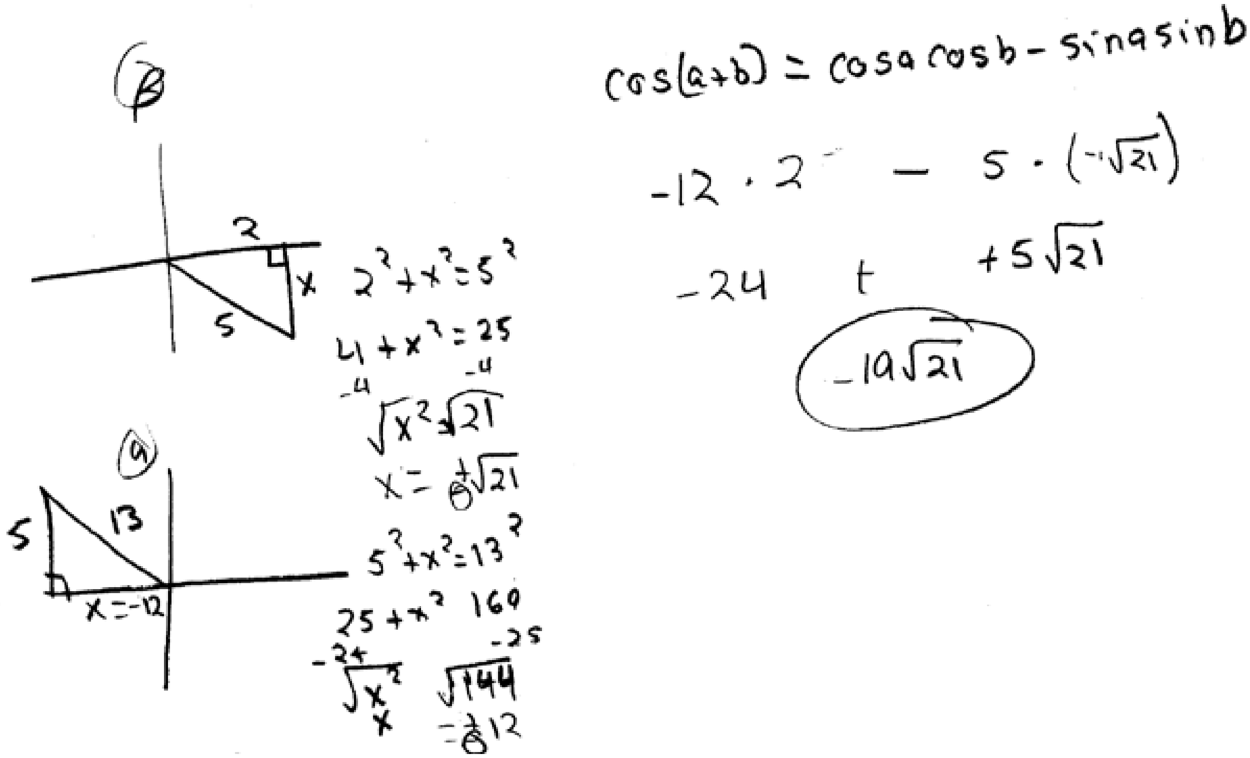
$2\left(\frac{5}{13}\right)\left(\frac{-12}{13}\right)$

$2\left(\frac{-60}{169}\right)$ $\boxed{\frac{-120}{169}}$

1 out of 1

Exemplar 2

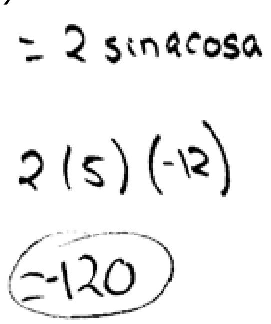
a)



1½ out of 3

+ ½ mark for x
+ ½ mark for y
+ 1 mark for substitution into the correct identity
– ½ mark for arithmetic error
E7 (notation error in lines 6 and 7)

b)



1 out of 1

+ 1 mark [marked consistently with a)]

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Question 15 R1

If $f(x) = x^3$ and $g(x) = 2x - 3$, what is the value of $\left(\frac{f}{g}\right)(-1)$?

Solution

$$f(-1) = (-1)^3$$
$$= -1$$

½ mark for substituting into $f(x)$ and $g(x)$

$$g(-1) = 2(-1) - 3$$
$$= -5$$

$$\left(\frac{f}{g}\right)(-1) = \frac{-1}{-5}$$
$$= \frac{1}{5}$$

½ mark for evaluating $\left(\frac{f}{g}\right)(-1)$

1 mark

Exemplar 1

$f(-1) = (-1)^3 = -1 \qquad g(-1) = 2(-1) - 3$

$= -2 - 3$

$= -5$

$$\left(\frac{f}{g}\right)(-1) = -5$$

½ out of 1

+ ½ mark for substitution into $f(x)$ and $g(x)$

Exemplar 2

$$\left(\frac{f(x)}{g(x)}\right)(-1) = \frac{x^3}{2x-3} = \frac{-1}{-2-3} = \frac{1}{5}$$

1 out of 1

award full marks
E7 (notation error)

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Booklet 2 Questions

Answer Key for Multiple-Choice Questions

Question	Answer	Learning Outcome
16	B	R3
17	C	R9
18	A	T3
19	C	R5
20	D	R12
21	A	R11
22	B	R7
23	D	R9
24	B	R13
25	A	R10

Question 16 R3

If the point $(4, -3)$ lies on the graph of $f(x)$, which point must lie on the graph of $2f(2x)$?

a) $(8, -6)$ b) $(2, -6)$ c) $\left(8, -\frac{3}{2}\right)$ d) $\left(2, -\frac{3}{2}\right)$

Question 17 R9

The graph of $y = \log_2(2x+6)$ intersects the graph of $y = 4$ at:

a) $x = -1$ b) $x = 1$ c) $x = 5$ d) $x = 14$

Question 18 T3

Given the point $A(-3, 5)$ on the terminal arm of an angle θ, identify the value of $\cot\theta$.

a) $-\frac{3}{5}$ b) $-\frac{5}{3}$ c) $-\frac{4}{5}$ d) $-\frac{5}{4}$

Question 19 R5

The graph of $y = \left(\frac{1}{2}\right)^x$ compared to the graph of $x = \left(\frac{1}{2}\right)^y$ is a:

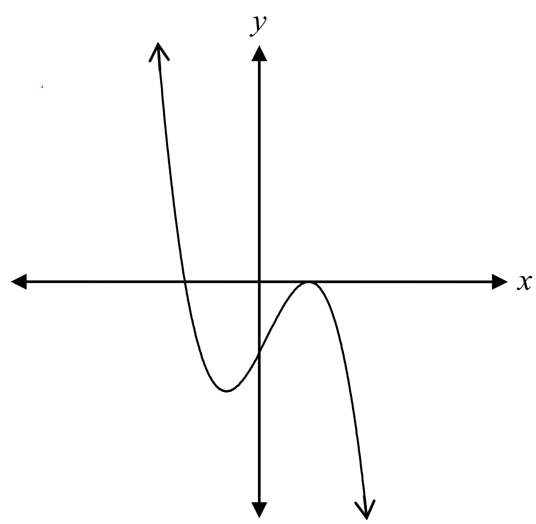
a) reflection in the x-axis

b) reflection in the y-axis

c) reflection in the line $y = x$

d) reciprocal function

Question 20 R12



Given the above graph of a polynomial function, which one of the following statements can be true?

a) The function has a degree of 4 with a positive leading coefficient.

b) The function has a degree of 4 with a negative leading coefficient.

c) The function has a degree of 3 with a positive leading coefficient.

d) The function has a degree of 3 with a negative leading coefficient.

Question 21 R11

Given that $(x+3)$ is a factor of polynomial $P(x)$, which of the following is true?

a) $P(-3) = 0$ b) $P(0) = -3$ c) $P(0) = 3$ d) $P(3) = 0$

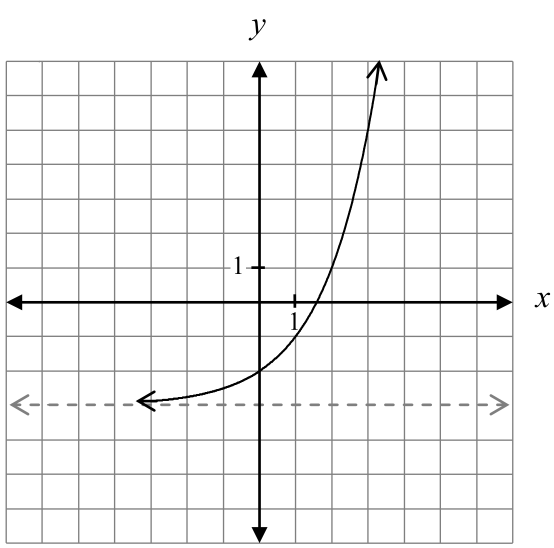
Question 22 R7

Which of the following is a reasonable estimate for the value of $\log 350$?

a) 2 b) 2.5 c) 2.8 d) 3

Question 23 R9

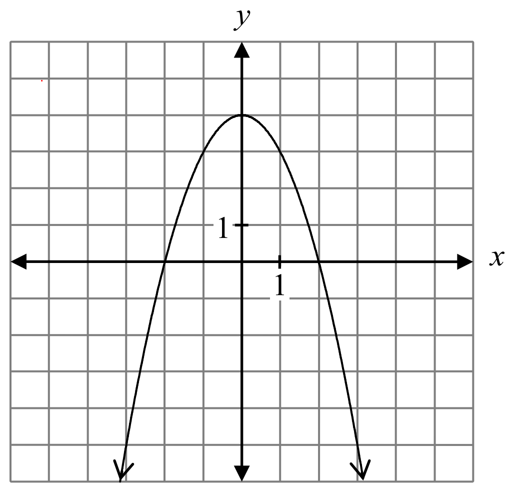
The graph of the function $f(x)$ shown below is best described by the equation:



a) $f(x) = 2^{x+3}$ b) $f(x) = 2^x + 3$ c) $f(x) = 2^{x-3}$ d) $f(x) = 2^x - 3$

Question 24 R13

Given the graph of $y = f(x)$, what is the domain of $\sqrt{f(x)}$?



a) $x \in \mathbb{R}$

b) $-2 \le x \le 2$

c) $x \le -2$ or $x \ge 2$

d) $0 \le x \le 4$

Question 25 R10

Solve:

$$e^{\ln(5-x)} = 7$$

a) -2

b) $-\ln 2$

c) $\ln 7 - \ln 5$

d) $\frac{7}{5}$

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One of the factors of $P(x) = x^3 - kx^2 - 7x + 10$ is $(x - 2)$.

Find the value of k.

Solution

Method 1

$x = 2$ — ½ mark for $x = 2$

$$\begin{aligned} 0 &= (2)^3 - k(2)^2 - 7(2) + 10 \\ 0 &= 8 - 4k - 14 + 10 \\ 0 &= 4 - 4k \\ 4k &= 4 \\ k &= 1 \end{aligned}$$

1 mark for remainder theorem

½ mark for solving for k

2 marks

Method 2

$x = 2$ — ½ mark for $x = 2$

$$\begin{array}{r|rrrr} 2 & 1 & -k & -7 & 10 \\ & & 2 & -2k+4 & -4k-6 \\ \hline & 1 & -k+2 & -2k-3 & -4k+4 \end{array}$$

½ mark for synthetic division

$$\begin{aligned} -4k + 4 &= 0 \\ 4k &= 4 \\ k &= 1 \end{aligned}$$

½ mark for equating the remainder to zero

½ mark for solving for k

2 marks

Exemplar 1

2 |	1	-1	-7	10
	↓	+2	+2	-10
	1	1	-5	0

Used guess and check to find K

$\_\_ + 2 = 1$

$K = 1$

2 out of 2

Exemplar 2

$P(2): 2^3 - K(2)^2 - 7(2) + 10$

$= 8 - K(4) - 14 + 10$

$= 0 - 4K - 4$

$= -4K + 4$

$\frac{-4}{4} = \frac{4K}{4}$

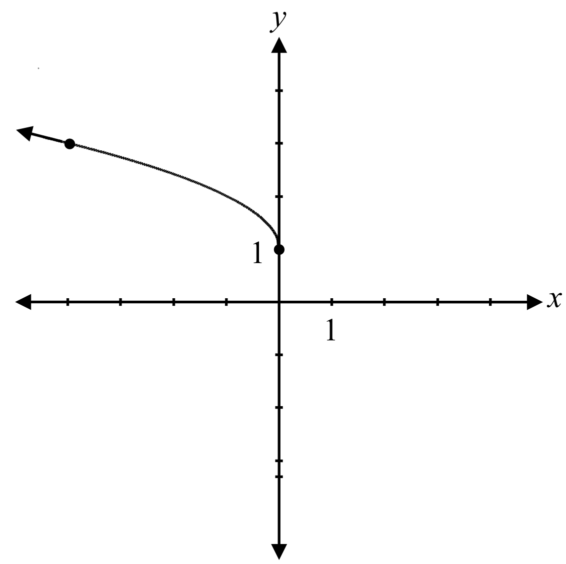
$-1 = K$

2 out of 2

award full marks
E7 (notation error in line 2 [left side should equal 0])
E7 (transcription error in line 5)

a) Sketch the graph of the function $y = \sqrt{-x} + 1$.

Solution

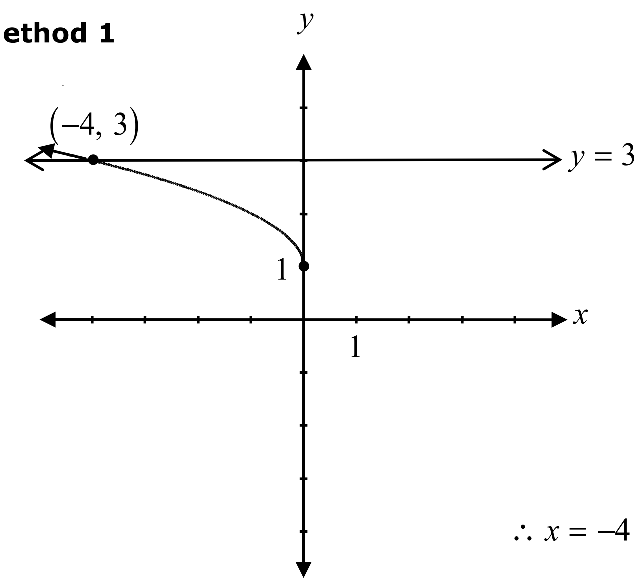


1 mark for general shape
1 mark for horizontal reflection
1 mark for vertical shift

3 marks

b) Determine the value of x when $y = 3$.

Method 1



$\therefore x = -4$

1 mark for consistent value of x

1 mark

Method 2

$y = \sqrt{-x} + 1$

$3 = \sqrt{-x} + 1$

$2 = \sqrt{-x}$

$4 = -x$

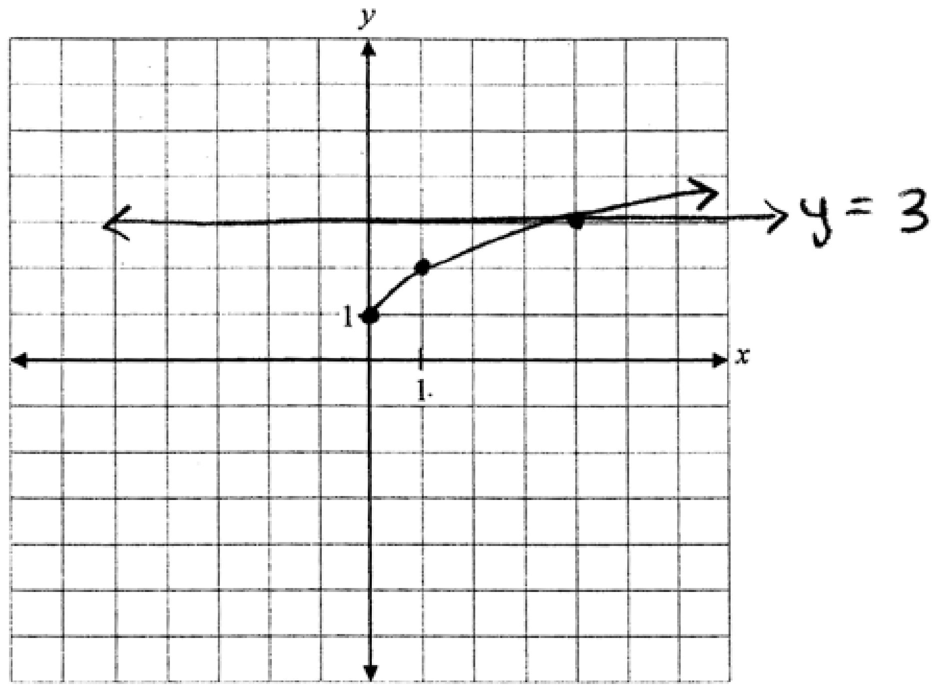
$x = -4$

1 mark for consistent value of x

1 mark

Exemplar

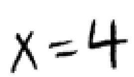
a)



2 out of 3

+ 1 mark for general shape
+ 1 mark for vertical shift

b)



1 out of 1

+ 1 mark for consistent value of x

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Question 28 — P2, P3

Solve the following equation:

$${}_nP_2 = {}_nC_3$$

Solution

Method 1

$$\frac{n!}{(n-2)!} = \frac{n!}{(n-3)!3!}$$

½ mark for substituting for ${}_nP_2$

$$\cancel{n!}(n-3)!3! = \cancel{n!}(n-2)!$$

½ mark for substituting for ${}_nC_3$

$$6 = \frac{(n-2)!}{(n-3)!}$$

1 mark for simplification

$$6 = \frac{(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}}$$

1 mark for expansion of $(n-2)!$

$$6 = n-2$$

$$8 = n$$

3 marks

Method 2

$${}_nP_2 = {}_nC_3; \text{ we know } n \geq 3$$

$$\frac{n!}{(n-2)!} = \frac{n!}{(n-3)!3!}$$

½ mark for substituting for ${}_nP_2$

½ mark for substituting for ${}_nC_3$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = \frac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}6}$$

½ mark for expansion of ${}_nP_2$

½ mark for expansion of ${}_nC_3$

$$6 = \frac{\cancel{n}\cancel{(n-1)}(n-2)}{\cancel{n}\cancel{(n-1)}}$$

we can divide by n and $(n-1)$ since we know $n \geq 3$

$$6 = n-2$$

1 mark for simplification

$$8 = n$$

3 marks

Question 28 P2, P3

Solution

Method 3

$$ {}_nP_2 = {}_nC_3 $$

$$ \frac{n!}{(n-2)!} = \frac{n!}{(n-3)!3!} $$

½ mark for substituting for ${}_nP_2$
½ mark for substituting for ${}_nC_3$

$$ \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = \frac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}3!} $$

½ mark for expansion of ${}_nP_2$
½ mark for expansion of ${}_nC_3$

$$ 6n(n-1) = n(n-1)(n-2) $$

$$ 6n(n-1) - n(n-1)(n-2) = 0 $$

$$ n(n-1)(6-(n-2)) = 0 $$

$$ n(n-1)(8-n) = 0 $$

1 mark for simplification

$$ \cancel{n=0} \quad \cancel{n=1} \quad n = 8 $$

3 marks

Note(s):

- deduct ½ mark for not rejecting non-permissible values

$$\frac{n!}{(n-2)!} = \frac{n!}{3!(n-3)!}$$

$$\frac{n(n-1)(n-2)!}{(n-2)!} = \frac{n(n-1)(n-2)(n-3)}{3!(n-3)!}$$

$$n(n-1) = \frac{n(n-1)(n-2)}{3}$$

$$n^2 - n = \frac{n(n^2-3n+2)}{3}$$

$$(3)n^2 - n = \frac{n^3-3n^2+2n}{3}(3)$$

$$\begin{aligned} 3n^2 - 3n &= n^3 - 3n^2 + 2n + 3n \\ -3n^2 \quad +3n & \quad\;\; -3n^2 \end{aligned}$$

$$0 = n^3 - 6n^2 + 5n$$

$$0 = n(n^2 - 6n + 5)$$

$$0 = n(n-1)(n-5)$$

$$n = 0, 1, 5$$

2 out of 3

award full marks
– ½ mark for arithmetic error in line 3
– ½ mark for non-permissible values of n
E7 (notation error in line 2)
E4 (missing brackets but still implied in line 5)

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Given $f(x) = x^2 - 1$ and $g(x) = \sqrt{x+1}$, sketch the graph of $y = f(g(x))$ and state its domain.

Solution

Method 1

$f(x) = x^2 - 1$

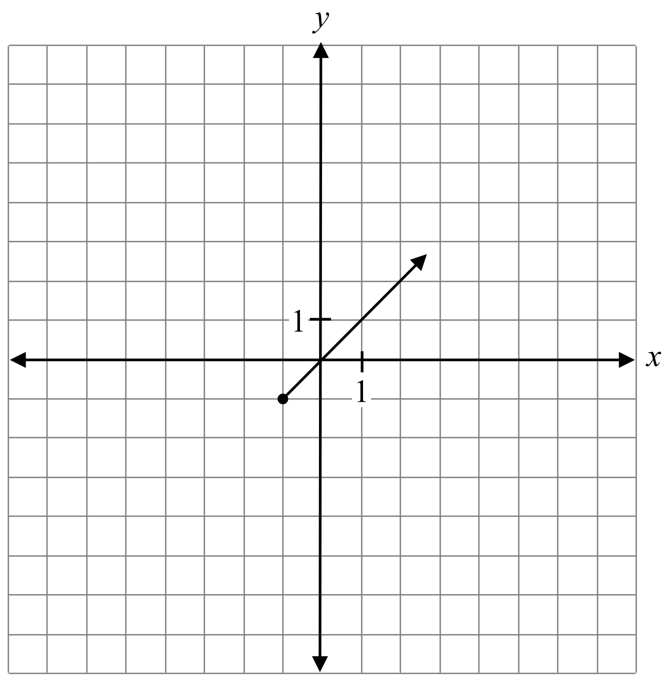
Domain: $x \in \mathbb{R}$

$g(x) = \sqrt{x+1}$

Domain: $[-1, \infty)$

$$\begin{aligned} f(g(x)) &= \left(\sqrt{x+1}\right)^2 - 1 \\ &= x + 1 - 1 \\ &= x \end{aligned}$$

1 mark for determining the function $f(g(x))$



1 mark for graph of composite function

Domain of $f(g(x))$: $[-1, \infty)$ **or** $\{x \mid x \geq -1, x \in \mathbb{R}\}$

1 mark for restricted domain

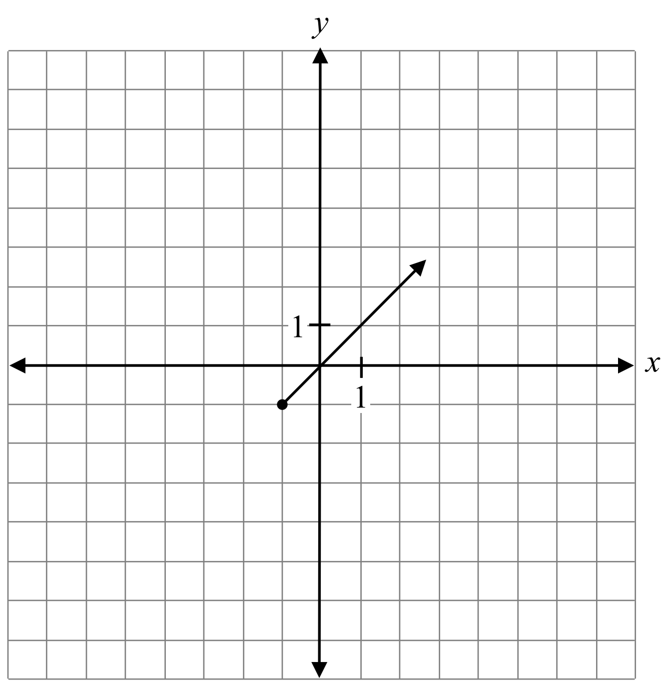
3 marks

Solution

Method 2

x	$g(x)$	$f(g(x))$
-2		
-1	0	-1
0	1	0
1		1
2		2
3	2	3

1 mark for table of values



1 mark for graph of composite function

Domain of $f(g(x))$: $[-1, \infty)$ **or** $\{x | x \geq -1,\ x \in \mathbb{R}\}$

1 mark for restricted domain

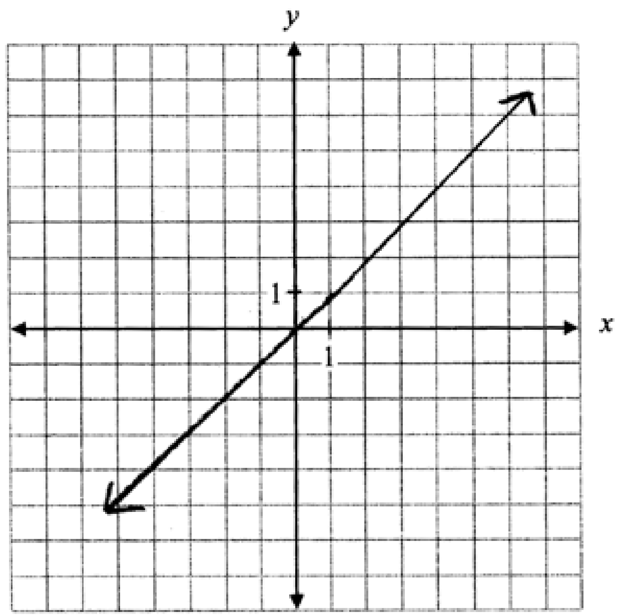
3 marks

$f(x) = x^2$

$g(x) = \sqrt{x+1}$

$f(g(x)) = (\sqrt{x+1})^2 - 1$

$= (x+1) - 1$



Domain: $\{x \in R\}$

2 out of 3

award full marks
– 1 mark (concept error for not restricting domain)

Question 30 R14

Write the equation of the horizontal asymptote for the function $f(x) = \dfrac{x-3}{x-2}$.

Solution

$y = 1$ 1 mark for equation of horizontal asymptote

1 mark

Exemplar

$HA = 1 \div 1$

$\boxed{HA = 1}$

1 out of 1

award full marks
E7 (notation error)

The x-intercept of $f(x)$ is 4 and the x-intercept of $g(x)$ is 4.

Benjamin concludes that the x-intercept of $f(x)+g(x)$ is 8.

Do you agree with Benjamin? Justify your answer.

Solution

No, I do not agree with Benjamin.

Benjamin added the x-values instead of adding the y-values.

If the x-intercept of $f(x)$ is 4, then $y = 0$.

If the x-intercept of $g(x)$ is 4, then $y = 0$.

$\therefore$ the x-intercept of $f(x)+g(x)$ is 4.

1 mark for justification

1 mark

Exemplar 1

No the x-intercept would still be 4.

1 out of 1

Exemplar 2

NO

0 out of 1

Solve the following equation:

$$2\log_4 x - \log_4(x+3) = 1$$

Solution

$$2\log_4 x - \log_4(x+3) = 1$$

$$\log_4\left(\frac{x^2}{x+3}\right) = 1$$

1 mark for power rule
1 mark for quotient rule

$$4^1 = \left(\frac{x^2}{x+3}\right)$$

1 mark for exponential form

$$4(x+3) = x^2$$

$$x^2 - 4x - 12 = 0$$

$$(x-6)(x+2) = 0$$

$$x = 6 \qquad \cancel{x = -2}$$

½ mark for solving for x
½ mark for rejecting extraneous root

4 marks

Exemplar 1

$$\log_4 2x - \log_4 (x+3) = 1$$

$$\log_4\left(\frac{2x}{x+3}\right) = 1$$

$$\frac{2x}{x+3} = 4$$

$$2x = 4(x+3)$$

$$2x = 4x + 12$$

$$-12 = 2x$$

$$x = -6$$

2½ out of 4

+ 1 mark for quotient rule
+ 1 mark for exponential form
+ ½ mark for solving for x

Exemplar 2

$$2\log_4 x - \log_4 (x+3) = 1$$

$$\log_4 x^2 - \log_4 (x+3) = 1$$

$$\log_4 \frac{x^2}{x+3} = 1$$

$$\frac{x^2}{x+3} = 4$$

$$x^2 = 4x + 12$$

$$x^2 - 4x - 12 = 0$$

$$(x-6)(x+2) = 0$$

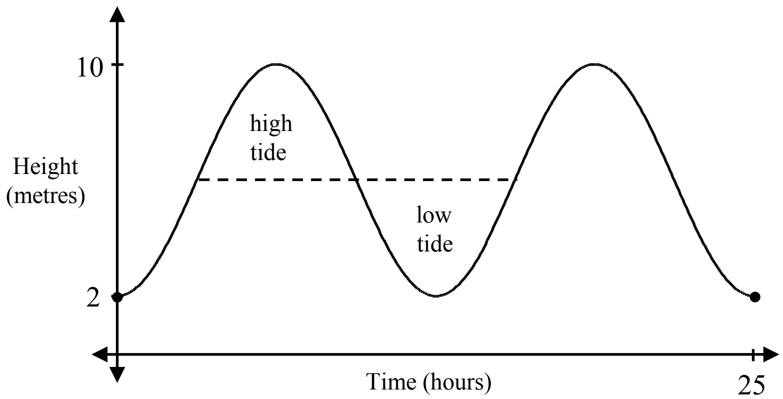
$$\boxed{x = 6}$$

3 out of 4

+ 1 mark for power rule
+ 1 mark for quotient rule
+ 1 mark for exponential form

Question 33 T4

The following graph represents tidal levels in the Bay of Fundy over a 25-hour period.



a) What is the average height of the water?

b) What is the period of the graph above?

Explain what the period represents in this situation.

Solution

a) 6 metres

1 mark

b) Period $= \dfrac{25}{2}$

$= 12.5$ hours

1 mark for period

The period represents the time to complete one cycle of tidal levels in the Bay of Fundy.

1 mark for explanation

2 marks

Exemplar

a)

$$\text{amplitude} = \frac{y\text{max} - y\text{min}}{2}$$
$$= \frac{10 - 2}{2} = 4$$
$$= \frac{8}{2}$$

∴ avorage height of the water is 4 metres

0 out of 1

b)

the period is 12.5

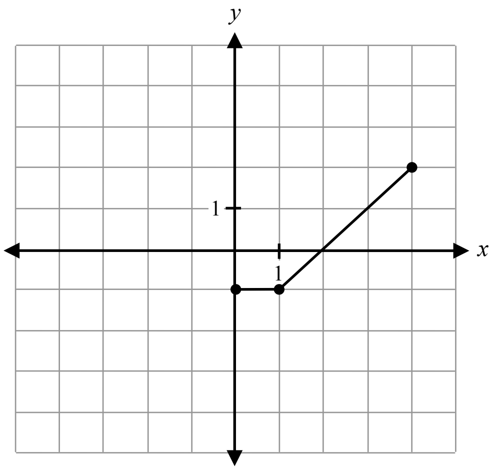
it represents the amount of time it takes the water to go from it's highest point to it's lowest and then back

2 out of 2

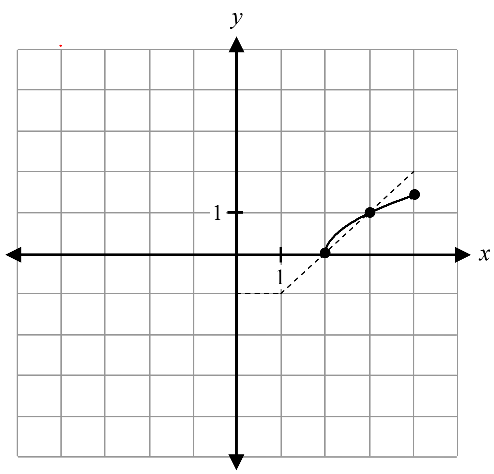
award full marks
E5 (missing unit of measure in line 1)

Question 34 R13

Given the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$.



Solution



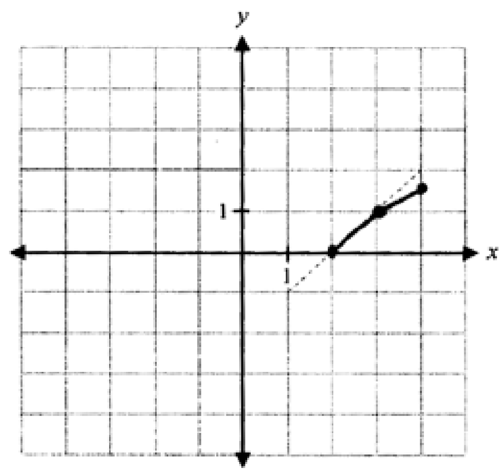
1 mark for restricting the domain

½ mark for graph above $y = f(x)$ over the range $[0, 1]$

½ mark for graph below $y = f(x)$ over the range $[1, 2]$

2 marks

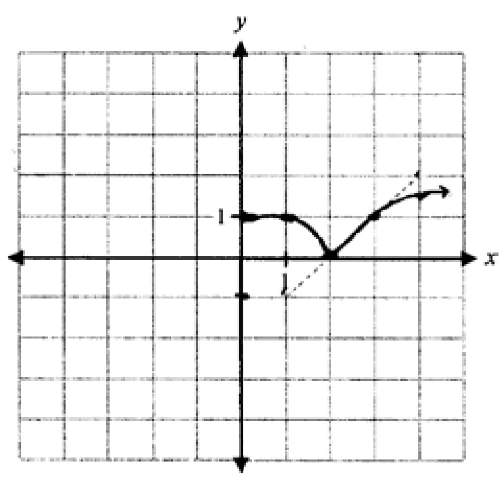
Exemplar 1



1½ out of 2

+ 1 mark for restricting the domain
+ ½ mark for the graph below $y = f(x)$ over the range $[1, 2]$

Exemplar 2



½ out of 2

+ ½ mark for graph below $y = f(x)$ over the range $[1, 2]$
E9 (incorrect endpoint)

Question 35 R11

When $P(x)$ is divided by $x-3,$ it has a quotient of $2x^2+x-6$ and a remainder of 4.

Determine $P(x)$.

Solution

$$P(x)=(x-3)\left(2x^2+x-6\right)+4$$

1 mark for polynomial $P(x)$

or

1 mark

$$P(x)=2x^3-5x^2-9x+22$$

Exemplar

$$(x-3)(2x^2+x-6)$$

$$2x^3 \quad x^2 \quad x-6 \quad -6x^2 \quad x-3 \quad +18$$

$$\boxed{2x^3-5x^2-9x+18}$$

0 out of 1

award full marks

– 1 mark for concept error of not including the remainder

Question 36 R14

Identify the domain and range of the following function:

$$f(x) = \frac{3}{x^2 + 1}$$

Solution

Domain: $\{x \in \mathbb{R}\}$ — 1 mark for domain

or

$(-\infty, \infty)$

Range: $\{y \in \mathbb{R} \mid 0 < y \leq 3\}$ — 1 mark for range

or

$(0, 3]$

2 marks

Exemplar

x^2 is always greater than zero

D: $[0, \infty)$

R: $(0, 3]$

1 out of 2

+ 1 mark for range

Question 37 T3

Evaluate:

$$\csc\left(\frac{11\pi}{6}\right) + \sin^2\left(-\frac{3\pi}{4}\right) + \cos\left(\frac{23\pi}{3}\right)$$

Solution

$$= (-2) + \left(-\frac{\sqrt{2}}{2}\right)^2 + \frac{1}{2}$$

$$= -2 + \frac{1}{2} + \frac{1}{2}$$

$$= -1$$

1 mark for $\csc\left(\frac{11\pi}{6}\right)$ (½ mark for quadrant, ½ mark for value)

1 mark for $\sin^2\left(-\frac{3\pi}{4}\right)$ (½ mark for quadrant, ½ mark for value)

1 mark for $\cos\left(\frac{23\pi}{3}\right)$ (½ mark for quadrant, ½ mark for value)

3 marks

Exemplar

$$\csc\left(-\frac{\sqrt{3}}{2}\right) + \sin^2\left(-\frac{1}{\sqrt{2}}\right) + \cos\left(\frac{1}{2}\right)$$

$$= \left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2}\right)$$

$$= -\frac{\sqrt{3}}{\sqrt{2}}$$

---

1 out of 3

+ 1 mark for $\sin^2\left(-\frac{3\pi}{4}\right)$

+ 1 mark for $\cos\left(\frac{23\pi}{3}\right)$

– ½ mark for arithmetic error in line 2

– ½ mark for arithmetic error in line 3

E7 (notation error in line 1)

Evaluate the coefficient of the term containing x^3 in the expansion of $(1+x)^7$.

Justify your answer.

Solution

Method 1

							1							
						1		1						
					1		2		1					
				1		3		3		1				
			1		4		6		4		1			
		1		5		10		10		5		1		
	1		6		15		20		15		6		1	
1		7		21		(35)		35		21		7		1

1 mark for justification

The coefficient of x^3 is 35.

1 mark for identifying coefficient

2 marks

Method 2

$$t_4 = {}_7C_3(1)^4(x)^3$$

1 mark for justification

$$ {}_7C_3 = \frac{7!}{3!4!}$$

$$= \frac{7 \bullet \cancel{6} \bullet 5 \bullet \cancel{4!}}{\cancel{3!}\,\cancel{4!}}$$

$$= 35$$

1 mark for evaluating coefficient

2 marks

The coefficient of x^3 is 35.

Exemplar 1

$_7C_4$

$$\frac{7 \times 6 \times 5}{3 \times 2}$$

$= 35$

2 out of 2

Exemplar 2

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 (35) 21 7 1

$35(x)^3(1)^4$

$35x^3$

2 out of 2

Question 39 R10

Which of the following equations could be solved without the use of logarithms?

Without actually solving the problem, explain your choice.

$$4^{x} = 10^{3x+1} \qquad \text{or} \qquad \left(\frac{1}{3}\right)^{2x+1} = 27^{4x-1}$$

Solution

$\left(\frac{1}{3}\right)^{2x+1} = 27^{4x-7}$ can be solved without the use of logarithms because $\frac{1}{3}$ and 27 can both be changed to a base of 3.

1 mark for explanation

1 mark

the second one because you know 3 is a factor of 27 and since its below 1 you know the exponent is negative. All you have to do is make the exponent = −3.

1 out of 1

Sketch the graph of $y = x^3 + x^2 - 5x + 3$ given that one of the x-intercepts is 1.

Identify the x-intercepts and y-intercept.

Solution

$x = 1$

$$\begin{array}{r|rrrr} 1 & 1 & 1 & -5 & 3 \\ & & 1 & 2 & -3 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$

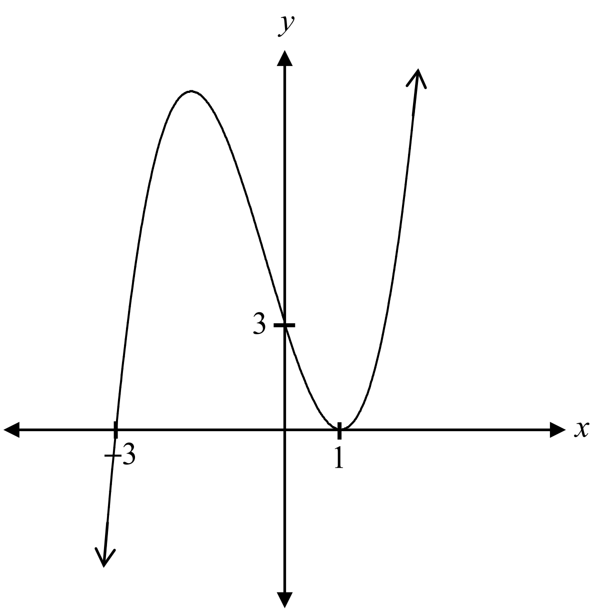
1 mark for synthetic division

$y = (x-1)\left(x^2 + 2x - 3\right)$

$y = (x-1)(x+3)(x-1)$

$y = (x+3)(x-1)^2$

1 mark for identifying the factors



2 marks for graph (½ mark for x-intercepts, ½ mark for multiplicity, ½ mark for y-intercept, ½ mark for end behaviour)

4 marks

Exemplar

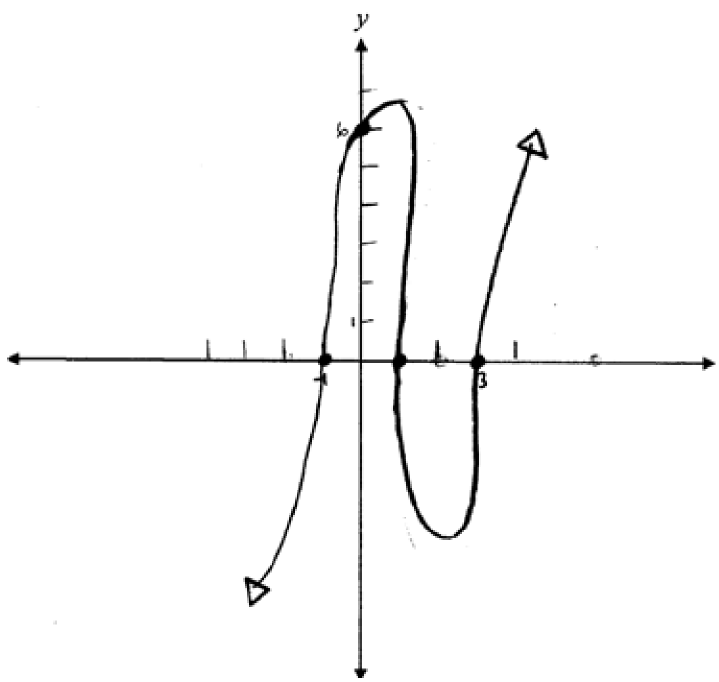
$x - 1$

1	1	1	−5	3
	+	1	2	−3
	1	2	−3	0

$(x-1)(x^2 - 2x - 3) = 0$ p: $-3 \cdot 1 = -3$ s: $-3 + 1 = -2$

$(x-1)(x-3)(x+1) = 0$

$x = 1$ $x = 3$ $x = -1$



3 out of 4

+ 1 mark for synthetic division
+ 1 mark for identifying factors
+ ½ mark for *x*-intercepts
+ ½ mark for end behaviour
+ ½ mark for multiplicity
– ½ mark for incorrect shape (not a function)
E7 (transcription error in line 3)

Question 41 R1

If $f(x) = \frac{1}{x-2}$ and $g(x) = x - 2$, what is the domain of $f(x) \bullet g(x)$?

Solution

$f(x) = \frac{1}{x-2}$ $g(x) = x - 2$

Domain: $x \in \mathbb{R},\ x \neq 2$ Domain: $x \in \mathbb{R}$

Domain of $f(x) \bullet g(x)$: $\{x \in \mathbb{R} \mid x \neq 2\}$

1 mark for domain of $f(x) \bullet g(x)$

1 mark

Exemplar

$f(x) = \frac{1}{x-2}$

$g(x) = x-2$

D; $x \neq 2$

1 out of 1

Given $f(x) = (x+1)^2$ for $x \le -1$, write the equation of $y = f^{-1}(x)$.

Solution

Method 1

$$y = (x+1)^2$$

$$x = (y+1)^2$$ 1 mark for inverse

$$y = \pm\sqrt{x} - 1$$ ½ mark for solving for y

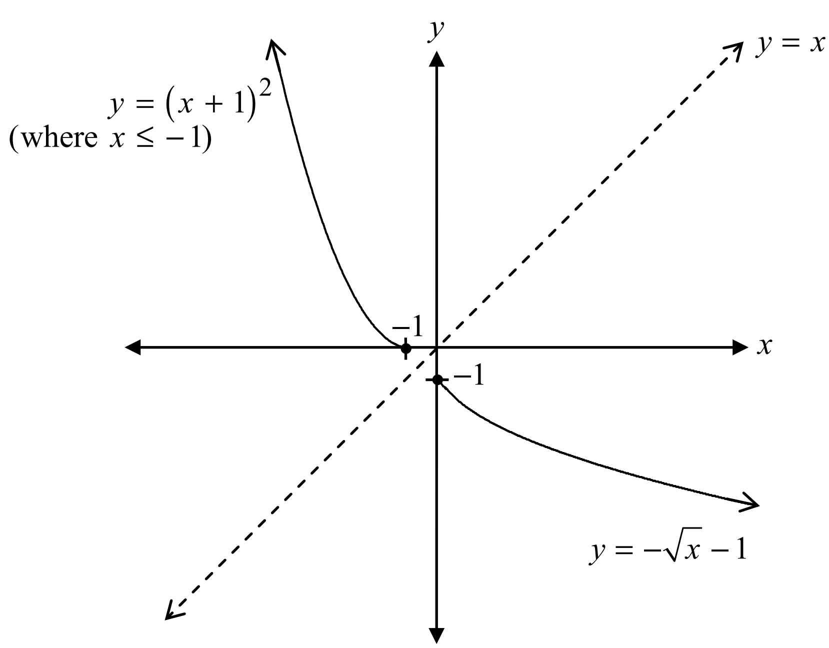
Since the domain of $f(x)$ is $x \le -1$,
the range of the inverse is $y \le -1$.

$$\therefore\ y = -\sqrt{x} - 1$$ ½ mark for rejecting $y = \sqrt{x}$

$$f^{-1}(x) = -\sqrt{x} - 1$$

2 marks

Method 2



1 mark for reflection over the line $y = x$

1 mark for correct equation

2 marks

Exemplar

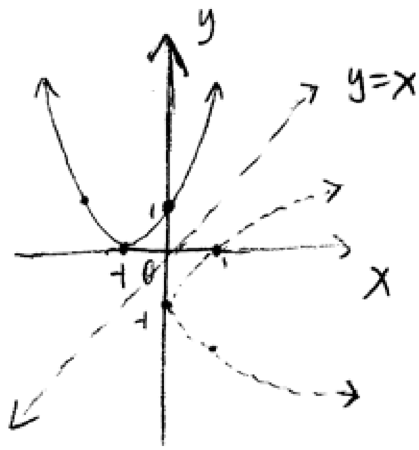
$y = (x+1)^2$

$x = (y+1)^2$

$\sqrt{x} = y+1$

$y = \sqrt{x} - 1$

$\therefore \boxed{f^{-1}(x) = \sqrt{x} - 1}$



Check: $f(f^{-1}(x)) = (\sqrt{x} - 1 + 1)^2 = \sqrt{x}^2 = x$ ✓

1 out of 2

Method 1

+ 1 mark for inverse

Question 43 T4

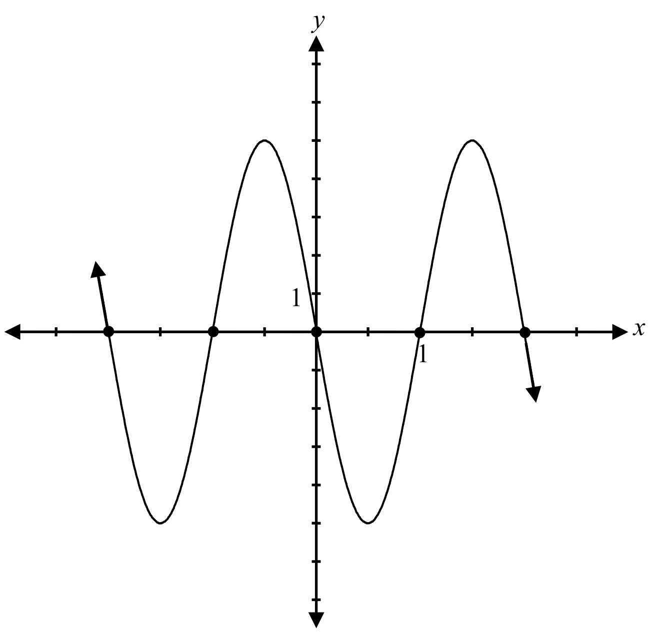
Sketch a graph of at least one period of the function $y = 5\sin\left[\pi(x+1)\right]$.

Clearly indicate the x-intercepts.

Solution

$b = \pi$

$\therefore$ period $= \dfrac{2\pi}{\pi} = 2$



1 mark for amplitude
1 mark for horizontal shift
1 mark for period
1 mark for clearly indicating at least two x-intercepts consistent with graph

4 marks

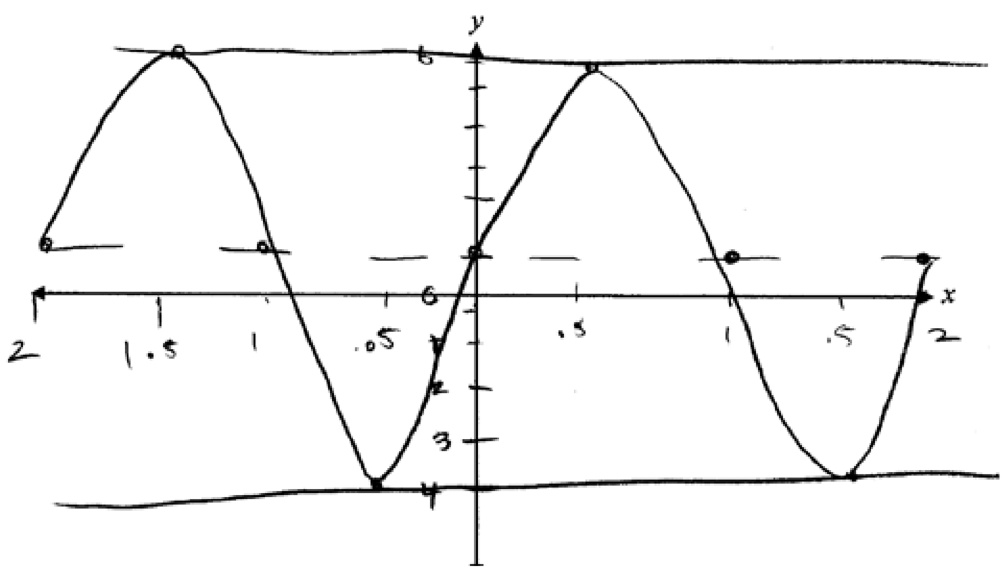
$(-1 \times 5,\ 1 \times 5)$

$(-5 + 1,\ 5 + 1)$

$(-4,\ 6)$

$\frac{2\pi}{\pi}$ period is 2.

$\frac{6 - 4}{2}$ $\frac{2}{2}$



2½ out of 4

+ 1 mark for amplitude
+ 1 mark for period
+ 1 mark for clearly indicating at least two *x*-intercepts consistent with graph
– ½ mark for incorrect shape of graph (solid lines at top and bottom)

Sketch the graph of the following function:

$$f(x) = \frac{x-2}{(2x-3)(x-2)}$$

Solution

$$f(x) = \frac{x-2}{(2x-3)(x-2)}$$

$$= \frac{1}{2x-3} \text{ with a point of discontinuity at } x = 2$$

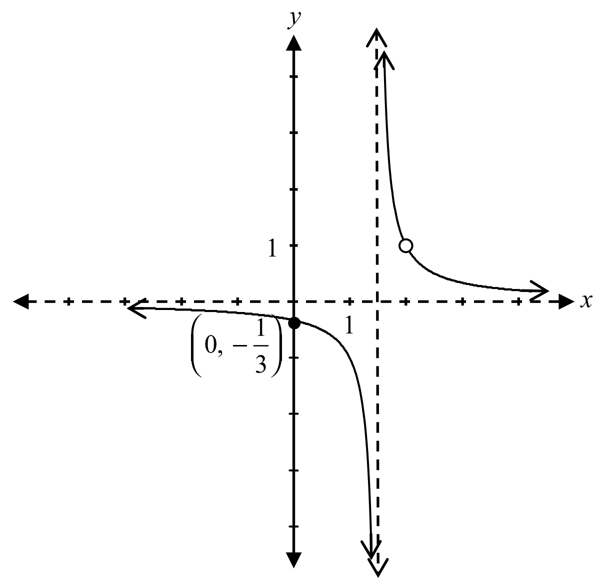
point of discontinuity: $f(2) = 1$

$\therefore$ there is a point of discontinuity at $(2, 1)$

y-intercept: $f(0) = \dfrac{0-2}{(2(0)-3)(0-2)}$

$$= -\frac{2}{6}$$

$$= -\frac{1}{3}$$



1 mark for horizontal asymptote at $y = 0$

1 mark for vertical asymptote at $x = \dfrac{3}{2}$

½ mark for graph left of vertical asymptote

½ mark for graph right of vertical asymptote

1 mark for point of discontinuity at $(2, 1)$; (½ mark for $x = 2$, ½ mark for $y = 1$)

4 marks

Exemplar 1

$f(x) = \dfrac{x-2}{(2x-3)(x-2)}$

$2x^2 - 7x + 6$

$x \neq \frac{3}{2}, 2$

y-int: $-\frac{1}{3}$

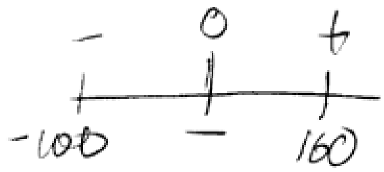
x-int: 2

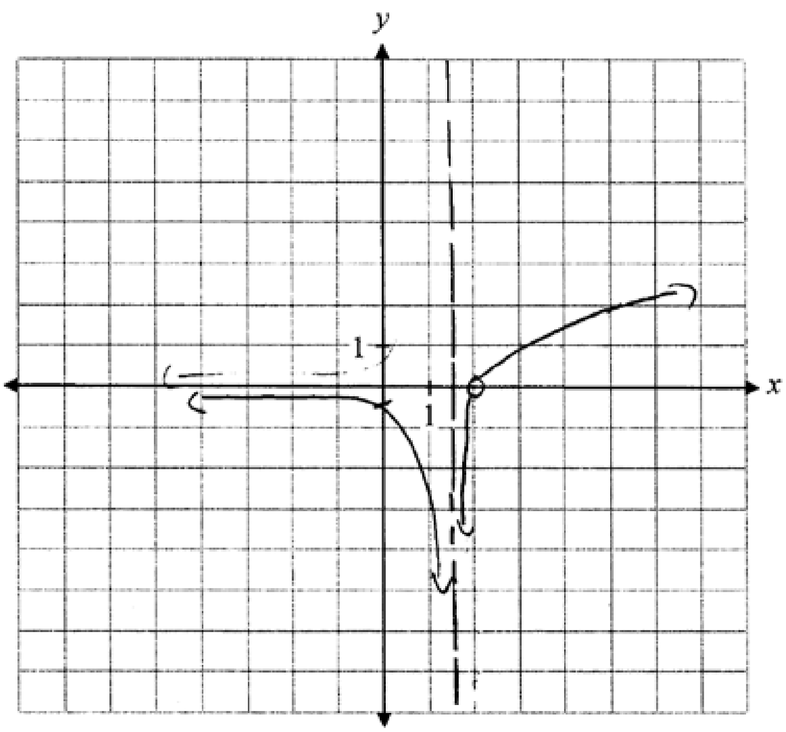
v asy: $x = \frac{3}{2}$

h asy: $y = 0$

$x \neq 2$

hole $x = 2$





3 out of 4

+ 1 mark for horizontal asymptote
+ 1 mark for vertical asymptote
+ ½ mark for graph left of vertical asymptote
+ ½ mark for point of discontinuity when $x = 2$

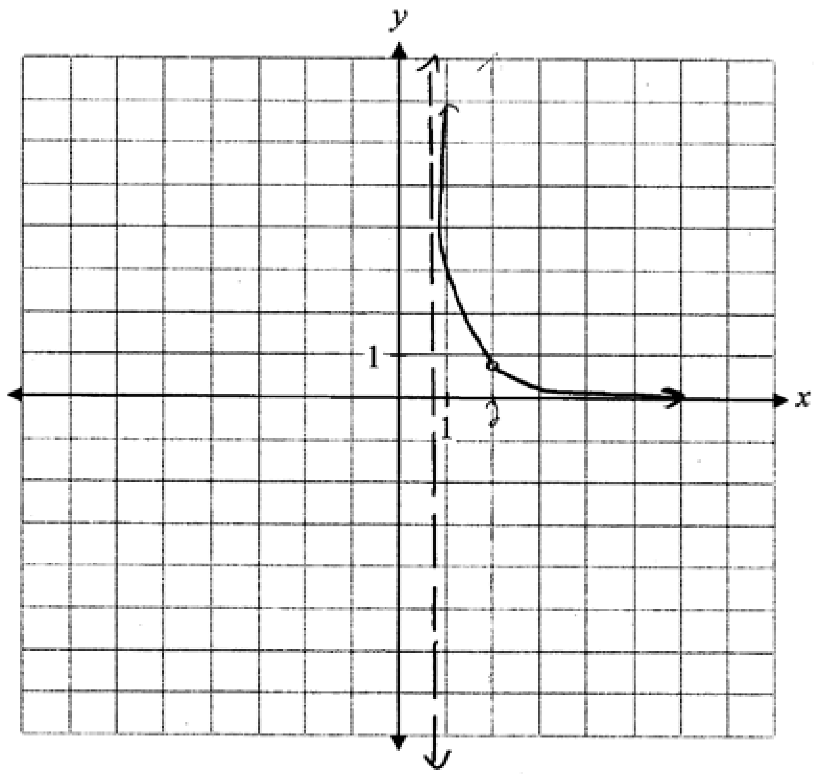
Exemplar 2

$\frac{x-2}{(2x-3)(x-2)}$ hole

$x \neq 2;\ \frac{3}{2}$ $\quad\frac{1}{\frac{3}{2}} \times 3 = \frac{1}{1,5}$

$y = \frac{1}{(2x-3)}$ $\quad y = 0,\overline{66}$

$x = 2$



1 out of 4

+ ½ mark for graph right of vertical asymptote
+ ½ mark for point of discontinuity when $x = 2$

Appendices

Appendix A

MARKING GUIDELINES

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply.

- arithmetic error
- procedural error
- terminology error in explanation
- lack of clarity in explanation
- incorrect shape of graph (only when marks are not allocated for shape)

Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the *Answer/Scoring Sheet*.

E1 final answer	▪ answer given as a complex fraction ▪ final answer not stated
E2 equation/expression	▪ changing an equation to an expression ▪ equating the two sides when proving an identity
E3 variables	▪ variable omitted in an equation or identity ▪ variables introduced without being defined
E4 brackets	▪ "$\sin x^2$" written instead of "$\sin^2 x$" ▪ missing brackets but still implied
E5 units	▪ missing units of measure ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa
E6 rounding	▪ rounding error ▪ rounding too early
E7 notation/transcription	▪ notation error ▪ transcription error
E8 domain/range	▪ answer included outside the given domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order
E9 graphing	▪ incorrect or missing endpoints or arrowheads ▪ scale values on axes not indicated ▪ coordinate points labelled incorrectly
E10 asymptotes	▪ asymptotes drawn as solid lines ▪ asymptotes missing but still implied ▪ graph crosses or curls away from asymptotes

Appendix B

IRREGULARITIES IN PROVINCIAL TESTS

A GUIDE FOR LOCAL MARKING

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the Department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all "NR") or only incorrect responses ("0")

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the Department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report*.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Booklet Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the Department along with the test materials.

Irregular Test Booklet Report

Test: ______________________________

Date marked: ______________________________

Booklet No.: ______________________________

Problem(s) noted: ______________________________

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Question(s) affected: ______________________________

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Action taken or rationale for assigning marks: ______________________________

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Follow-up: ______________________________

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Decision: ______________________________

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Marker's Signature: ______________________________

Principal's Signature: ______________________________

For Department Use Only—After Marking Complete

Consultant: ______________________________

Date: ______________________________

Appendix C

Table of Questions by Unit and Learning Outcome

Unit A: Transformations of Functions		
Question	**Learning Outcome**	**Mark**
8	R4	1
10	R1	2
13	R2	2
16	R3	1
19	R5	1
29	R1	3
31	R1	1
15	R1	1
41	R1	1
42	R5, R6	2
Unit B: Trigonometric Functions		
Question	**Learning Outcome**	**Mark**
1	T1	2
7	T3, T5, T6	4
9	T1	1
18	T3	1
33	T4	3
37	T3	3
43	T4	4
Unit C: Binomial Theorem		
Question	**Learning Outcome**	**Mark**
4	P4	2
6	P2	2
11	P3	2
28	P2, P3	3
38	P4	2
Unit D: Polynomial Functions		
Question	**Learning Outcome**	**Mark**
20	R12	1
21	R11	1
26	R11	2
35	R11	1
40	R12	4

Unit E: Trigonometric Equations and Identities		
Question	**Learning Outcome**	**Mark**
2	T5	3
14	T6	4
12 a)	T6	2
12 b)	T5, T6	2
Unit F: Exponents and Logarithms		
Question	**Learning Outcome**	**Mark**
3	R10	2
5	R8	3
17	R9	1
22	R7	1
23	R9	1
25	R10	1
32	R10	4
39	R10	1
Unit G: Radicals and Rationals		
Question	**Learning Outcome**	**Mark**
24	R13	1
27 a)	R13	3
27 b)	R13	1
30	R14	1
34	R13	2
36	R14	2
44	R14	4