PEARSON

Math Makes Sense 9

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Table of Contents

UNIT 3 Rational Numbers

UNIT 4 Linear Relations

UNIT 5 Polynomials

UNIT 6 Linear Equations and Inequalities

UNIT 7 Similarity and Transformations

UNIT 8 Circle Geometry

UNIT 9

Probability and Statistics

Welcome to
Pearson Math Makes Sense 9

Math helps you understand your world.

This book will help you improve your problem-solving skills and show you how you can use your math now, and in your future career.

The opening pages of **each unit** are designed to help you prepare for success.

UNIT 1

Square Roots and Surface Area

Which geometric objects can you name?

How could you determine their surface areas?

What You'll Learn

- Determine the square roots of fractions and decimals that are perfect squares.
- Approximate the square roots of fractions and decimals that are non-perfect squares.
- Determine the surface areas of composite 3-D objects to solve problems.

Why It's Important

Real-world measures are often expressed as fractions or decimals.
We use the square roots of these measures when we work with formulas such as the Pythagorean Theorem.
An understanding of surface area allows us to solve practical problems such as calculating:
the amount of paper needed to wrap a gift;
the number of cans of paint needed to paint a room;
and the amount of siding needed to cover a building

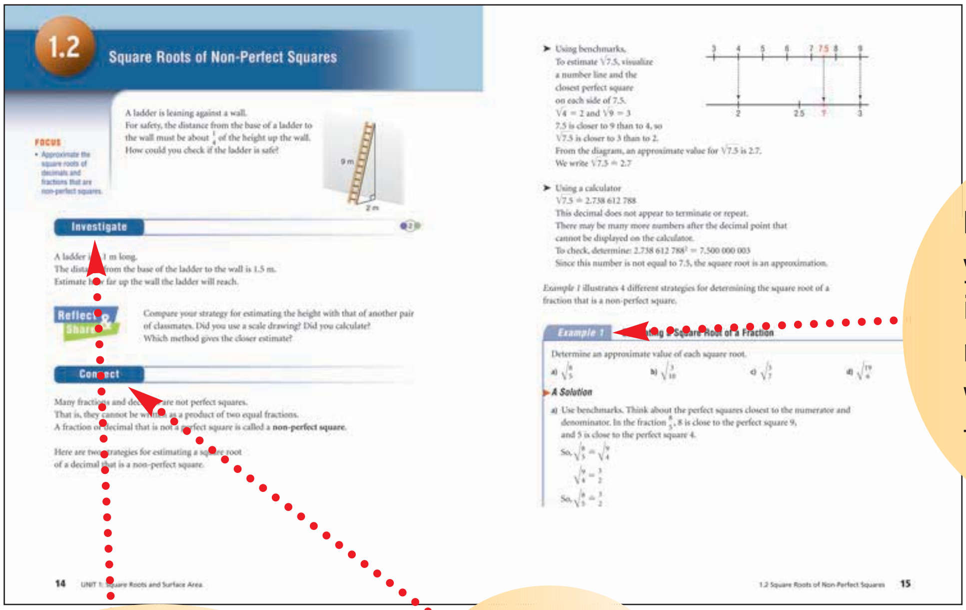
4

Key Words

- perfect square
- non-perfect square
- composite object

5

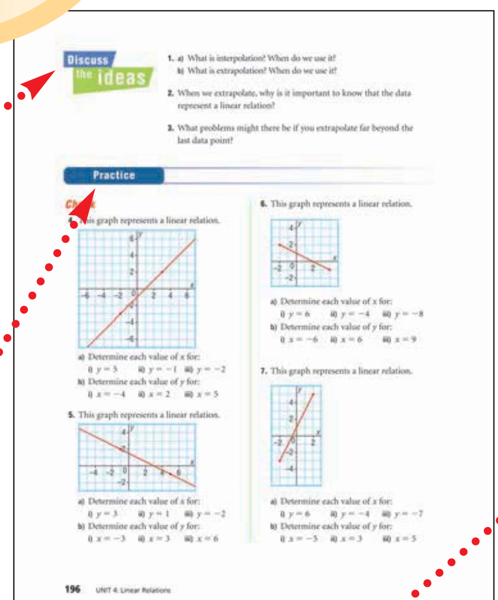
Find out **What You'll Learn** and **Why It's Important**. Check the list of **Key Words**.

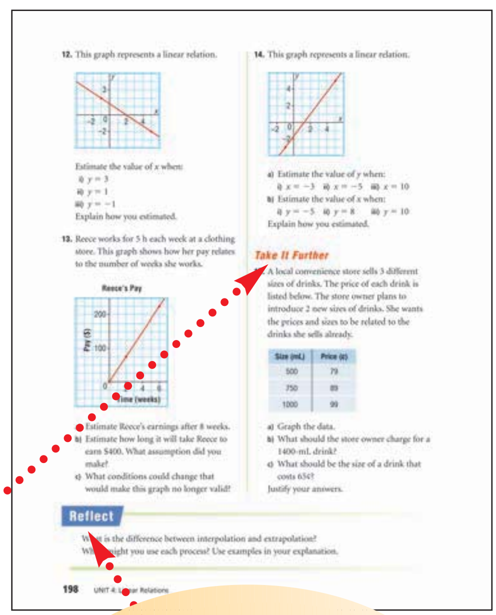


Examples show you how to use the ideas and that there may be different ways to approach the question.

Investigate an idea or problem, usually with a partner, and often using materials.

Connect summarizes the math.



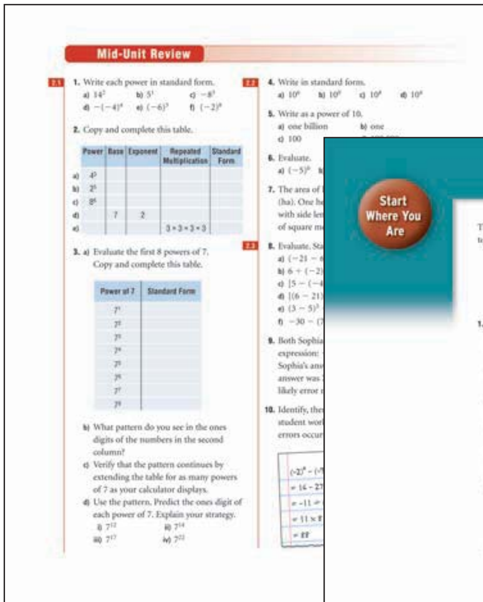


Discuss the Ideas invites you to talk about the math.

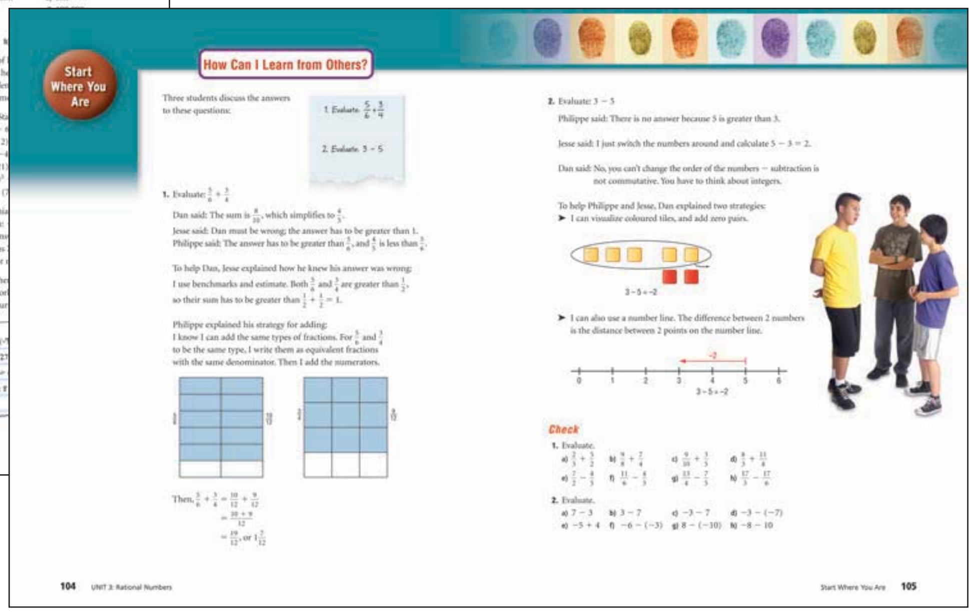
Practice questions reinforce the math.

Take It Further questions offer enrichment and extension.

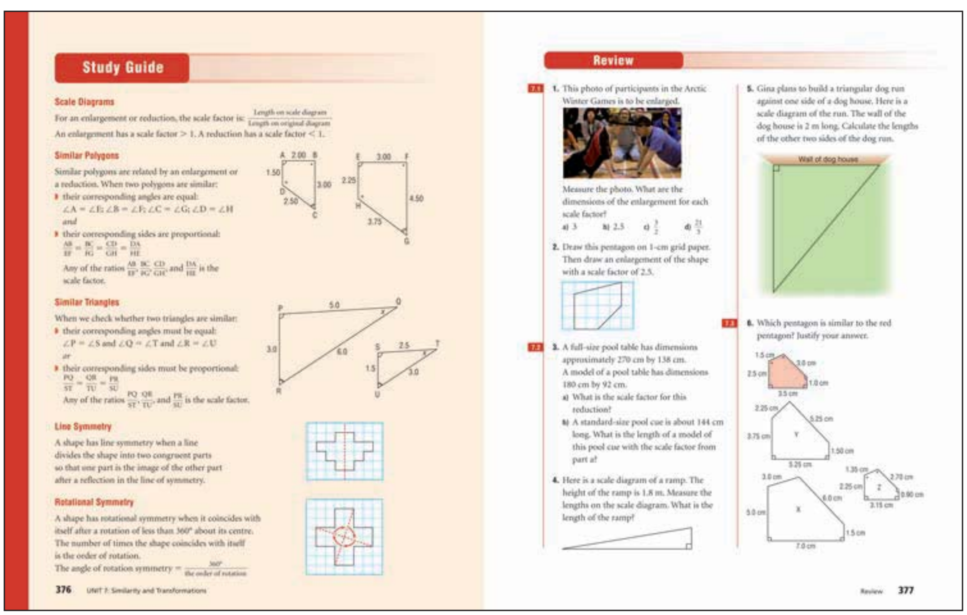
Reflect on the big ideas of the lesson. Think about your learning style and strategies.



Use the **Mid-Unit Review** to refresh your memory of key concepts.



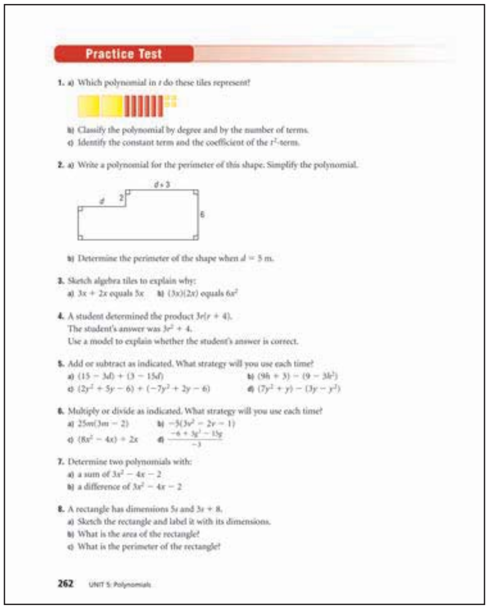
Start Where You Are illustrates strategies you may use to show your best performance.



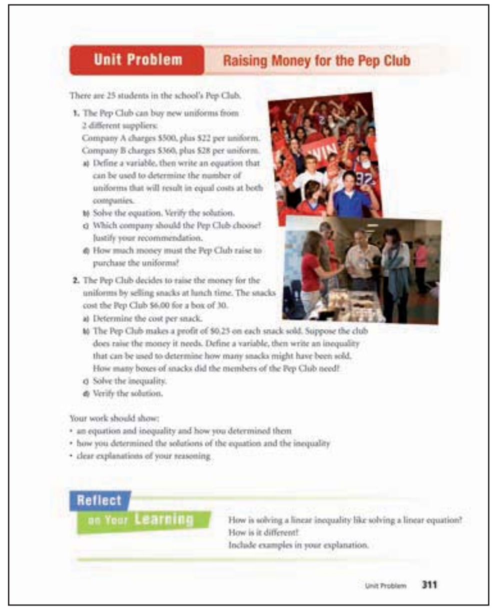
Study Guide summarizes key ideas from the unit.

Review questions allow you to find out if you are ready to move on.

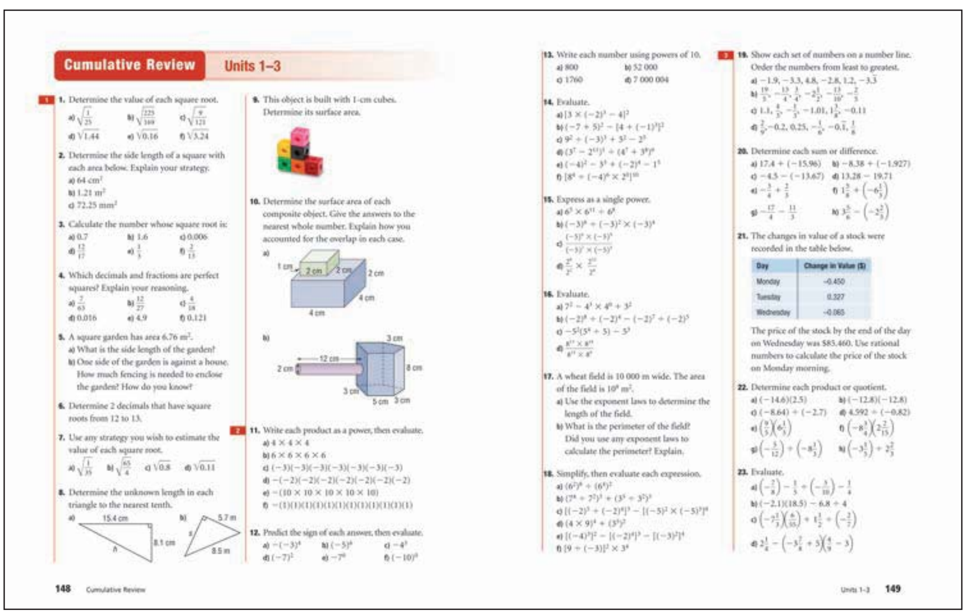
The *Practice and Homework Book* provides additional support.



The **Practice Test** models the kind of test your teacher might give.

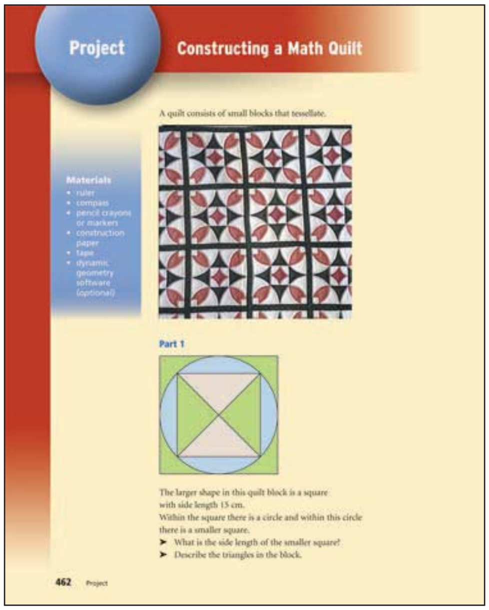


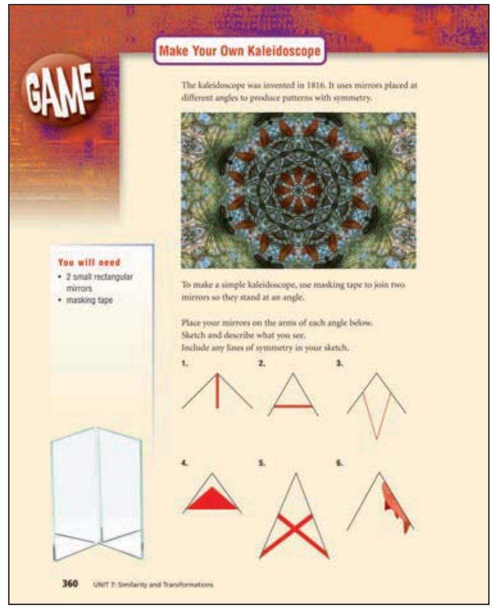
The **Unit Problem** presents problems to solve, or a project to do, using the math of the unit.



Keep your skills sharp with **Cumulative Review**.

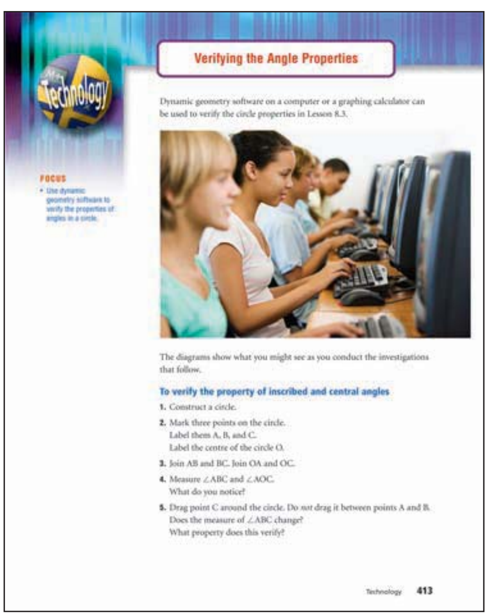
Explore some interesting math when you do the **Projects**.





Play a **Game** with your classmates or at home to reinforce your skills.

Icons remind you to use **technology**. Follow the instructions for using a computer or calculator to do math.



The **Illustrated Glossary** is a dictionary of important math words.

Project

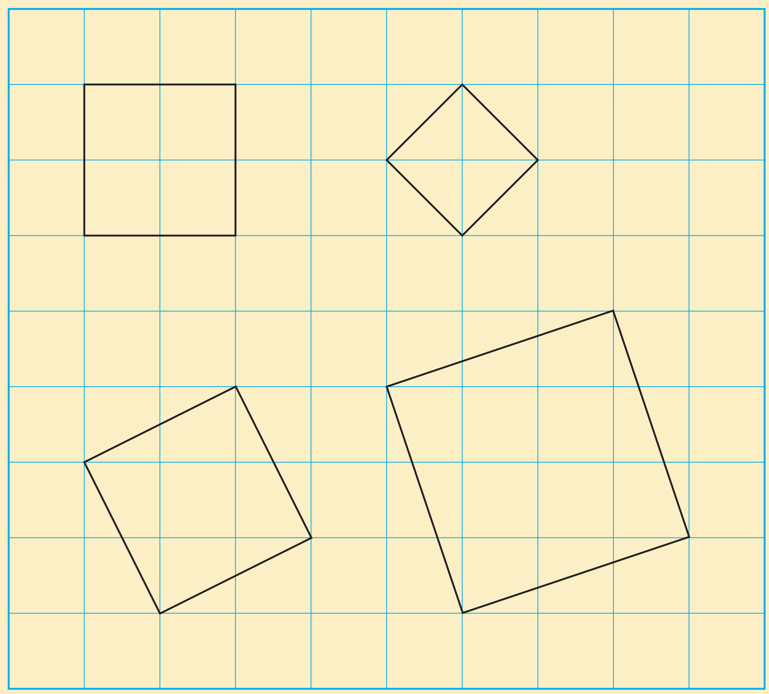
Making Squares into Cubes

Materials

- 1-cm grid paper
- wooden or plastic cubes
- ruler
- 1-cm grid card stock
- scissors
- tape

Part 1

➤ Copy the squares below onto 1-cm grid paper.



➤ Determine the area of each square and its side length.

➤ Describe your method and why you know it works.

➤ Compare methods with your classmates.
Write a description of a method that is different from your method.

Part 2

➤ Choose one of these areas:
32 cm^2, 40 cm^2, 45 cm^2
Draw a square with the area you chose.

➤ Label the area of your square.
Determine its side length.

- ➤ How could you fill your square with copies of one of the squares in Part 1? Use as many copies of one square in Part 1 as you like. Sketch what you find.
- ➤ How is the smaller square related to the larger square?

Part 3

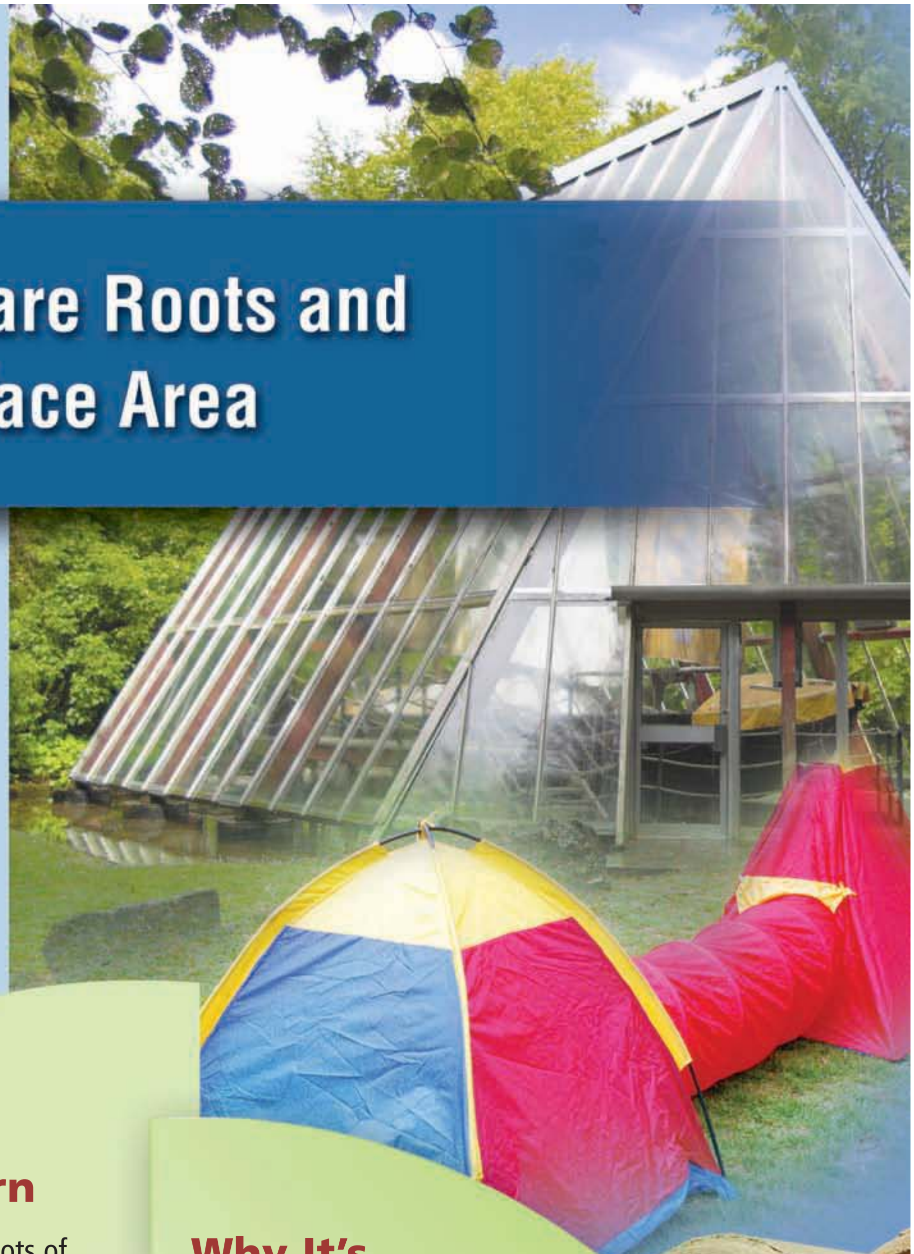
- ➤ Sketch a cube. Explain how you would determine the surface area of the cube. Share your strategy for determining the surface area with a classmate. Did both of you come up with the same strategy? If not, will both strategies work?
- ➤ Build a cube from 1-cm grid card stock. Each face of your cube should be the square you drew in Part 2. Use tape to assemble your cube.
- ➤ Draw a net of your cube.
- ➤ Calculate the surface area of your cube. Show your calculations.
- ➤ Describe how you would calculate the volume of your cube.
- ➤ Is the volume of your cube less than or greater than the volume of a cube with edge length 6 cm? Explain.

Take It Further

- ➤ Draw squares with these areas: 5 cm^2, 20 cm^2, 45 cm^2, 80 cm^2, and 125 cm^2 How are these squares related? What makes these squares a "family"?
- ➤ What other families of squares could you draw? Draw 3 squares from that family. Describe the family of squares. Determine the side length of each square.

UNIT 1

Square Roots and Surface Area



Which geometric objects can you name?

How could you determine their surface areas?

What You'll Learn

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and the amount of siding needed to cover a building

Key Words

- perfect square
- non-perfect square
- composite object

1.1 Square Roots of Perfect Squares

FOCUS

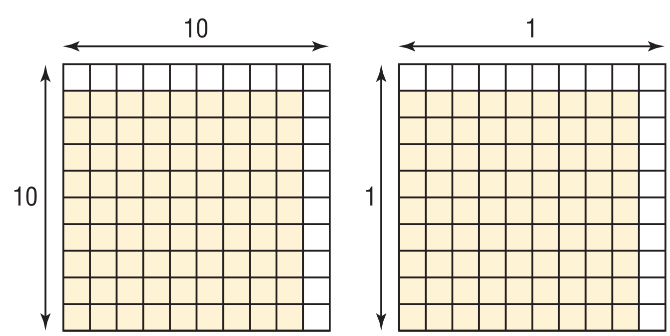
- Determine the square roots of decimals and fractions that are perfect squares.

A children's playground is a square with area 400 m^2.
What is the side length of the square?
How much fencing is needed to go around the playground?

Investigate

2

Each square below has been divided into 100 equal parts.
In each diagram, what is the area of one small square?



For the shaded square on the left:

➤ What is its area?
➤ Write this area as a product.
➤ How can you use a square root to relate the side length and area?

For the shaded square on the right:

➤ What is its area?
➤ Write this area as a product of fractions.
➤ How can you use a square root to relate the side length and area?

For the area of each square in the table:

➤ Write the area as a product.

➤ Write the side length as a square root.

Area as a Product	Side Length as a Square Root
$49 =$	
$\frac{49}{100} =$	
$64 =$	
$\frac{64}{100} =$	
$121 =$	
$\frac{121}{100} =$	
$144 =$	
$\frac{144}{100} =$	

Reflect & Share

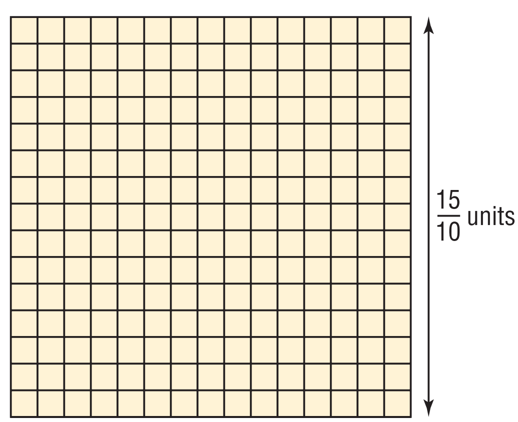
Compare your results with those of your classmates.
How can you use the square roots of whole numbers to determine the square roots of fractions?
Suppose each fraction in the table is written as a decimal.
How can you use the square roots of whole numbers to determine the square roots of decimals?

Connect

To determine the area of a square, we multiply the side length by itself.
That is, we *square* the side length.

$$\text{Area} = \left(\frac{15}{10}\right)^2$$
$$= \frac{15}{10} \times \frac{15}{10}$$
$$= \frac{225}{100}$$

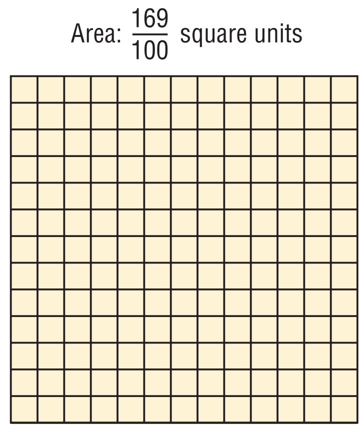
The area is $\frac{225}{100}$ square units.



To determine the side length of a square, we calculate the square root of its area.

$$\text{Side length} = \sqrt{\frac{169}{100}}$$
$$= \sqrt{\frac{13}{10} \times \frac{13}{10}}$$
$$= \frac{13}{10}$$

The side length is $\frac{13}{10}$ units.



Squaring and taking the square root are opposite, or inverse, operations.
The side length of a square is the square root of its area.

That is, $\sqrt{\frac{225}{100}} = \frac{15}{10}$ and $\sqrt{\frac{169}{100}} = \frac{13}{10}$

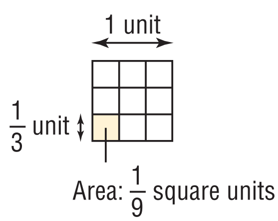
We can rewrite these equations using decimals:

$$\sqrt{2.25} = 1.5 \text{ and } \sqrt{1.69} = 1.3$$

1.5 and 1.3 are terminating decimals.

The square roots of some fractions are repeating decimals.
To determine the side length of the shaded square, take the square root of $\frac{1}{9}$:

$$\sqrt{\frac{1}{9}} = \sqrt{\frac{1}{3} \times \frac{1}{3}}$$
$$= \frac{1}{3}$$
$$= 0.333\ 333\ 333 \ldots$$
$$= 0.\overline{3}$$



When the area of a square is $\frac{1}{9}$ square units, its side length is $\frac{1}{3}$, or $0.\overline{3}$ of a unit.

A fraction in simplest form is a **perfect square** if it can be written as a product of two equal fractions.

When a decimal can be written as a fraction that is a perfect square, then the decimal is also a perfect square. The square root is a terminating or repeating decimal.

Example 1 Determining a Perfect Square Given its Square Root

Calculate the number whose square root is:

a) $\frac{3}{8}$ **b)** 1.8

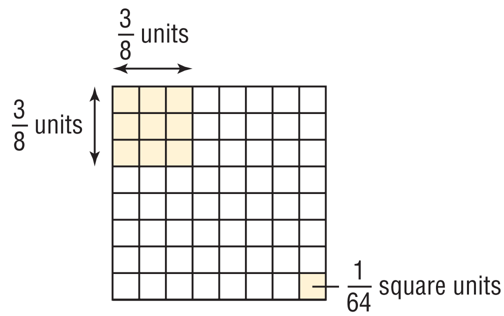
A Solution

a) Visualize $\frac{3}{8}$ as the side length of a square.

The area of the square is: $\left(\frac{3}{8}\right)^2 = \frac{3}{8} \times \frac{3}{8}$

$= \frac{9}{64}$

So, $\frac{3}{8}$ is a square root of $\frac{9}{64}$.

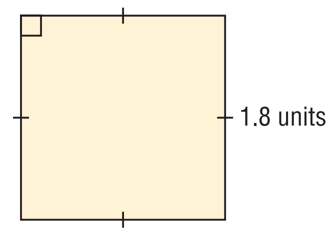


b) Visualize 1.8 as the side length of a square.

The area of the square is: $1.8^2 = 1.8 \times 1.8$

$= 3.24$

So, 1.8 is a square root of 3.24.



Example 2 Identifying Fractions that Are Perfect Squares

Is each fraction a perfect square? Explain your reasoning.

a) $\frac{8}{18}$ **b)** $\frac{16}{5}$ **c)** $\frac{2}{9}$

A Solution

a) $\frac{8}{18}$

Simplify the fraction first. Divide the numerator and denominator by 2.

$\frac{8}{18} = \frac{4}{9}$

Since $4 = 2 \times 2$ and $9 = 3 \times 3$, we can write:

$\frac{4}{9} = \frac{2}{3} \times \frac{2}{3}$

Since $\frac{4}{9}$ can be written as a product of two equal fractions, it is a perfect square.

So, $\frac{8}{18}$ is also a perfect square.

b) $\frac{16}{5}$

The fraction is in simplest form.
So, look for a fraction that when multiplied by itself gives $\frac{16}{5}$.

The numerator can be written as $16 = 4 \times 4$, but the denominator cannot be written as a product of equal factors.

So, $\frac{16}{5}$ is not a perfect square.

c) $\frac{2}{9}$

The fraction is in simplest form.
So, look for a fraction that when multiplied by itself gives $\frac{2}{9}$.

The denominator can be written as $9 = 3 \times 3$, but the numerator cannot be written as a product of equal factors.

So, $\frac{2}{9}$ is not a perfect square.

Example 3 Identifying Decimals that Are Perfect Squares

Is each decimal a perfect square? Explain your reasoning.

a) 6.25 **b)** 0.627

Solutions

Method 1

a) Write 6.25 as a fraction.

$6.25 = \frac{625}{100}$

Simplify the fraction. Divide the numerator and denominator by 25.

$6.25 = \frac{25}{4}$

$\frac{25}{4}$ can be written as $\frac{5}{2} \times \frac{5}{2}$.

So, $\frac{25}{4}$, or 6.25 is a perfect square.

b) Write 0.627 as a fraction.

$0.627 = \frac{627}{1000}$

This fraction is in simplest form.
Neither 627 nor 1000 can be written as a product of equal factors, so 0.627 is not a perfect square.

Method 2

Use a calculator.
Use the square root function.

a) $\sqrt{6.25} = 2.5$

The square root is a terminating decimal, so 6.25 is a perfect square.

b) $\sqrt{0.627} \doteq 0.791\ 833\ 316$

The square root appears to be a decimal that neither terminates nor repeats, so 0.627 is not a perfect square. To be sure, write the decimal as a fraction, then determine if the fraction is a perfect square, as shown in *Method 1*.

Discuss the ideas

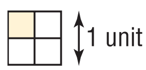
1. How can you tell if a decimal is a perfect square?
2. How can you tell if a fraction is a perfect square?

Practice

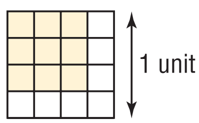
Check

3. Use each diagram to determine the value of the square root.

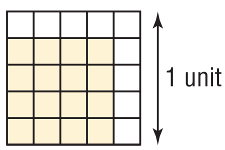
a) $\sqrt{0.25}$



b) $\sqrt{\frac{9}{16}}$



c) $\sqrt{\frac{16}{25}}$



4. a) List all the whole numbers from 1 to 100 that are perfect squares.
   b) Write a square root of each number you listed in part a.

5. Use your answers to question 4. Determine the value of each square root.
   a) $\sqrt{0.36}$ b) $\sqrt{0.49}$
   c) $\sqrt{0.81}$ d) $\sqrt{0.16}$
   e) $\sqrt{\frac{1}{36}}$ f) $\sqrt{\frac{25}{9}}$
   g) $\sqrt{\frac{64}{100}}$ h) $\sqrt{\frac{36}{16}}$

6. a) List all the whole numbers from 101 to 400 that are perfect squares.
   b) Write a square root of each number you listed in part a.

7. Use your answers to questions 4 and 6. Determine the value of each square root.
   a) $\sqrt{\frac{169}{16}}$ b) $\sqrt{\frac{400}{196}}$
   c) $\sqrt{\frac{256}{361}}$ d) $\sqrt{\frac{225}{289}}$
   e) $\sqrt{144}$ f) $\sqrt{0.0225}$
   g) $\sqrt{0.0121}$ h) $\sqrt{3.24}$
   i) $\sqrt{0.0324}$ j) $\sqrt{0.0169}$

Apply

8. Which decimals and fractions are perfect squares? Explain your reasoning.
   a) 0.12 b) 0.81 c) 0.25
   d) 1.69 e) $\frac{9}{12}$ f) $\frac{36}{81}$
   g) $\frac{81}{49}$ h) $\frac{75}{27}$ i) 0.081
   j) $\frac{25}{10}$ k) 2.5 l) $\frac{8}{50}$

9. Calculate the number whose square root is:
   a) 0.3 b) 0.12
   c) 1.9 d) 3.1
   e) $\frac{2}{3}$ f) $\frac{5}{6}$
   g) $\frac{1}{7}$ h) $\frac{2}{5}$

10. Determine the value of each square root.
   a) $\sqrt{12.25}$ b) $\sqrt{30.25}$
   c) $\sqrt{20.25}$ d) $\sqrt{56.25}$

11. a) Write each decimal as a fraction.
Which fractions are perfect squares?

i) 36.0 **ii)** 3.6 **iii)** 0.36
iv) 0.036 **v)** 0.0036 **vi)** 0.000 36

b) To check your answers to part a, use a calculator to determine a square root of each decimal.

c) What patterns do you see in your answers to parts a and b?

d) When can you use the square roots of perfect squares to determine the square roots of decimals?

12. a) Use the fact that $\sqrt{9} = 3$ to write the value of each square root.

i) $\sqrt{90\ 000}$ **ii)** $\sqrt{900}$
iii) $\sqrt{0.09}$ **iv)** $\sqrt{0.0009}$

b) Use the fact that $\sqrt{25} = 5$ to write the value of each square root.

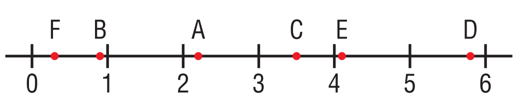
i) $\sqrt{0.0025}$ **ii)** $\sqrt{0.25}$
iii) $\sqrt{2500}$ **iv)** $\sqrt{250\ 000}$

c) Use the patterns in parts a and b. Choose a whole number whose square root you know. Use that number and its square root to write 3 decimals and their square roots. How do you know the square roots are correct?

13. Assessment Focus

a) Which letter on the number line below corresponds to each square root?
Justify your answers.

i) $\sqrt{12.25}$ **ii)** $\sqrt{\frac{121}{25}}$ **iii)** $\sqrt{16.81}$
iv) $\sqrt{\frac{81}{100}}$ **v)** $\sqrt{0.09}$ **vi)** $\sqrt{\frac{841}{25}}$



b) Sketch the number line in part a. Write 3 different decimals, then use the letters G, H, and J to represent their square roots. Place each letter on the number line. Justify its placement.

14. A square has area 5.76 cm^2.

a) What is the side length of the square?

b) What is the perimeter of the square?
How do you know?

15. A square piece of land has an area not less than 6.25 km^2 and not greater than 10.24 km^2.

a) What is the least possible side length of the square?

b) What is the greatest possible side length of the square?

c) A surveyor determined that the side length is 2.8 km. What is the area of the square?

16. A student said that $\sqrt{0.04} = 0.02$.
Is the student correct?
If your answer is yes, how could you check that the square root is correct?
If your answer is no, what is the correct square root? Justify your answer.

17. Look at the perfect squares you wrote for questions 4 and 6.
The numbers 36, 64, and 100 are related:
$36 + 64 = 100$, or $6^2 + 8^2 = 10^2$
These numbers form a *Pythagorean triple.*
a) Why do you think this name is appropriate?
b) How many other Pythagorean triples can you find? List each triple.

Take It Further

18. Are there any perfect squares between 0.64 and 0.81? Justify your answer.

19. A student has a rectangular piece of paper 7.2 cm by 1.8 cm. She cuts the paper into parts that can be rearranged and taped to form a square.
a) What is the side length of the square?
b) What are the fewest cuts the student could have made? Justify your answer.

Reflect

Explain the term *perfect square*. List some whole numbers, fractions, and decimals that are perfect squares. Determine a square root of each number.

Math Link

History

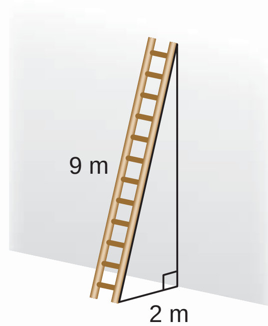
The Pythagorean Theorem is named for the Greek philosopher, Pythagoras, because he was the first person to record a proof for the theorem, around 540 BCE. However, clay tablets from around 1700 BCE show that the Babylonians knew how to calculate the length of the diagonal of a square. And, around 2000 BCE, it is believed that the Egyptians may have used a knotted rope that formed a triangle with side lengths 3, 4, and 5 to help design the pyramids.

1.2 Square Roots of Non-Perfect Squares

FOCUS

- Approximate the square roots of decimals and fractions that are non-perfect squares.

A ladder is leaning against a wall.
For safety, the distance from the base of a ladder to the wall must be about $\frac{1}{4}$ of the height up the wall.
How could you check if the ladder is safe?



Investigate

2

A ladder is 6.1 m long.
The distance from the base of the ladder to the wall is 1.5 m.
Estimate how far up the wall the ladder will reach.

Reflect & Share

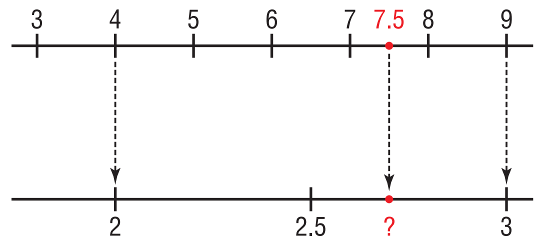
Compare your strategy for estimating the height with that of another pair of classmates. Did you use a scale drawing? Did you calculate?
Which method gives the closer estimate?

Connect

Many fractions and decimals are not perfect squares.
That is, they cannot be written as a product of two equal fractions.
A fraction or decimal that is not a perfect square is called a **non-perfect square**.

Here are two strategies for estimating a square root of a decimal that is a non-perfect square.

- Using benchmarks,
  To estimate $\sqrt{7.5}$, visualize a number line and the closest perfect square on each side of 7.5.
  $\sqrt{4} = 2$ and $\sqrt{9} = 3$
  7.5 is closer to 9 than to 4, so $\sqrt{7.5}$ is closer to 3 than to 2.



  From the diagram, an approximate value for $\sqrt{7.5}$ is 2.7.
  We write $\sqrt{7.5} \doteq 2.7$

- Using a calculator
  $\sqrt{7.5} \doteq 2.738\,612\,788$
  This decimal does not appear to terminate or repeat.
  There may be many more numbers after the decimal point that cannot be displayed on the calculator.
  To check, determine: $2.738\,612\,788^2 = 7.500\,000\,003$
  Since this number is not equal to 7.5, the square root is an approximation.

Example 1 illustrates 4 different strategies for determining the square root of a fraction that is a non-perfect square.

Example 1 Estimating a Square Root of a Fraction

Determine an approximate value of each square root.

a) $\sqrt{\frac{8}{5}}$ **b)** $\sqrt{\frac{3}{10}}$ **c)** $\sqrt{\frac{3}{7}}$ **d)** $\sqrt{\frac{19}{6}}$

A Solution

a) Use benchmarks. Think about the perfect squares closest to the numerator and denominator. In the fraction $\frac{8}{5}$, 8 is close to the perfect square 9, and 5 is close to the perfect square 4.

So, $\sqrt{\frac{8}{5}} \doteq \sqrt{\frac{9}{4}}$

$\sqrt{\frac{9}{4}} = \frac{3}{2}$

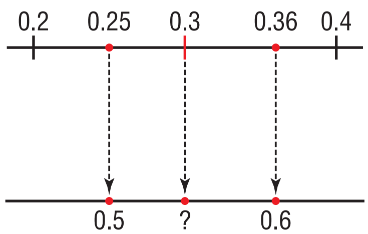
So, $\sqrt{\frac{8}{5}} \doteq \frac{3}{2}$

b) Write the fraction as a decimal, then think about benchmarks.

Write $\frac{3}{10}$ as a decimal: 0.3

Think of the closest perfect squares on either side of 0.3.

$\sqrt{0.25} = 0.5$ and $\sqrt{0.36} = 0.6$



0.3 is approximately halfway between 0.25 and 0.36, so choose 0.55 as a possible estimate for a square root.

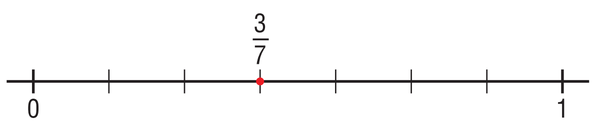
To check, evaluate:

$0.55^2 = 0.3025$

0.3025 is close to 0.3, so 0.55 is a reasonable estimate.

So, $\sqrt{\frac{3}{10}} \doteq 0.55$

c) Choose a fraction close to $\frac{3}{7}$ that is easier to work with.



$\frac{3}{7}$ is a little less than $\frac{1}{2}$.

$\frac{1}{2} = 0.5$

$\sqrt{0.5} \doteq \sqrt{0.49}$

And, $\sqrt{0.49} = 0.7$

So, $\sqrt{\frac{3}{7}} \doteq 0.7$

d) Use the square root function on a calculator.

$\sqrt{\frac{19}{6}} \doteq 1.779\ 513\ 042$

To the nearest hundredth, $\sqrt{\frac{19}{6}} \doteq 1.78$

Example 2 Finding a Number with a Square Root between Two Given Numbers

Identify a decimal that has a square root between 10 and 11. Check the answer.

Solutions

Method 1	Method 2
The number with a square root of 10 is: $10^2 = 100$ The number with a square root of 11 is: $11^2 = 121$ So, any number between 100 and 121 has a square root between 10 and 11. A decimal between 100 and 121 is 105.6. So, $\sqrt{105.6}$ is between 10 and 11. Use a calculator to check. $\sqrt{105.6} \doteq 10.276\ 186\ 06$ So, the decimal 105.6 is one correct answer.	One decimal between 10 and 11 is 10.4. To determine the number whose square root is 10.4, evaluate: $10.4^2 = 108.16$ So, $\sqrt{108.16}$ is between 10 and 11. Use a calculator to check. $\sqrt{108.16} = 10.4$ So, the decimal 108.16 is one correct answer.

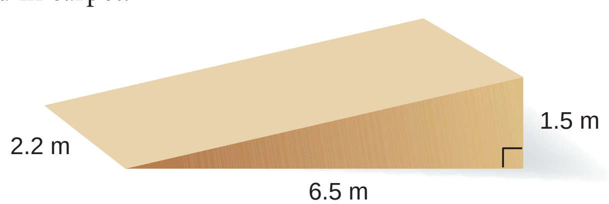
Example 3 Applying the Pythagorean Theorem

The sloping face of this ramp is to be covered in carpet.

a) Estimate the length of the ramp to the nearest tenth of a metre.

b) Use a calculator to check the answer.

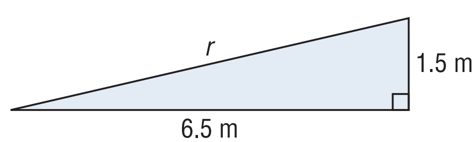
c) Calculate the area of carpet needed.



A Solution

a) The ramp is a right triangular prism with a base that is a right triangle.
The base of the prism is its side view.
To calculate the length of the ramp, r, use the Pythagorean Theorem.

$$r^2 = 6.5^2 + 1.5^2$$
$$= 42.25 + 2.25$$
$$= 44.5$$
$$r = \sqrt{44.5}$$



44.5 is between the perfect squares 36 and 49, and closer to 49.
So, $\sqrt{44.5}$ is between 6 and 7, and closer to 7.
Estimate $\sqrt{44.5}$ as 6.7.
To check, evaluate: $6.7^2 = 44.89$
This is very close to 44.5, so $r \doteq 6.7$
The ramp is about 6.7 m long.

Since the dimensions of the ramp were given to the nearest tenth, the answer is also written in this form.

b) Use a calculator to check: $\sqrt{44.5} \doteq 6.670\ 832\ 032$
This number is 6.7 to the nearest tenth, so the answer is correct.

c) The sloping face of the ramp is a rectangle with dimensions 6.7 m by 2.2 m.
The area of the rectangle is about: $6.7 \times 2.2 = 14.74$
Round the answer up to the nearest square metre to ensure there is enough carpet.
So, about 15 m^2 of carpet are needed.

Discuss the ideas

1. Explain the term *non-perfect square.*

2. Name 3 perfect squares and 3 non-perfect squares between the numbers 0 and 10. Justify your answers.

3. Why might the square root shown on a calculator be an approximation?

Practice

Check

4. For each square root, name the two closest perfect squares and their square roots.

a) $\sqrt{3.5}$ **b)** $\sqrt{13.5}$
c) $\sqrt{53.5}$ **d)** $\sqrt{73.5}$
e) $\sqrt{93.5}$ **f)** $\sqrt{113.5}$

5. For each square root, name the two closest perfect squares and their square roots.

a) $\sqrt{\frac{5}{10}}$ **b)** $\sqrt{\frac{55}{10}}$
c) $\sqrt{\frac{95}{10}}$ **d)** $\sqrt{\frac{595}{10}}$
e) $\sqrt{\frac{795}{10}}$ **f)** $\sqrt{\frac{1095}{10}}$

Apply

6. Use benchmarks to estimate a fraction for each square root.
State the benchmarks you used.

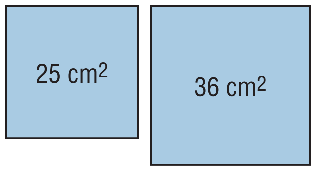
a) $\sqrt{\frac{8}{10}}$ **b)** $\sqrt{\frac{17}{5}}$
c) $\sqrt{\frac{7}{13}}$ **d)** $\sqrt{\frac{29}{6}}$

7. Use benchmarks to approximate each square root to the nearest tenth.
State the benchmarks you used.

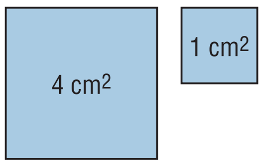
a) $\sqrt{4.5}$ **b)** $\sqrt{14.5}$
c) $\sqrt{84.5}$ **d)** $\sqrt{145.5}$
e) $\sqrt{284.5}$ **f)** $\sqrt{304.5}$

8. Use each pair of squares to approximate each square root.
Explain your strategy.

a) $\sqrt{29.5}$



b) $\sqrt{\frac{5}{2}}$



9. Which of the following square roots are correct to the nearest tenth?
How do you know? Correct the square roots that are incorrect.

a) $\sqrt{4.4} \doteq 2.2$ b) $\sqrt{0.6} \doteq 0.3$

c) $\sqrt{6.6} \doteq 2.6$ d) $\sqrt{0.4} \doteq 0.2$

10. Find 2 decimals that have square roots between each pair of numbers.
Justify your answers.

a) 3 and 4

b) 7 and 8

c) 12 and 13

d) 1.5 and 2.5

e) 4.5 and 5.5

11. Use any strategy you wish to estimate the value of each square root. Explain why you used the strategy you did.

a) $\sqrt{4.5}$ b) $\sqrt{\frac{17}{2}}$ c) $\sqrt{0.15}$ d) $\sqrt{\frac{10}{41}}$

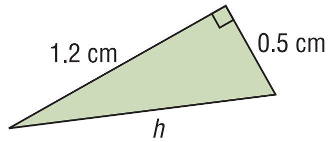
e) $\sqrt{0.7}$ f) $\sqrt{\frac{8}{45}}$ g) $\sqrt{0.05}$ h) $\sqrt{\frac{90}{19}}$

12. Approximate each square root to the nearest tenth. Explain your strategy.

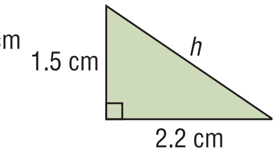
a) $\sqrt{\frac{3}{8}}$ b) $\sqrt{\frac{5}{12}}$ c) $\sqrt{\frac{13}{4}}$ d) $\sqrt{\frac{25}{3}}$

13. In each triangle, determine the unknown length.

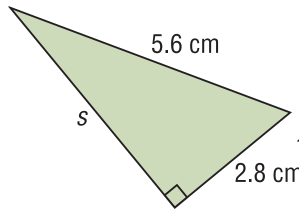
a)



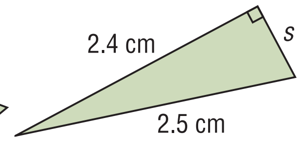
b)



c)



d)



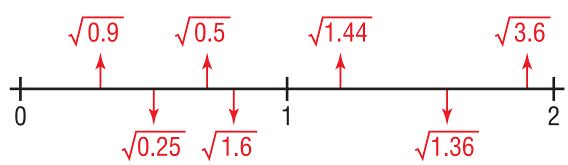
14. Assessment Focus How many decimals and fractions can you find with square roots between 0.5 and 0.6?
List the decimals and fractions.
Justify your answers. Show your work.

15. Sketch a number line from 0 to 10.
Place each square root on the number line to show its approximate value.

a) $\sqrt{0.1}$ b) $\sqrt{56.3}$

c) $\sqrt{0.6}$ d) $\sqrt{0.03}$

16. a) Which square roots are correctly placed on the number line below?
How do you know?



b) Sketch a number line from 0 to 2. On the number line, correctly place the square roots that were incorrectly placed in part a.

17. Use a calculator to determine each square root. Which square roots are approximate? How do you know?

a) $\sqrt{52.9}$ **b)** $\sqrt{5.29}$ **c)** $\sqrt{2.25}$ **d)** $\sqrt{22.5}$

18. Look at the numbers and their square roots you have determined in this lesson.
How would you describe the numbers whose square roots are:

a) less than the number?
b) equal to the number?
c) greater than the number?

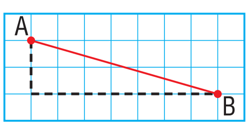
Justify your answer.

19. Determine a decimal or a fraction whose square root is between each pair of numbers.

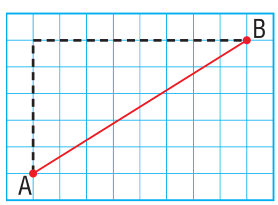
a) 0 and 1 **b)** 1.5 and 2
c) $\frac{1}{2}$ and $\frac{3}{4}$ **d)** $3\frac{3}{4}$ and 4

20. On each grid below, the side length of each square represents 0.25 km.
Determine the length of AB to the nearest hundredth of a kilometre.

a)



b)



21. a) Use a calculator to approximate each square root.

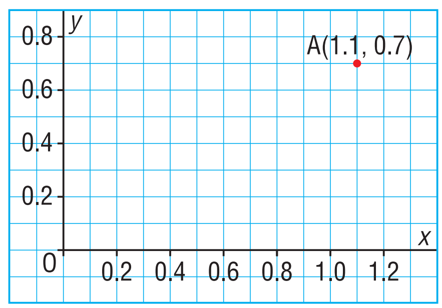
i) $\sqrt{0.005}$ **ii)** $\sqrt{0.5}$ **iii)** $\sqrt{50}$
iv) $\sqrt{5000}$ **v)** $\sqrt{500\ 000}$

b) What patterns do you see in the square roots in part a? Use the patterns to write the previous two square roots less than $\sqrt{0.005}$ and the next two square roots greater than $\sqrt{500\ 000}$.

Take It Further

22. Are there any square numbers between 0.6 and 0.61? How do you know?

23. The grid below shows point A(1.1, 0.7) that is one vertex of a square with area 0.25 square units. What are the coordinates of the other three vertices of the square? Justify your answer.



24. The side length of a square photograph is 5.5 cm. An enlargement of the photograph is a square with an area that is twice the area of the smaller photograph.

a) Estimate the side length of the larger photograph. Justify your answer.
b) Why is the side length of the larger photograph not twice the side length of the smaller photograph?

Reflect

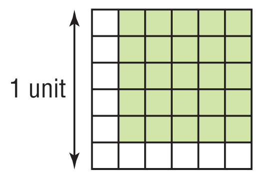
Explain why the square root of a non-perfect square displayed on a calculator is only an approximation. Include examples in your explanation.

Mid-Unit Review

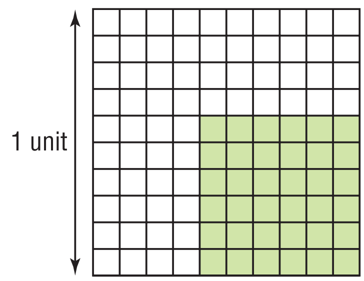
1.1

1. Explain how you can use each diagram to determine the square root.

a) $\sqrt{\frac{25}{36}}$



b) $\sqrt{0.36}$



2. Calculate the number whose square root is:

a) 1.4 **b)** $\frac{3}{8}$ **c)** $\frac{7}{4}$ **d)** 0.5

3. Determine the value of each square root.

a) $\sqrt{0.04}$ **b)** $\sqrt{\frac{1}{16}}$ **c)** $\sqrt{1.96}$ **d)** $\sqrt{\frac{4}{81}}$

e) $\sqrt{1.69}$ **f)** $\sqrt{\frac{121}{49}}$ **g)** $\sqrt{0.09}$ **h)** $\sqrt{\frac{289}{100}}$

4. Determine the value of each square root.

a) $\sqrt{3.24}$ **b)** $\sqrt{90.25}$ **c)** $\sqrt{2.56}$

5. A square has area 148.84 cm^2.

a) What is the side length of the square?

b) What is the perimeter of the square?

6. A student said that $\sqrt{0.16} = 0.04$.
Is the student correct?
If your answer is yes, how could you check that the square root is correct?
If your answer is no, explain how to get the correct square root.

7. Which decimals and fractions are perfect squares? Explain your reasoning.

a) $\frac{9}{64}$ **b)** 3.6 **c)** $\frac{6}{9}$ **d)** 5.76

1.2

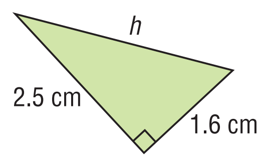
8. Use benchmarks to estimate each square root.

a) $\sqrt{5.6}$ **b)** $\sqrt{\frac{9}{10}}$ **c)** $\sqrt{42.8}$

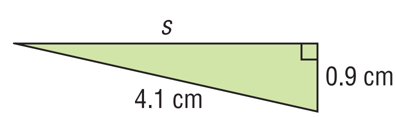
d) $\sqrt{\frac{356}{10}}$ **e)** $\sqrt{0.056}$ **f)** $\sqrt{\frac{9}{100}}$

9. In each triangle, determine the unknown length.

a)



b)



10. Which of the following square roots are correct to the nearest tenth?
How do you know? Correct the square roots that are incorrect.

a) $\sqrt{0.09} \doteq 0.3$ **b)** $\sqrt{1.7} \doteq 0.4$

c) $\sqrt{8.5} \doteq 2.9$ **d)** $\sqrt{27.5} \doteq 5.2$

11. Find 2 decimals that have square roots between each pair of numbers.
Justify your answers.

a) 4 and 8 **b)** 0.7 and 0.9

c) 1.25 and 1.35 **d)** 0.25 and 0.35

e) 4.5 and 5.5 **f)** 0.05 and 0.1

Start Where You Are

How Can I Begin?

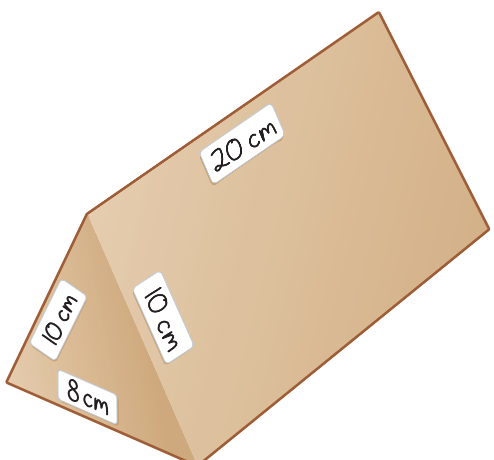
Suppose I have to solve this problem:

A right triangular prism is 20 cm long.
Each base is an isosceles triangle with side lengths 10 cm, 10 cm, and 8 cm.
What is the surface area of the prism?

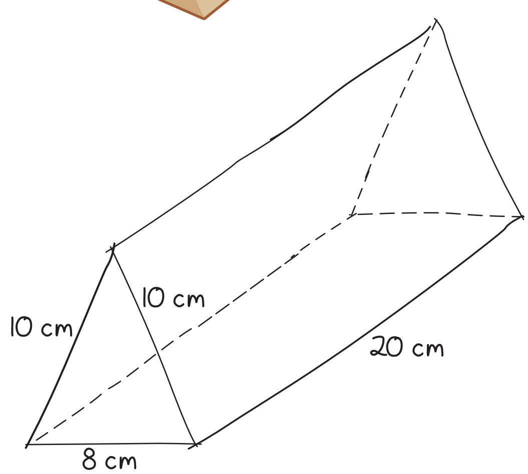
➤ What is my first step?

- I could use a model.
- I could sketch a diagram.
- I could visualize the prism in my mind.

If I use a model, I can place stickers on the prism to label its dimensions.



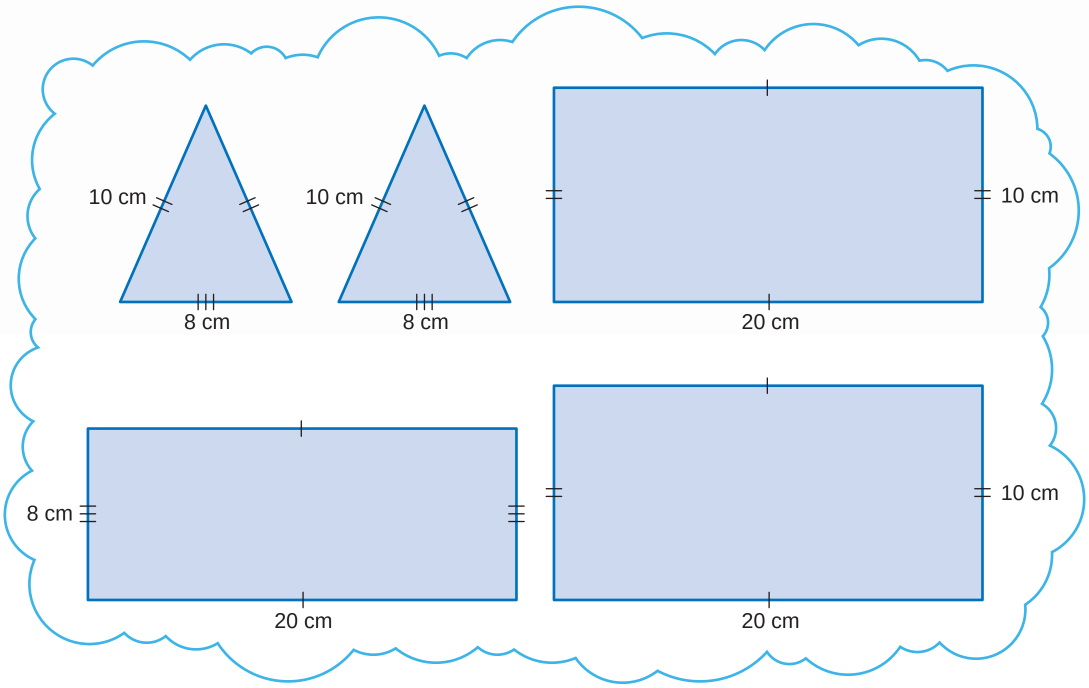
The model should have the shape of a triangular prism, but the dimensions of the prism do not have to match the given dimensions.



If I sketch a diagram, I label it with the given dimensions.

The diagram does not have to be drawn to scale.

If I visualize the prism, I picture its faces.



➤ What do I already know?

- a strategy to find the area of a rectangle
- a strategy to find the height of an isosceles triangle when the lengths of its sides are known
- a strategy to find the area of an isosceles triangle

Use strategies *you* know to find the surface area of the right triangular prism.

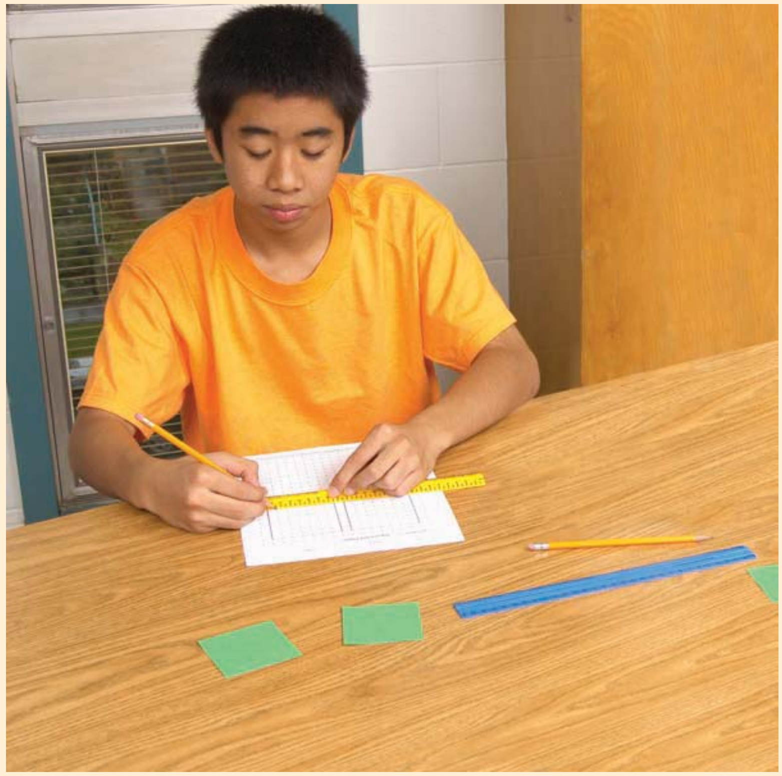
Check

1. A right triangular prism is 35 cm high. Its bases are equilateral triangles, with side lengths 12 cm. What is the surface area of the prism?

2. A right cylinder is 35 cm long. Its diameter is 12 cm. What is the surface area of the cylinder?

GAME

Making a Larger Square from Two Smaller Squares



You will need
- 2 congruent square pieces of paper
- scissors
- square dot paper

Number of Players
- 2

Goal of the Game
- To cut two congruent squares and rearrange the pieces to form one larger square

Before you cut the squares, sketch them on square dot paper.
Draw possible cuts you could make.
Imagine joining all the pieces with no overlap.
Do the pieces form a larger square?

Check your prediction by cutting the squares and arranging the pieces to form a larger square. If your prediction did not work, try again using another two congruent squares.

Share your method with another pair of students.
Are there other possible ways of forming the larger square?
How could you do this by making the fewest cuts possible?

Suppose the area of each congruent square is 1 square unit.
- What is the area of the larger square?
- What is the side length of the larger square, to the nearest tenth?

Suppose the area of each congruent square is 2 square units.
Determine the area and side length of the larger square.

1.3 Surface Areas of Objects Made from Right Rectangular Prisms

FOCUS

- Determine the surface areas of composite objects made from cubes and other right rectangular prisms.

These cube houses were built in Rotterdam, Netherlands.
Suppose you wanted to determine the surface area of one of these houses.
What would you need to know?

Investigate

2

Each of you needs 5 linking cubes.
Assume each face of a linking cube has area 1 unit^2.

➤ What is the surface area of 1 cube?
Put 2 cubes together to make a "train."
What is the surface area of the train?
Place another cube at one end of your train.
What is its surface area now?
Continue to place cubes at one end of the train, and determine its surface area.
Copy and complete this table.

Number of Cubes	Surface Area (square units)
1	
2	
3	
4	
5	

What patterns do you see in the table?
What happens to the surface area each time you place another cube on the train?
Explain why the surface area changes this way.

➤ With the 5 cubes, build an object that is different from the train and different from your partner's object.
Determine its surface area.
Compare the surface area of your object with that of your partner's object.

Reflect & Share

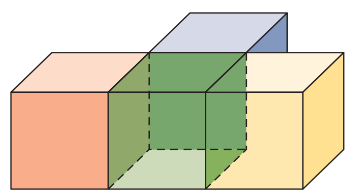
Compare your objects with those of another pair of students who made different objects. Are any of the surface areas different?
If your answer is yes, explain how they can be different when all the objects are made with 5 cubes.

Connect

Here is an object made from 4 unit cubes.
Each face of a cube is a square with area 1 unit^2.

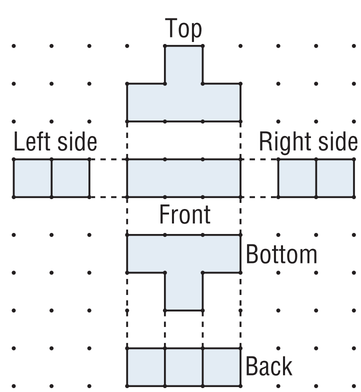
Here are 2 strategies for determining the surface area of the object.

➤ Count the square faces of all the cubes, then subtract 2 faces for each surface where the cubes are joined.
We say the faces *overlap*.



The object has 4 cubes. Each cube has 6 faces.
So, the number of faces is: $6 \times 4 = 24$
There are 3 places where the faces overlap,
so subtract: 3×2, or 6 faces
The surface area, in square units, is: $24 - 6 = 18$

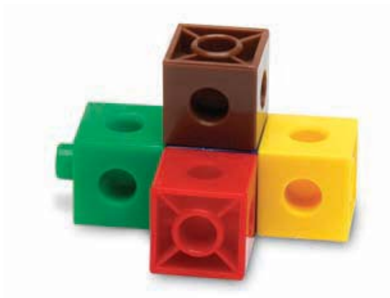
➤ Count the squares on each of the 6 views.
There are:
4 squares on the top,
4 squares on the bottom,
3 squares on the front,
3 squares on the back,
2 squares at the right,
and 2 squares at the left.
The surface area, in square units, is:
$4 + 4 + 3 + 3 + 2 + 2 = 18$



An object like that on page 26 is called a **composite object** because it is made up, or *composed*, of other objects.

Example 1 Determining the Surface Area of a Composite Object Made from Cubes

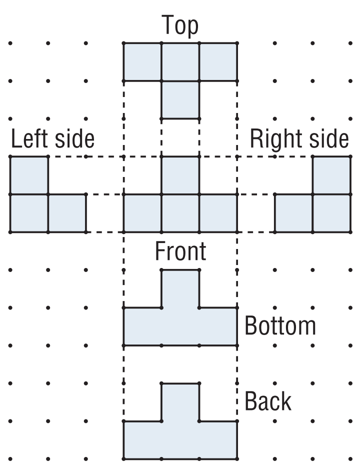
Determine the surface area of this composite object.
Each cube has edge length 2 cm.



Solutions

Method 1

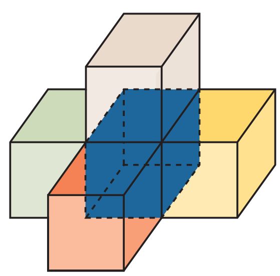
Count the squares on each of the 6 views:



Each of the front, back, top, and bottom views has 4 squares.
Each of the right and left views has 3 squares.
The surface area, in squares, is:
$(4 \times 4) + (3 \times 2) = 22$
Each square has area: $2 \text{ cm} \times 2 \text{ cm} = 4 \text{ cm}^2$
So, the surface area is: $22 \times 4 \text{ cm}^2 = 88 \text{ cm}^2$

Method 2

The composite object has 5 cubes.
Each cube has 6 square faces.
So, the total number of squares is: $5 \times 6 = 30$



The cubes overlap at 4 places,
so there are 4×2, or 8 squares
that are not part of the surface area.
The surface area, in squares, is: $30 - 8 = 22$
Each square has area: $2 \text{ cm} \times 2 \text{ cm} = 4 \text{ cm}^2$
So, the surface area is: $22 \times 4 \text{ cm}^2 = 88 \text{ cm}^2$

We can use the surface area of composite objects to solve problems outside the classroom.

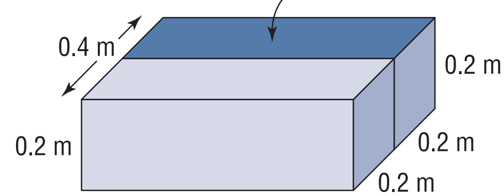
Example 2 Determining the Surface Area of a Composite Object Made from Right Rectangular Prisms

Renee uses 3 pieces of foam to make this chair.
Each piece of foam is a right rectangular prism with dimensions 60 cm by 20 cm by 20 cm.
Can Renee cover the chair with 2 m^2 of fabric? Explain.

A Solution

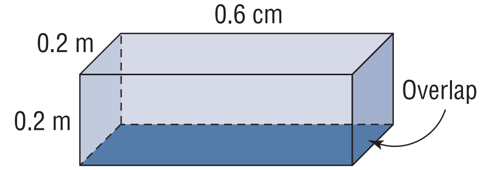
Convert each measurement to metres, then the surface area is measured in square metres.
60 cm = 0.6 m 20 cm = 0.2 m

Determine the surface area of the rectangular prism that is the base of the chair.



Area of top and bottom faces: $2(0.6 \times 0.4) = 0.48$
Area of front and back faces: $2(0.6 \times 0.2) = 0.24$
Area of left and right faces: $2(0.2 \times 0.4) = 0.16$
Surface area of the base of the chair:
$0.48 + 0.24 + 0.16 = 0.88$

Determine the surface area of the rectangular prism that is the back rest.



Area of top, bottom, front, and back:
$4(0.6 \times 0.2) = 0.48$
Area of left and right faces: $2(0.2 \times 0.2) = 0.08$
Surface area of back rest: $0.48 + 0.08 = 0.56$

Add the two surface areas, then subtract twice the area of the overlap because neither of these areas is part of the surface area of the chair:

$$0.88 + 0.56 - 2(0.6 \times 0.2) = 1.44 - 0.24$$
$$= 1.2$$

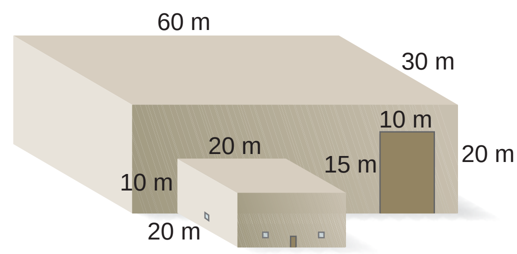
The surface area that is to be covered in fabric is 1.2 m^2.
Since 2 m^2 > 1.2 m^2, Renee can cover the chair with 2 m^2 of fabric.

Example 3 Solving Problems Involving the Surface Area of a Composite Object

A warehouse measures 60 m by 30 m by 20 m. An office attached to one wall of the warehouse measures 20 m by 20 m by 10 m.

a) Determine the surface area of the building.

b) A contractor quotes to paint the exterior of the building at a rate of $2.50/m^2.
These parts of the building are not to be painted: the 2 roofs; the office door with area 2 m^2; 3 loading doors, each measuring 10 m by 15 m; and 4 windows on the office, each with area 1 m^2.
How much would it cost to paint the building?



A Solution

The surface area is measured in square metres.

a) The 4 walls and roof of the warehouse form its surface area.
Area of roof: $60 \times 30 = 1800$
Area of left and right side walls: $2(30 \times 20) = 1200$
Area of the front and back walls: $2(60 \times 20) = 2400$
So, the surface area of the warehouse is: $1800 + 1200 + 2400 = 5400$

The 3 walls and roof of the office form its surface area.
Area of roof: $20 \times 20 = 400$
Area of front, left, and right side walls: $3(20 \times 10) = 600$
So, the surface area of the office is: $400 + 600 = 1000$

For the surface area of the building, add the surface areas of the warehouse and the office, then subtract the area of the overlap.
The area of the overlap, which is the back of the office, is: $20 \times 10 = 200$
So, the surface area of the building is: $5400 \text{ m}^2 + 1000 \text{ m}^2 - 200 \text{ m}^2 = 6200 \text{ m}^2$

b) To calculate the area to be painted, subtract the areas of the roofs, doors, and windows from the surface area of the building.
Area of roofs: $1800 + 400 = 2200$
Area of loading doors: $3(10 \times 15) = 450$
Area of office door and windows: $2 + 4(1) = 6$
So, the area to be painted is: $6200 \text{ m}^2 - 2200 \text{ m}^2 - 450 \text{ m}^2 - 6 \text{ m}^2 = 3544 \text{ m}^2$
The cost to paint the building is: $3544 \times \$2.50 = \$8860.00$

Discuss the ideas

1. When a composite object is made from right rectangular prisms, why is the surface area of the object not the sum of the surface areas of the individual prisms?

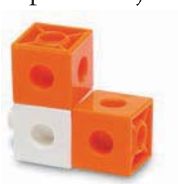
2. The surface area of an object is the area of a net of the object. How would drawing a net help you determine the surface area of a composite object?

3. In *Example 3*, why are the bases of the warehouse and office not included in the surface area?

Practice

Check

4. Make each composite object with cubes. Assume each face of a cube has area 1 unit^2. Determine the surface area of each composite object.

a) 

b) 

c) 

d) 

e) 

f) 

Apply

5. These are 1-cm cubes.

a) Determine the surface area of the composite object formed by placing cube 4 on top of each indicated cube.
   i) cube 1 ii) cube 2 iii) cube 3

b) Why are the surface areas in part a equal?

6. These are 1-cm cubes.

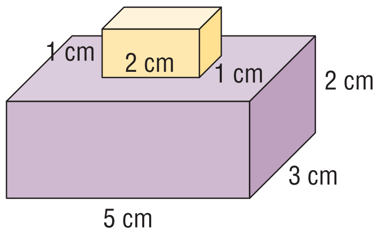
a) Determine the surface area of the composite object formed by placing cube 5 on top of each indicated cube.
   i) cube 1 ii) cube 2 iii) cube 3

b) Why are all the surface areas in part a not equal?

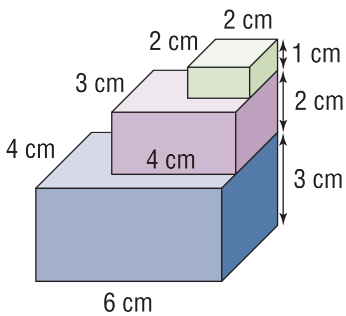
7. Why could you not use 6 views to determine the surface area of this composite object?

8. Determine the surface area of each composite object.
What effect does the overlap have on the calculation of the surface area?

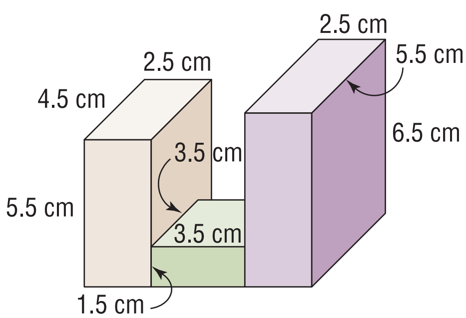
a)



b)



c)

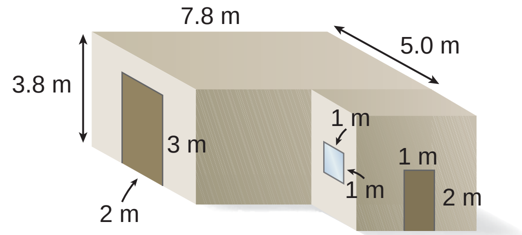


9. Work with a partner. Tape a tissue box on a shoebox to form a composite object.

a) What is the area of the overlap?
How did you calculate it?

b) Determine the surface area of the object.
How did you use the area of the overlap in your calculation?

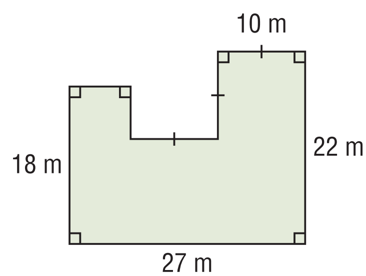
10. Assessment Focus A garage has the dimensions shown. The attached shed has the same height as the garage, but is one-half as long and one-half as wide.



a) What is the surface area of the building?

b) Vinyl siding costs $\$15/\text{m}^2$. The doors, windows, and roof will not be covered with siding. How much will it cost to cover this building with siding?

11. This is a floor plan of a building that is 8 m tall. It has a flat roof. What is the surface area of the building, including its roof?



12. Use 27 small cubes to build a large cube.

a) Determine and record its surface area.

b) How many ways can you remove one cube without changing the surface area? Explain your work.

c) Suppose you painted the large cube. How many small cubes would have paint on:

i) exactly 1 face? **ii)** exactly 2 faces?
iii) exactly 3 faces? **iv)** 0 faces?
v) more than 3 faces?

How could you check your answers?

13. Every January, the Ice Magic Festival is held at Chateau Lake Louise in Banff National Park. An ice castle is constructed from huge blocks of ice.

a) Suppose you have 30 blocks of ice measuring 25 cm by 50 cm by 100 cm. Sketch a castle with no roof that could be built with some or all of these blocks.

b) Determine the surface area of your castle, inside and out.

Take It Further

14. Use 6 centimetre cubes.

a) Build a composite object. Sketch the object, then determine and record its surface area.

b) Use the cubes to build other objects with different surface areas. Sketch each object and record its surface area.

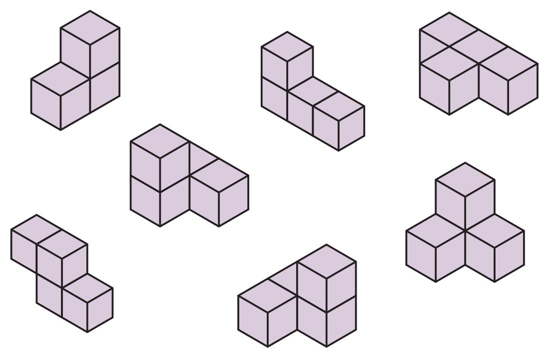
c) Determine all the different surface areas for a composite object of 6 cubes.

d) Describe the object with the greatest surface area. Describe the object with the least surface area.

15. Use centimetre cubes. Build, then sketch all possible composite objects that have a surface area of 16 cm^2.

16. A pyramid-like structure is made with 1-m^3 wooden cubes. The bottom layer of the structure is a rectangular prism with a square base and a volume of 25 m^3. The next layer has a volume of 16 m^3. The pattern of layers continues until the top layer, which has a volume of 1 m^3. Determine the surface area of the structure. Describe any patterns you find.

17. The SOMA Puzzle was invented by a Danish poet and scientist named Piet Hein in 1936. The object of the puzzle is to arrange these 7 pieces to form one large cube:



a) Determine the surface area of each piece.

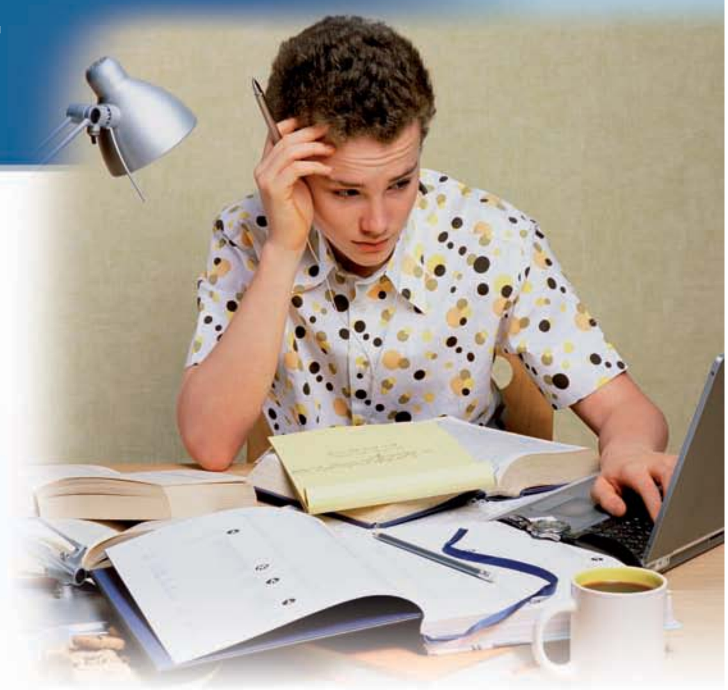
b) Use linking cubes to make your own pieces and arrange them to form a large cube.

c) Suppose you painted the large cube. How many faces of the original 7 pieces would not be painted? How do you know?

Reflect

Why is it important to consider the areas of overlap when determining the surface area of a composite object? Include an example in your explanation.

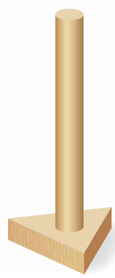
1.4 Surface Areas of Other Composite Objects



FOCUS

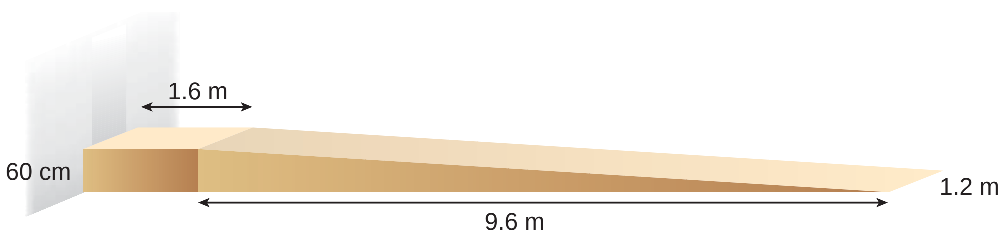
- Determine the surface areas of composite objects made from right prisms and right cylinders.

A student designed this stand for a table lamp. How could the student determine the surface area of this stand? What would he need to know?



Investigate

To meet safety regulations, a wheelchair ramp must be followed by a landing.
This wheelchair ramp and landing lead into an office building.
Calculate the surface area of the ramp and landing.



Reflect & Share

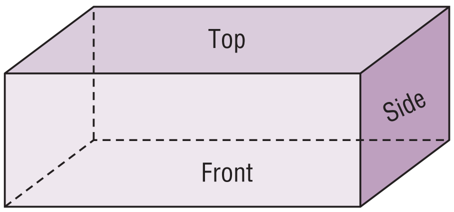
What strategies did you use to determine the surface area?
What assumptions did you make?
Compare your strategy and calculations with those of another pair of students.
How many different ways can you determine the surface area? Explain.

Connect

We use the strategies from Lesson 1.3 to determine the surface area of a composite object made from right cylinders and right triangular prisms. That is, consider each prism or cylinder separately, add their surface areas, then account for the overlap.

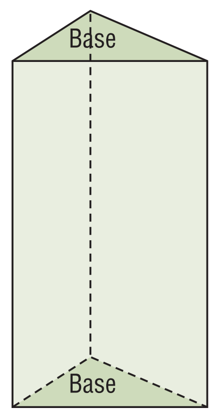
For composite objects involving right prisms, we can use word formulas to determine the surface areas of the prisms.

➤ A right rectangular prism has 3 pairs of congruent faces:
- the top and bottom faces
- the front and back faces
- the left side and right side faces



The surface area is the sum of the areas of the faces:
Surface area = 2 × area of top face + 2 × area of front face + 2 × area of side face

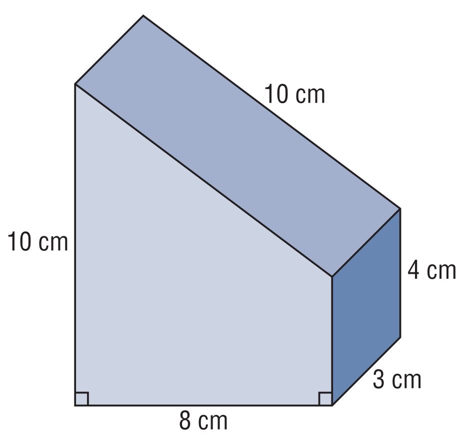
➤ A right triangular prism has 5 faces:
- 2 congruent triangular bases
- 3 rectangular faces



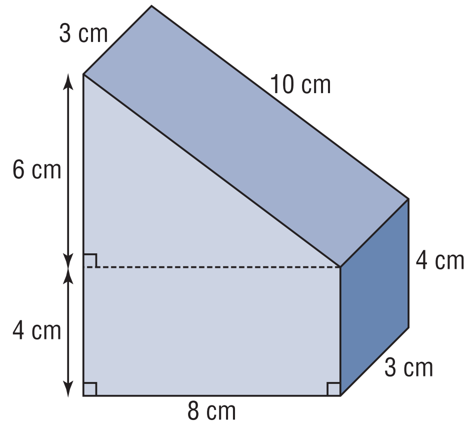
The surface area is the sum of the areas of the triangular bases and the rectangular faces:
Surface area = 2 × area of base + areas of 3 rectangular faces

Example 1 Determining the Surface Area of a Composite Object Made from Two Prisms

Determine the surface area of this object.

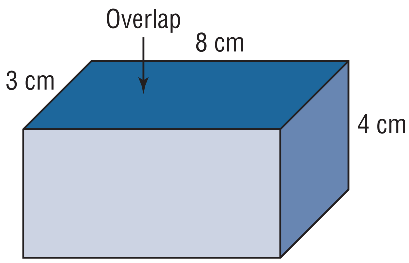


A Solution



The object is composed of a right triangular prism on top of a right rectangular prism. The surface area is measured in square centimetres.

For the surface area of the rectangular prism:



Surface area $= 2 \times$ area of top face $+ 2 \times$ area of front face $+ 2 \times$ area of side face

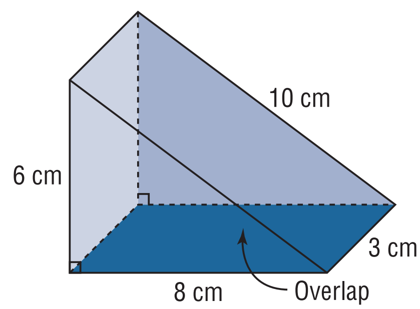
$= (2 \times 8 \times 3) + (2 \times 8 \times 4) + (2 \times 3 \times 4)$ Use the order of operations.

$= 48 + 64 + 24$

$= 136$

The surface area of the right rectangular prism is 136 cm^2.

For the surface area of the triangular prism:
Each base of the prism is a right triangle, with base 8 cm and height 6 cm.



Surface area $= 2 \times$ area of base $+$ areas of 3 rectangular faces

$= (2 \times \frac{1}{2} \times 8 \times 6) + (3 \times 6) + (3 \times 8) + (3 \times 10)$ Use the fact that $2 \times \frac{1}{2} = 1$.

$= (1 \times 8 \times 6) + (3 \times 6) + (3 \times 8) + (3 \times 10)$ Use the order of operations.

$= 48 + 18 + 24 + 30$

$= 120$

The surface area of the right triangular prism is 120 cm^2.

Add the two surface areas, then subtract twice the area of the overlap.

Surface area $= 136 + 120 - (2 \times 8 \times 3)$

$= 136 + 120 - 48$

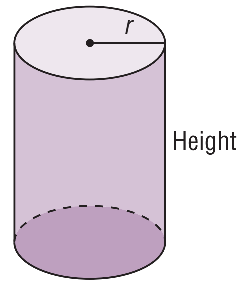
$= 208$

The surface area of the object is 208 cm^2.

When a composite object includes a right cylinder, we can use a formula to determine its surface area.
A cylinder has 2 congruent bases and a curved surface.
Each base is a circle, with radius r and area πr^2.
The curved surface is formed from a rectangle with:

- one side equal to the circumference of the circular base, and
- one side equal to the height of the cylinder

The circumference of the circular base is $2\pi r$.



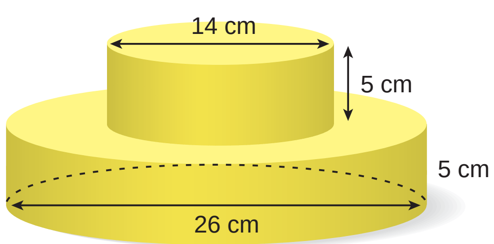
Surface area = area of two circular bases + curved surface area
= 2 × area of one circular base + circumference of base × height of cylinder
$= 2 \times \pi r^2 + 2\pi r \times$ height

Sometimes, one base of the cylinder is not included in the surface area calculation because the cylinder is sitting on its base. Then,

Surface area = area of one base + circumference of base × height of cylinder
$= \pi r^2 + 2\pi r \times$ height

Example 2 Determining the Surface Area of a Composite Object Made from Two Cylinders

Two round cakes have diameters of 14 cm and 26 cm, and are 5 cm tall.
They are arranged as shown. The cakes are covered in frosting. What is the area of frosting?



Solutions

Method 1

Calculate the surface area of each cake.
Do not include the base it sits on because this will not be frosted.
The surface area is measured in square centimetres.

For the smaller cake:

The diameter is 14 cm, so the radius, r, is 7 cm. The height is 5 cm.

$$\begin{aligned}\text{Surface area} &= \text{area of one base} + \text{circumference of base} \times \text{height of cylinder}\\ &= \pi r^2 + 2\pi r \times \text{height}\\ &= (\pi \times 7^2) + (2 \times \pi \times 7 \times 5) \quad \text{Use a calculator and the order of operations.}\\ &\doteq 373.85\end{aligned}$$

For the larger cake:

The diameter is 26 cm, so the radius, r, is 13 cm. The height is 5 cm.

$$\begin{aligned}\text{Surface area} &= \text{area of one base} + \text{circumference of base} \times \text{height of cylinder}\\ &= \pi r^2 + 2\pi r \times \text{height}\\ &= (\pi \times 13^2) + (2 \times \pi \times 13 \times 5) \quad \text{Use a calculator.}\\ &\doteq 939.34\end{aligned}$$

To calculate the area of frosting, add the two surface areas, then subtract the area of the overlap; that is, the area of the base of the smaller cake: $\pi \times 7^2$

$$\begin{aligned}\text{Area of frosting} &\doteq 373.85 + 939.34 - (\pi \times 7^2) \quad \text{Use a calculator.}\\ &\doteq 1159.25\end{aligned}$$

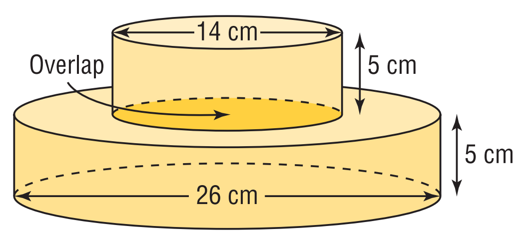
The area of frosting is about 1159 cm^2.

Since the dimensions were given to the nearest centimetre, the surface area is given to the nearest square centimetre.

Method 2

Calculate the surface area directly.

The overlap is the area of the base of the smaller cake. So, instead of calculating the area of the top of the smaller cake, then subtracting that area as the overlap, we calculate only the curved surface area of the smaller cake.



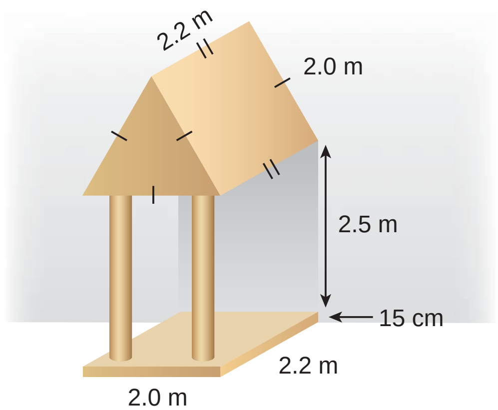
$$\begin{aligned}\text{Area of frosting} &= \text{curved surface area of smaller cake}\\ &\quad + \text{surface area of larger cake, without one base}\\ &= (2 \times \pi \times 7 \times 5) + [(\pi \times 13^2) + (2 \times \pi \times 13 \times 5)] \quad \text{Use a calculator.}\\ &\doteq 1159.25\end{aligned}$$

The area of frosting is about 1159 cm^2.

When some of the lengths on a right triangular prism are not given, we may need to use the Pythagorean Theorem to calculate them.

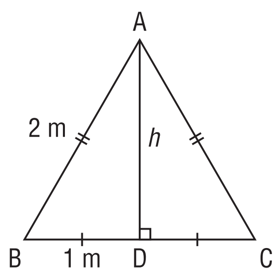
Example 3 Using the Pythagorean Theorem in Surface Area Calculations

The roof, columns, and base of this porch are to be painted.
The radius of each column is 20 cm.
What is the area to be painted, to the nearest square metre?



A Solution

The roof is a triangular prism with its base an equilateral triangle.
To determine the area of the triangular base, we need to know the height of the triangle.
Let the height of the triangle be h.



The height, AD, bisects the base, BC.
Use the Pythagorean Theorem in ΔABD.

$h^2 + 1^2 = 2^2$

$h^2 + 1 = 4$ Solve for h^2.

$h^2 = 4 - 1$

$= 3$

$h = \sqrt{3}$ Determine the square root.

$\doteq 1.732$

The height of the equilateral triangle is about 1.7 m.

Since one base of the triangular prism is against the house, it will not be painted.
The rectangular faces are congruent because they have the same length and width.

So, for the roof:

Surface area = area of one triangular base + areas of 3 congruent rectangular faces

$\doteq \left(\frac{1}{2} \times 2.0 \times 1.732\right) + [3 \times (2.0 \times 2.2)]$

$= 1.732 + 13.2$

$= 14.932$

The base of the porch is a right rectangular prism with only the front, top, and 2 side faces to be painted. The units must match, so convert 15 cm to 0.15 m.

Surface area = area of front face + area of top face + 2 × area of side face

$= (2.0 \times 0.15) + (2.0 \times 2.2) + [2 \times (2.2 \times 0.15)]$

$= 0.3 + 4.4 + 0.66$

$= 5.36$

The two columns are cylinders. Only the curved surfaces need to be painted. The radius is 20 cm, which is 0.2 m.

Surface area = 2 × (circumference of base × height of cylinder)

$= 2 \times (2\pi r \times \text{height})$

$= 2 \times (2 \times \pi \times 0.2 \times 2.5)$

$\doteq 6.283$

To calculate the area to be painted, add the surface areas of the roof, base, and columns, then subtract the area of overlap at the top and bottom of the columns. The area of overlap is 4 times the area of the base of one column.

The area of each circular base is:

$\pi r^2 = \pi \times 0.2^2$

$\doteq 0.126$

Surface area = area of roof + area of base + area of cylinders − 4 × area of circular base of column

$\doteq 14.932 + 5.36 + 6.283 - (4 \times 0.126)$

$= 26.071$

The area to be painted is about 26 m^2.

Discuss the ideas

1. What can you use to calculate an unknown length when the base of a right prism is a right triangle? Explain why.

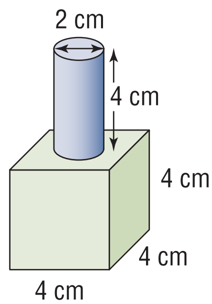
2. When do you think it is not helpful to draw a net to calculate the surface area of a composite object?

Practice

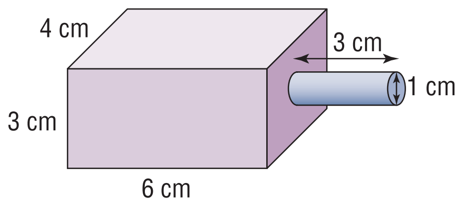
Check

3. Determine the surface area of each composite object. Give the answers to the nearest whole number.

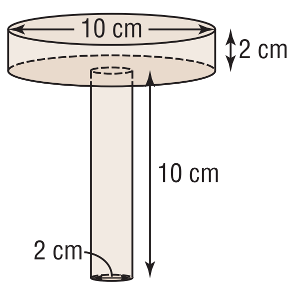
a) cylinder on a cube



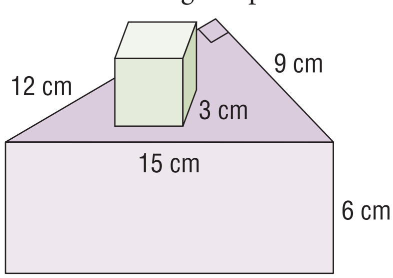
b) cylinder on a rectangular prism



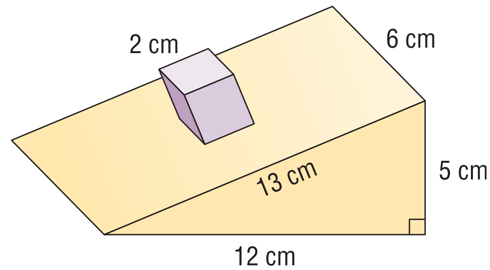
c) cylinder on a cylinder



d) cube on a triangular prism

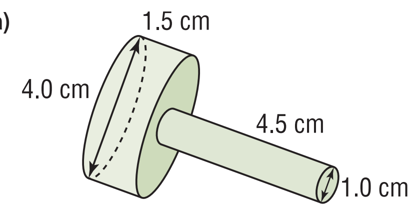


e) cube on a triangular prism

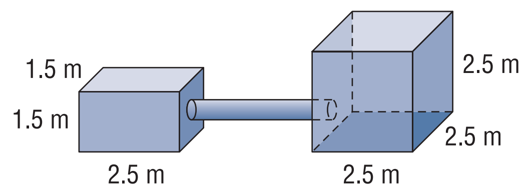


4. Determine the surface area of each composite object. Give the answers to the nearest tenth.

a)

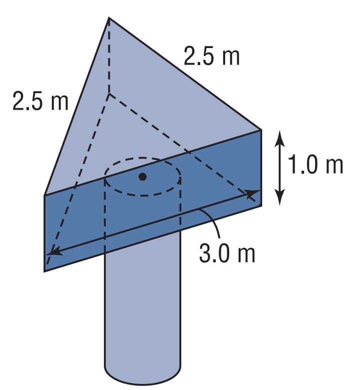


b) The cylinder is 3.5 m long with diameter 0.5 m.

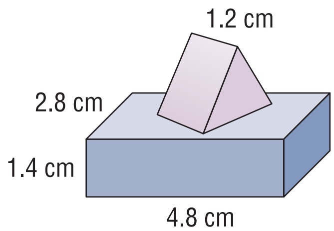


5. Determine the surface area of each composite object.

a) The cylinder is 2.5 m long with radius 0.5 m.

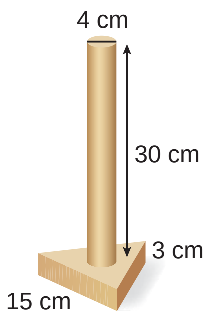


b) The base of the triangular prism is an equilateral triangle with side length 2.8 cm.



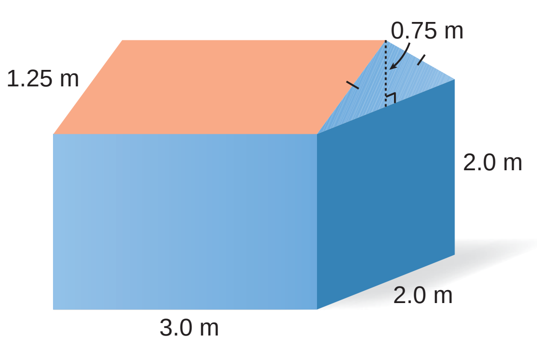
Apply

6. Here is the lamp stand from the top of page 33. The base of the lamp is a triangular prism with an equilateral triangle base. The surface of the stand is to be painted. What is the area that will be painted? Give the answer to the nearest whole number.



7. Assessment Focus

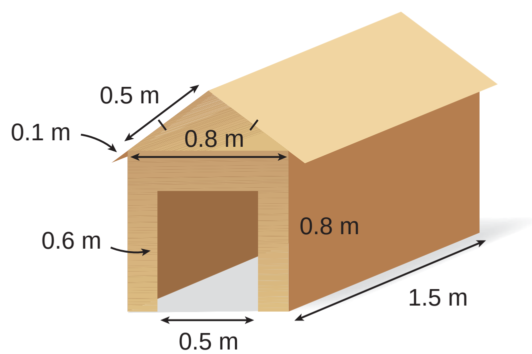
a) A playhouse has the shape of a rectangular prism with a triangular prism roof. Determine the surface area of the playhouse.



b) What are possible dimensions for a door and 2 windows? Explain how including these features will affect the surface area of the playhouse.

c) Determine the surface area of the playhouse not including its doors and windows.

8. Jemma has built this doghouse. The roof is a triangular prism with an isosceles triangle base. There is an overhang of 0.1 m. There is an opening for the doorway.



a) Determine the surface area of the doghouse.

b) The doghouse is to be covered with 2 coats of wood stain. Wood stain can be bought in 1-L or 4-L cans. One litre of stain covers 6 m^2. How many cans of either size are needed? Explain your thinking.

9. Each layer of a three-layer cake is a cylinder with height 7.5 cm.
The bottom layer has diameter 25 cm.
The middle layer has diameter 22.5 cm.
The top layer has diameter 20 cm.
The surface of the cake is frosted.

a) Sketch the cake.

b) What area of the cake is frosted?

10. In question 9, you determined the surface area of a three-layer cake.

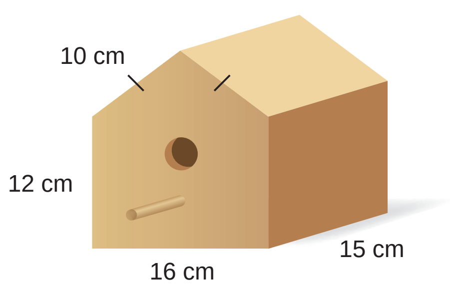
a) Suppose a fourth layer, with diameter 27.5 cm, is added to the bottom of the cake. What is the surface area of cake that will be frosted now?

b) Suppose a fifth layer, with diameter 30 cm, is added to the bottom of the cake. What is the surface area of cake that will be frosted now?

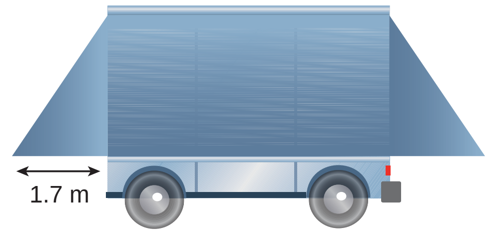
c) How does the surface area change when each new layer is added?

Give all the answers to the nearest tenth.

11. Rory will paint this birdhouse he built for his backyard. The perch is a cylinder with length 7 cm and diameter 1 cm. The diameter of the entrance is 3 cm. What is the area that needs to be painted? Give the answer to the nearest whole number.

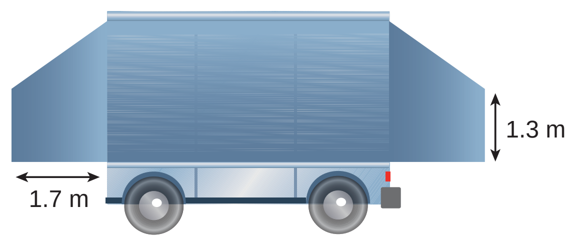


12. Shael and Keely are camping with their parents at Waskesiu Lake in Prince Albert National Park. Their tent trailer is 5 m long and 2.5 m wide. When the trailer is set up, the canvas expands to a height of 2.5 m. At each end, there is a fold out bed that is 1.7 m wide, in a space that is shaped like a triangular prism. The diagram shows a side view of the trailer.



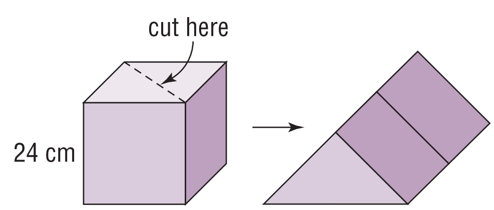
a) Determine the surface area of the canvas on the trailer.

b) Two parallel bars, 1.3 m high, are placed vertically at each end to support the canvas and provide more space in the beds. Does the surface area of the canvas change when the bars are inserted? Explain how you know.



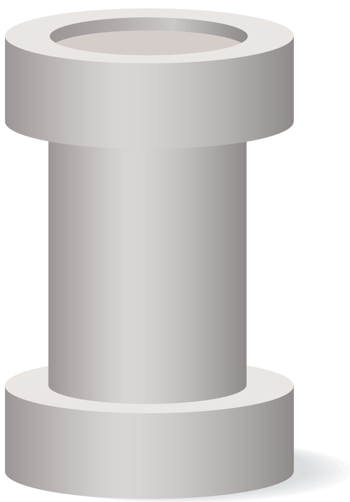
13. a) What is the surface area of a cube with edge length 24 cm?

b) The cube is cut along a diagonal of one face to form two triangular prisms. These prisms are glued together to form a longer triangular prism. What is the surface area of this prism? Give the answer to the nearest whole number.



c) Why do the cube and the triangular prism have different surface areas?

14. A birdbath and stand are made from 3 cylinders. The top and bottom cylinders have radius 22 cm and height 13 cm. The middle cylinder has radius 15 cm and height 40 cm. The "bath" has radius 15 cm and depth 2 cm. The birdbath and stand are to be tiled. Calculate the area to be tiled.



Take It Further

15. **a)** What is the surface area of a cylinder that is 50 cm long and has diameter 18 cm?

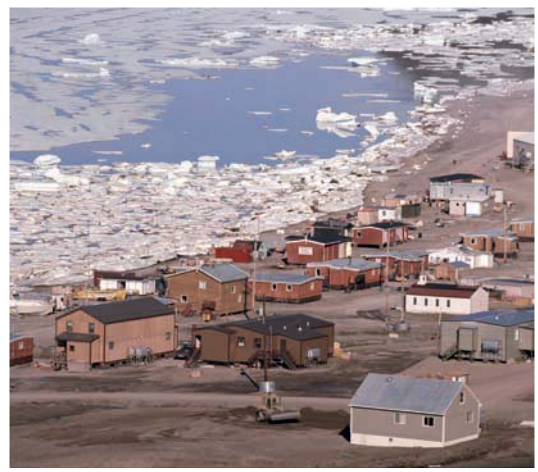
b) The cylinder is cut in half along its length and the two pieces are glued together end to end.

i) Sketch the composite object.

ii) What is its surface area?

Give the answers to the nearest whole number.

16. Grise Fiord, Nunavut, is Canada's northernmost Inuit community and it is home to 150 residents. In Inuktitut, this hamlet is called Aujuittuq, which means "the place that never thaws." Although the ground is frozen most of the year, it softens in the summer. The freezing and thawing of the ground would ruin a house foundation. The houses are made of wood, and are built on platforms. The homes are compact and have few windows.



a) Design and sketch the exterior of a home that could fit on a platform that is 10 m wide and 20 m long.

b) Determine the surface area of this home.

c) Every outside face needs to be insulated. Insulation costs $\$4.25/m^2$.
How much will it cost to insulate this home?

Reflect

Sketch a building or structure in your community that is made up of two or more prisms or cylinders. Explain how you would determine its surface area.

Study Guide

Perfect Squares

When a fraction can be written as a product of two equal fractions, the fraction is a perfect square.

For example, $\frac{144}{25}$ is a perfect square because $\frac{144}{25} = \frac{12}{5} \times \frac{12}{5}$; and $\sqrt{\frac{144}{25}} = \frac{12}{5}$

When a decimal can be written as a fraction that is a perfect square, then the decimal is also a perfect square.

The square root is a terminating or repeating decimal.

For example, 12.25 is a perfect square because $12.25 = \frac{1225}{100}$, and $\sqrt{\frac{1225}{100}} = \frac{35}{10}$, or 3.5

Non-Perfect Squares

A fraction or decimal that is not a perfect square is a non-perfect square.

To estimate the square roots of a non-perfect square, use perfect squares as benchmarks or use a calculator.

For example, $\sqrt{\frac{143}{25}} \doteq \sqrt{\frac{144}{25}}$, which is $\frac{12}{5}$, or 2.4

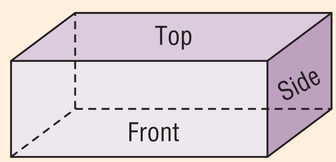
And, $\sqrt{6.4} \doteq 2.5$ to the nearest tenth

Surface Area of a Composite Object

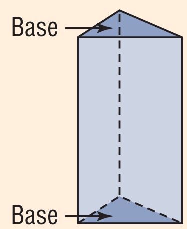
This is the sum of the surface areas of the objects that make up the composite object, minus the overlap.

The objects that make up the composite object can be:

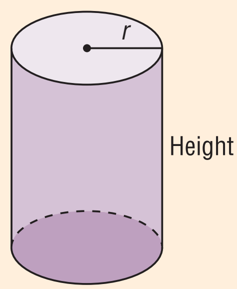
- A right rectangular prism with
  Surface area = 2 × area of top face + 2 × area of front face + 2 × area of side face



- A right triangular prism with
  Surface area = 2 × area of base + areas of 3 rectangular faces



- A right cylinder, radius r, with
  Surface area = 2 × area of one circular base + circumference of base × height of cylinder
  $= 2\pi r^2 + 2\pi r \times$ height



Review

1.1

1. Use grid paper to illustrate each square root as the side length of a square, then determine the value of the square root.

a) $\sqrt{1.21}$ **b)** $\sqrt{\frac{9}{25}}$ **c)** $\sqrt{0.64}$

d) $\sqrt{\frac{81}{16}}$ **e)** $\sqrt{2.56}$ **f)** $\sqrt{\frac{1}{36}}$

g) $\sqrt{0.25}$ **h)** $\sqrt{\frac{100}{64}}$ **i)** $\sqrt{3.61}$

j) $\sqrt{\frac{4}{121}}$ **k)** $\sqrt{2.89}$ **l)** $\sqrt{\frac{36}{49}}$

2. Determine each square root.

a) $\sqrt{\frac{144}{25}}$ **b)** $\sqrt{\frac{225}{64}}$

c) $\sqrt{\frac{196}{81}}$ **d)** $\sqrt{\frac{324}{121}}$

e) $\sqrt{0.0196}$ **f)** $\sqrt{0.0289}$

g) $\sqrt{1.69}$ **h)** $\sqrt{4.41}$

3. Which fractions and decimals are perfect squares? Explain your reasoning.

a) $\frac{48}{120}$ **b)** 1.6 **c)** $\frac{49}{100}$

d) 0.04 **e)** $\frac{144}{24}$ **f)** 2.5

g) $\frac{50}{225}$ **h)** 1.96 **i)** $\frac{63}{28}$

4. Calculate the number whose square root is:

a) $\frac{3}{5}$ **b)** 1.6 **c)** $\frac{9}{7}$ **d)** 0.8

5. Determine the side length of a square with each area below. Explain your strategy.

a) 0.81 m^2 **b)** 0.01 m^2

c) 4.84 cm^2 **d)** 6.25 cm^2

e) 0.16 km^2 **f)** 1.44 km^2

1.2

6. Use benchmarks to approximate each square root to the nearest tenth. State the benchmarks you used.

a) $\sqrt{3.8}$ **b)** $\sqrt{33.8}$

c) $\sqrt{133.8}$ **d)** $\sqrt{233.8}$

7. Use benchmarks to estimate a fraction for each square root. State the benchmarks you used.

a) $\sqrt{\frac{77}{10}}$ **b)** $\sqrt{\frac{18}{11}}$ **c)** $\sqrt{\frac{15}{39}}$

d) $\sqrt{\frac{83}{19}}$ **e)** $\sqrt{\frac{28}{103}}$ **f)** $\sqrt{\frac{50}{63}}$

8. Use any strategy you wish to estimate the value of each square root. Explain why you used the strategy you did.

a) $\sqrt{5.9}$ **b)** $\sqrt{\frac{7}{20}}$ **c)** $\sqrt{0.65}$

d) $\sqrt{\frac{21}{51}}$ **e)** $\sqrt{23.2}$ **f)** $\sqrt{\frac{88}{10}}$

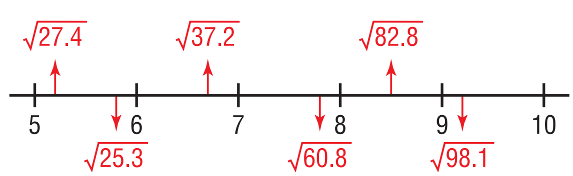
9. Which of the following square roots are correct to the nearest tenth? How do you know? Correct the square roots that are incorrect.

a) $\sqrt{2.4} \doteq 1.5$ **b)** $\sqrt{1.6} \doteq 0.4$

c) $\sqrt{156.8} \doteq 15.6$ **d)** $\sqrt{47.8} \doteq 6.9$

e) $\sqrt{0.5} \doteq 0.7$ **f)** $\sqrt{0.7} \doteq 0.5$

10. Which square roots are correctly placed on the number line below? How do you know?



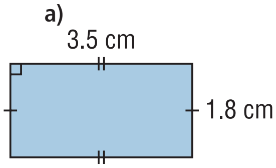
11. Use the square roots listed below. Which square roots are between each pair of numbers? Justify your answers.

a) 1 and 2 **b)** 11 and 12
c) 3.5 and 4.5 **d)** 1.5 and 2.5
e) 4.5 and 5.5 **f)** 14.5 and 15.5

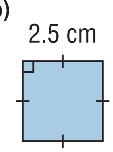
$\sqrt{12.9}$ $\sqrt{4.8}$ $\sqrt{134.5}$ $\sqrt{1.2}$
$\sqrt{21.2}$ $\sqrt{15.2}$ $\sqrt{222.1}$ $\sqrt{9.6}$
$\sqrt{3.2}$ $\sqrt{237.1}$ $\sqrt{2.3}$ $\sqrt{213.1}$
$\sqrt{125.4}$ $\sqrt{23.1}$ $\sqrt{129.9}$ $\sqrt{2.8}$
$\sqrt{5.7}$ $\sqrt{29.1}$

12. Determine the length of a diagonal of each rectangle.

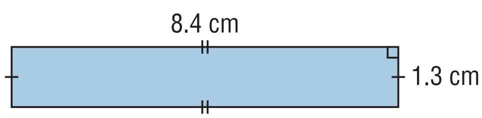
a)



b)



c)



13. Determine a decimal or a fraction whose square root is between each pair of numbers.

a) $\frac{1}{3}$ and 1 **b)** 0.2 and 0.3
c) 1.4 and 1.41 **d)** $\frac{1}{10}$ and $\frac{3}{10}$

14. a) Use a calculator to approximate each square root.

i) $\sqrt{0.0015}$ **ii)** $\sqrt{0.15}$ **iii)** $\sqrt{15}$
iv) $\sqrt{1500}$ **v)** $\sqrt{150\,000}$

b) What patterns do you see in the square roots in part a? Use the patterns to write the previous two square roots less than $\sqrt{0.0015}$ and the next two square roots greater than $\sqrt{150\,000}$.

1.3

15. Each object is built with 1-cm cubes. Determine its surface area.

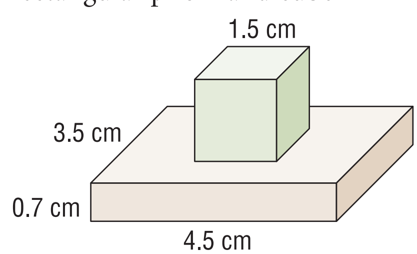
a)

b)

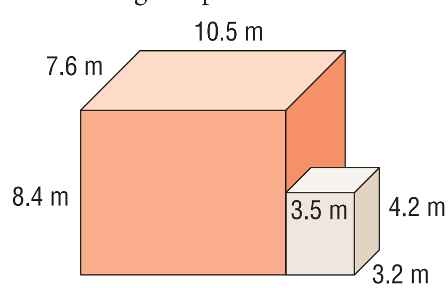
c)

16. Determine the surface area of each composite object. What effect does the overlap have on the surface area?

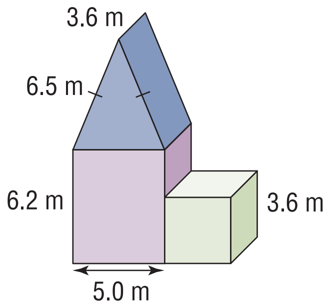
a) rectangular prism and cube



b) two rectangular prisms



c) triangular prism, rectangular prism, and cube



17. A desk top is a rectangular prism with dimensions 106 cm by 50 cm by 2 cm. Each of 4 legs of the desk is a rectangular prism with dimensions 75 cm by 3 cm by 3 cm.

a) Sketch the desk.

b) Determine the surface area of the desk.

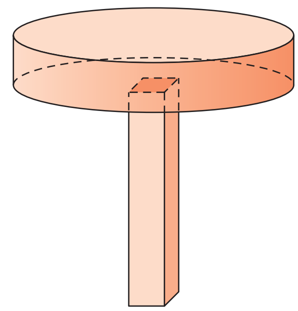
18. An Inukshuk is a human-like object constructed from stone by Canada's Inuit People. Traditionally, Inukshuks were used as markers during the Caribou hunt. The Inukshuk is now a symbol of leadership, cooperation, and human spirit. Each stone is separate; the stones are balanced to make the Inukshuk. This giant Inukshuk in Igloolik, Nunavut was built to commemorate the new millenium.

Construct an Inukshuk of cardboard boxes or other materials. Determine its surface area.

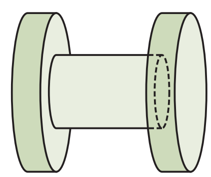
1.4

19. Determine the surface area of each composite object. Give the answers to the nearest tenth.

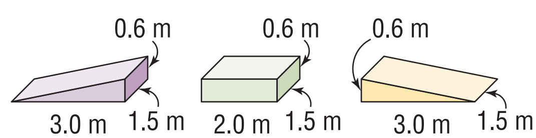
a) The rectangular prism has dimensions 2.5 cm by 2.5 cm by 15.0 cm. The cylinder is 3.5 cm high with radius 9.6 cm.



b) Each of the two congruent cylinders is 2.8 cm long, with radius 7.8 cm. The middle cylinder is 10.4 cm long, with radius 3.6 cm.



20. There are 2 wooden ramps, each of which is a triangular prism with a right triangle base; and a platform that is a rectangular prism. The ramps are joined to the platform to make one larger ramp for a BMX bike. This ramp will be painted completely.

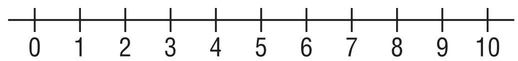


a) Calculate the surface area to be painted.

b) The paint costs $19.95 for one 3.78-L container. This will cover 35 m^2. The surface area needs 2 coats of paint. How much paint is needed and how much will it cost?

Practice Test

1. Sketch this number line.



a) Do *not* use a calculator. Determine or estimate each square root. Where necessary, write the square root to the nearest tenth. Place each square root on the number line.

i) $\sqrt{\frac{49}{4}}$ **ii)** $\sqrt{6.25}$ **iii)** $\sqrt{\frac{64}{9}}$ **iv)** $\sqrt{98.5}$ **v)** $\sqrt{\frac{9}{100}}$ **vi)** $\sqrt{\frac{9}{10}}$

b) How can you use benchmarks to determine or estimate square roots?

2. a) Use a calculator to determine or estimate each square root. Where necessary, write the square root to the nearest hundredth.

i) $\sqrt{\frac{3}{7}}$ **ii)** $\sqrt{52.5625}$ **iii)** $\sqrt{\frac{576}{25}}$ **iv)** $\sqrt{213.16}$ **v)** $\sqrt{135.4}$

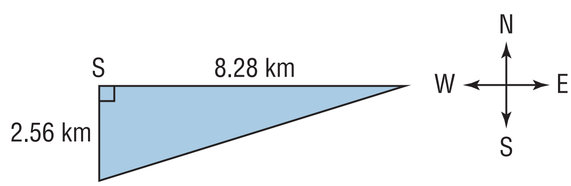
b) Which square roots in part a are exact? Which are approximate?

c) Explain why a square root shown on a calculator display may be approximate.

3. a) Identify a perfect square between 0 and 0.5.
How do you know the number is a perfect square?

b) Identify a number whose square root is between 0 and 0.5.
How can you check your answer?

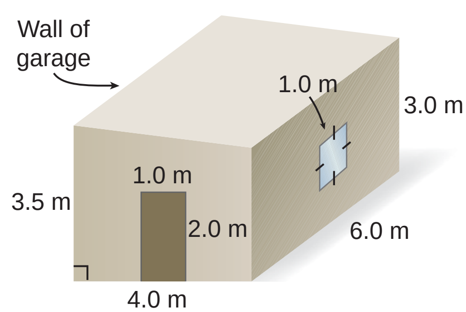
4. One canoe is 2.56 km due south of a small island, S. Another canoe is 8.28 km due east of the island. How far apart are the canoes? How do you know?



5. A garden shed is built against one wall of a garage. The shed has a sloping roof.

a) What is the surface area of the shed, not including its door and window?

b) The shed is to be painted with 2 coats of paint. Paint costs \$3.56/L. One litre covers 10 m^2. What will it cost to paint the shed?



6. Each of two congruent cubes has volume 64 cm^3. The cubes are joined at their faces to a cylinder that is 5 cm long and has radius 2 cm.

a) Sketch the object. **b)** What is the surface area of this object?

Unit Problem — Design a Play Structure

You will design a play structure for young children, constructed of light-weight nylon fabric and fibreglass poles.

Your budget for this project is \$800. A student has offered to sew the fabric for a donation of \$125 toward upgrading the school sewing machines.

The design can only include cylinders, rectangular prisms, and triangular prisms.
There should be between 6 and 8 objects, with at least one of each type.
The objects can be connected face to face.
Keep in mind that cylinders and openings need to allow enough movement space to safely accommodate a small child.

The fabric is available in three different colours:
Red costs \$10/m^2. Yellow costs \$11/m^2. Blue costs \$12/m^2.

The skeleton of the structure is made from fibreglass poles that cost \$3/m.
A fabric cylinder needs flexible circular supports every 1 m for reinforcement.
These cost \$4/m.
Joiners are included at no cost.

Your work should show:

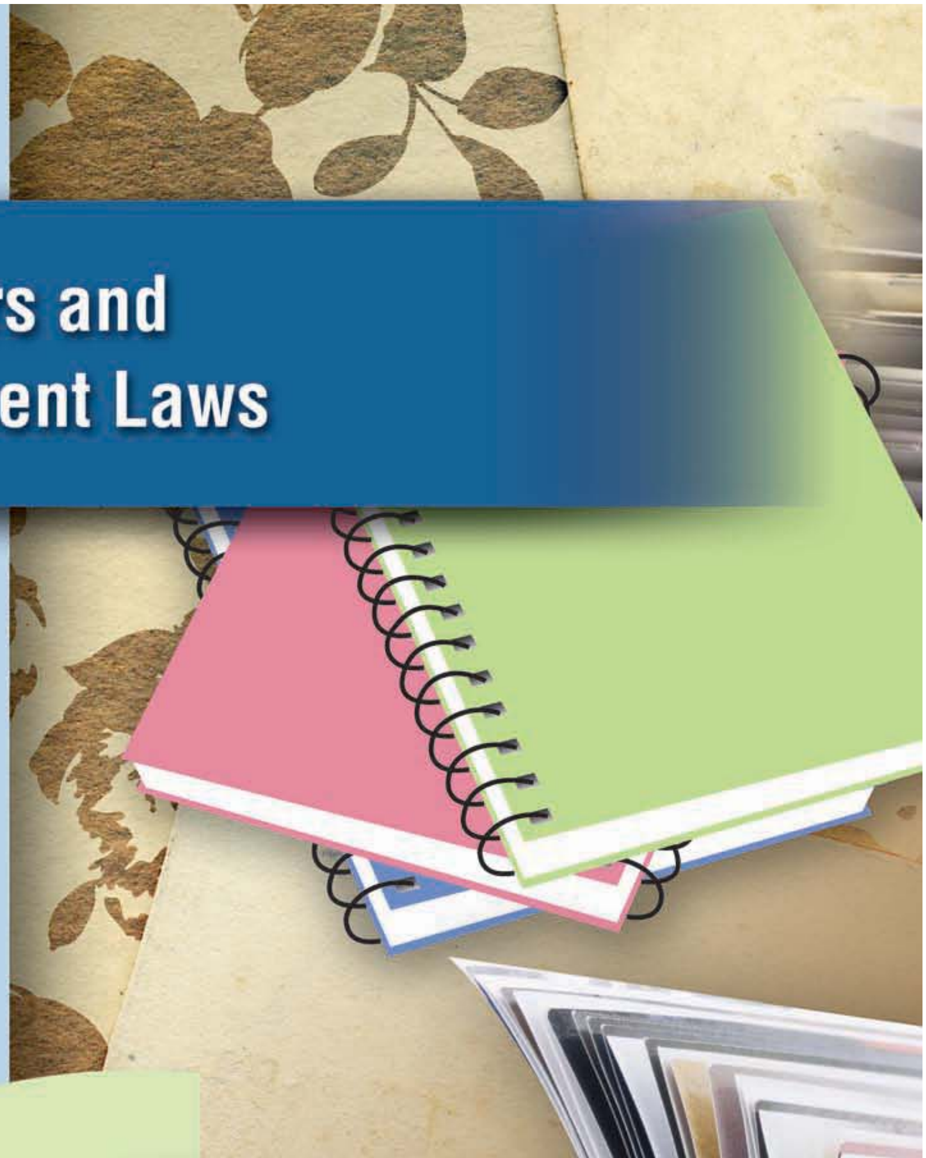
- models or sketches of your design
- the surface area of each object
- the cost of each object
- how you calculated the total surface area and the cost of the project
- an explanation of any unique features of your structure and why you included them

Reflect on Your Learning

What have you learned about perfect squares, non-perfect squares, and square roots?
How are square roots used when you calculate surface area?

UNIT 2

Powers and Exponent Laws



Imagine folding a piece of paper in half to form 2 layers.

Imagine folding it in half again to form 4 layers.

Try this with a sheet of paper. How many times can you fold the paper before it is impossible to make another fold?

What You'll Learn

- Use powers to represent repeated multiplication.
- Use patterns to understand a power with exponent 0.
- Solve problems involving powers.
- Perform operations with powers.
- Explain and apply the order of operations with exponents.

Why It's Important

Powers provide an efficient way to record our work. The properties of powers lead to even more efficient ways to perform some calculations. Powers are used in many formulas with applications in science, construction, and design.

Key Words

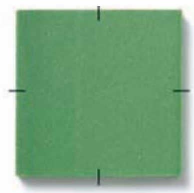
- power
- base
- exponent
- square number
- cube number
- power of a power
- power of a product
- power of a quotient

2.1 What Is a Power?

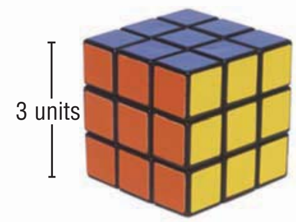
FOCUS

- Use powers to represent repeated multiplication.

What is the area of this square?
Write the area as a product.



What is the volume of this cube?
Write the volume as a product.



Investigate

2

You will need congruent square tiles and congruent cubes.

➤ Use the tiles to make as many different-sized larger squares as you can.
Write the area of each square as a product. Record your results in a table.

Number of Tiles	Area (square units)	Side Length (units)	Area as a Product
1	1	1	1×1

➤ Use the cubes to make as many different-sized larger cubes as you can.
Write the volume of each cube as a product. Record your results in a table.

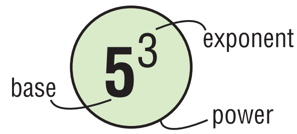
Number of Cubes	Volume (cubic units)	Edge Length (units)	Volume as a Product
1	1	1	$1 \times 1 \times 1$

Reflect & Share

What patterns do you see in the tables?
Use the patterns to predict the areas of the next 3 squares and the volumes of the next 3 cubes.
How are these areas and volumes the same? How are they different?

Connect

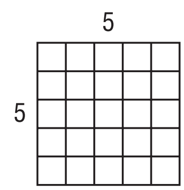
When an integer, other than 0, can be written as a product of equal factors, we can write the integer as a **power**.
For example, $5 \times 5 \times 5$ is 5^3.
5 is the **base**.
3 is the **exponent**.
5^3 is the *power*.
5^3 is a power of 5.



We say: 5 to the 3rd, or 5 cubed

➤ A power with an integer base and exponent 2 is a **square number**.
When the base is a positive integer, we can illustrate a square number.

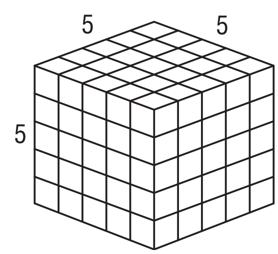
Here are 3 ways to write 25.
Standard form: 25
As repeated multiplication: 5×5
As a power: 5^2



$5 \times 5 = 5^2$
$= 25$
25 is a square number.

➤ A power with an integer base and exponent 3 is a **cube number**.
When the base is a positive integer, we can illustrate a cube number.

Here are 3 ways to write 125.
Standard form: 125
As repeated multiplication: $5 \times 5 \times 5$
As a power: 5^3



$5 \times 5 \times 5 = 5^3$
$= 125$
125 is a cube number.

Example 1 Writing Powers

Write as a power.
a) $3 \times 3 \times 3 \times 3 \times 3 \times 3$ **b)** 7

A Solution

a) $3 \times 3 \times 3 \times 3 \times 3 \times 3$
The base is 3. There are 6 equal factors, so the exponent is 6.
So, $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

b) 7
The base is 7. There is only 1 factor, so the exponent is 1.
So, $7 = 7^1$

Example 2 Evaluating Powers

Write as repeated multiplication and in standard form.

a) 3^5 b) 7^4

A Solution

a) $3^5 = 3 \times 3 \times 3 \times 3 \times 3$ As repeated multiplication
$= 243$ Standard form

b) $7^4 = 7 \times 7 \times 7 \times 7$ As repeated multiplication
$= 2401$ Standard form

Examples 1 and *2* showed powers with positive integer bases.
A power can also be negative or have a base that is a negative integer.

Example 3 Evaluating Expressions Involving Negative Signs

Identify the base of each power, then evaluate the power.

a) $(-3)^4$ b) -3^4 c) $-(-3^4)$

A Solution

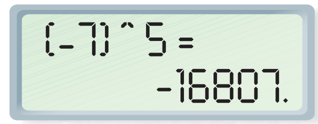
a) The base of the power is -3.
$(-3)^4 = (-3) \times (-3) \times (-3) \times (-3)$ As repeated multiplication
Apply the rules for multiplying integers:
The sign of a product with an even number of negative factors is positive.
So, $(-3)^4 = 81$ Standard form

b) The base of the power is 3.
The exponent applies only to the base 3, and not to the negative sign.
$-3^4 = -(3^4)$
$= -(3 \times 3 \times 3 \times 3)$
$= -81$

c) From part b, we know that $-3^4 = -81$.
So, $-(-3^4) = -(-81)$ $-(-81)$ is the opposite of -81, which is 81.
$= 81$

We may write the product of integer factors without the multiplication sign.
In *Example 3a*, we may write $(-3) \times (-3) \times (-3) \times (-3)$ as $(-3)(-3)(-3)(-3)$.

A calculator can be used to evaluate a power such as $(-7)^5$ in standard form.



Discuss the ideas

1. Can every integer, other than 0, be written as a power? Explain.

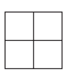
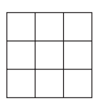
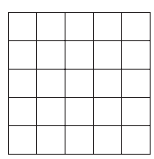
2. Why is -3^4 negative but $(-3)^4$ positive? Give another example like this.

3. Two students compared the calculator key sequences they used to evaluate a power. Why might the sequences be different?

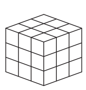
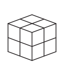
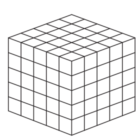
Practice

Check

4. Write the number of unit squares in each large square as a power.

a)  **b)**  **c)** 

5. Write the number of unit cubes in each large cube as a power.

a)  **b)**  **c)** 

6. Use grid paper. Draw a picture to represent each square number.

a) 4^2 **b)** 6×6 **c)** 49
d) 10^2 **e)** 81 **f)** 12×12

7. Write the base of each power.

a) 2^7 **b)** 4^3
c) 8^2 **d)** $(-10)^5$
e) $(-6)^7$ **f)** -8^3

8. Write the exponent of each power.

a) 2^5 **b)** 6^4
c) 9^1 **d)** -3^2
e) $(-2)^9$ **f)** $(-8)^3$

9. Write each power as repeated multiplication.

a) 3^2 **b)** 10^4
c) 8^5 **d)** $(-6)^5$
e) -6^5 **f)** -4^2

10. a) Explain how to build models to show the difference between 3^2 and 2^3.

b) Why is one number called a square number and the other number called a cube number?

11. Use repeated multiplication to show why 6^4 is not the same as 4^6.

12. Write as a power.

a) $4 \times 4 \times 4 \times 4$

b) $2 \times 2 \times 2$

c) $5 \times 5 \times 5 \times 5 \times 5 \times 5$

d) $10 \times 10 \times 10$

e) $(-79)(-79)$

f) $-(-2)(-2)(-2)(-2)(-2)(-2)(-2)(-2)$

Apply

13. Write each product as a power, then evaluate.

a) 5×5

b) $3 \times 3 \times 3 \times 3$

c) $10 \times 10 \times 10 \times 10 \times 10$

d) $-(9 \times 9 \times 9)$

e) $(-2)(-2)(-2)$

f) $-(-4)(-4)(-4)$

g) $(-5)(-5)(-5)(-5)$

h) $-(5)(5)(5)(5)$

i) $-(-5)(-5)(-5)(-5)$

14. Predict whether each answer is positive or negative, then evaluate.

a) 2^3 b) 10^6 c) 3^1 d) -7^3

e) $(-7)^3$ f) $(-2)^8$ g) -2^8 h) -6^4

i) $(-6)^4$ j) $-(-6)^4$ k) $(-5)^3$ l) -4^4

15. Canada Post often creates special postage stamps to celebrate important events and honour famous people.

a) Captain George Vancouver was a Dutch explorer who named almost 400 Canadian places. To commemorate his 250th birthday in 2007, Canada Post created a $1.55 stamp.

i) How many stamps are in a 3 by 3 block? Write the number of stamps as a power.

ii) What is the value of these stamps?

b) In July 2007, Canada hosted the FIFA U-20 World Cup Soccer Championships. Canada Post issued a 52¢ stamp to honour all the players and fans.

i) How many stamps are in a 4 by 4 block? Write the number of stamps as a power.

ii) What is the value of these stamps?

16. Evaluate.

a) 3^{12} b) -7^7

c) 5^{11} d) $-(-4)^{10}$

e) $(-9)^8$ f) 2^{23}

17. Assessment Focus

a) Write as repeated multiplication and in standard form.

i) 4^3 ii) -4^3 iii) $-(-4^3)$ iv) (-4^3)

b) Which products in part a are positive? Why? Which products are negative? Why?

c) Write as repeated multiplication and in standard form.

i) 4^2 ii) -4^2 iii) $-(-4^2)$ iv) (-4^2)

d) Which products in part c are positive? Why? Which products are negative? Why?

e) Write other sets of powers like those in parts a and c. Explain how you know if each product is positive or negative before you write the power in standard form.

18. a) Is the value of -3^5 different from the value of $(-3)^5$ or (-3^5)? What purpose do the brackets serve?

b) Is the value of -4^6 different from the value of $(-4)^6$ or (-4^6)? What purpose do the brackets serve?

19. a) When does a negative base in a power produce a negative product? Give 3 examples.

b) When does a negative base in a power produce a positive product? Give 3 examples.

Take It Further

20. Write each number as a power with base 2. Explain your method.

a) 4 **b)** 16 **c)** 64
d) 256 **e)** 32 **f)** 128

21. a) Write each number as a power in as many ways as possible.

i) 16 **ii)** 81 **iii)** 256

b) Find other numbers that can be written as a power in more than one way. Show your work.

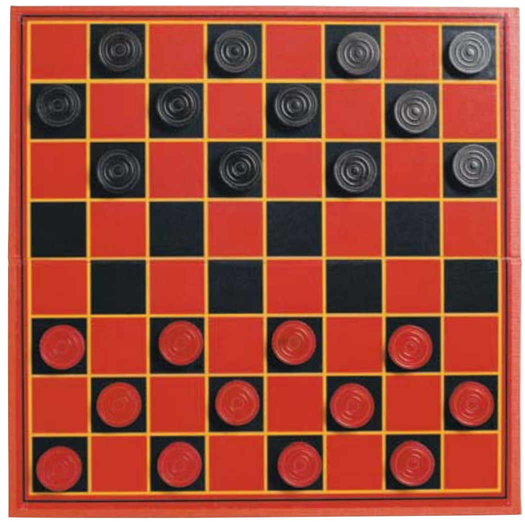
22. a) How are the powers in each pair the same? How are they different?

i) 2^3 or 3^2 **ii)** 2^5 or 5^2
iii) 3^4 or 4^3 **iv)** 5^4 or 4^5

b) In part a, which is the greater power in each pair? Explain how you know.

23. Without evaluating all the powers, write them in order from greatest to least:
3^5, 5^2, 3^4, 6^3
Explain your strategy.

24.



How many squares of each side length are there on a checkerboard? Write each number as a power.

a) 1 unit **b)** 2 units
c) 3 units **d)** 4 units
e) 5 units **f)** 6 units
g) 7 units **h)** 8 units

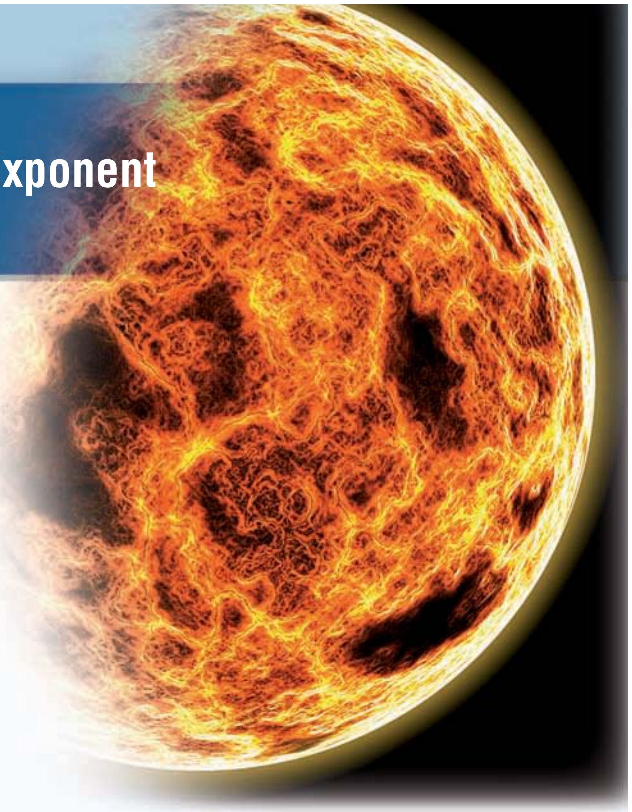
What patterns do you see in the answers?

25. Explain how to tell if a number is a square number, or a cube number, or neither. Give examples.

Reflect

What is a power?
Why are brackets used when a power has a negative base?

2.2 Powers of Ten and the Zero Exponent



FOCUS

- Explore patterns and powers of 10 to develop a meaning for the exponent 0.

Nuclear reactions in the core of the sun create solar energy.
For these reactions to take place, extreme temperature and pressure are needed.
The temperature of the sun's core is about 10^7 °C.
What is this temperature in millions of degrees Celsius?

Investigate

2

Choose a number between 1 and 10 as the base of a power.
Use the exponents 5, 4, 3, 2, and 1.
Use your base and each exponent to write a power.
Then write the power as repeated multiplication and in standard form.
Record your results in a table.

Exponent	Power	Repeated Multiplication	Standard Form
5			
4			
3			
2			
1			

Describe any patterns in your table.
Continue the patterns to complete the entries in the last row.

Reflect & Share

Compare your tables and patterns with those of other pairs of students.
What do you think is the value of a power with exponent 0?
Use a calculator to check your answer for different integer bases.

Connect

This table shows decreasing powers of 10.

Number in Words	Standard Form	Power
One billion	1 000 000 000	10^9
One hundred million	100 000 000	10^8
Ten million	10 000 000	10^7
One million	1 000 000	10^6
One hundred thousand	100 000	10^5
Ten thousand	10 000	10^4
One thousand	1 000	10^3
One hundred	100	10^2
Ten	10	10^1
One	1	10^0

← We use the pattern in the exponents to write 1 as 10^0.

We could make a similar table for the powers of any integer base except 0.
So, 1 can be written as any power with exponent 0.
For example, $1 = 2^0$
$1 = 13^0$
$1 = (-5)^0$

Zero Exponent Law

A power with an integer base, other than 0, and an exponent 0 is equal to 1.
$n^0 = 1, \quad n \neq 0$

Example 1 Evaluating Powers with Exponent Zero

Evaluate each expression.

a) 4^0 **b)** -4^0 **c)** $(-4)^0$

A Solution

A power with exponent 0 is equal to 1.

a) $4^0 = 1$ **b)** $-4^0 = -1$ **c)** $(-4)^0 = 1$

We can use the zero exponent and powers of 10 to write a number.

Example 2 Writing Numbers Using Powers of Ten

Write 3452 using powers of 10.

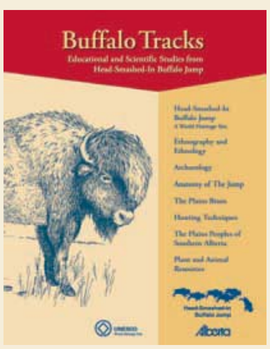
A Solution

Use a place-value chart.

Thousands	Hundreds	Tens	Ones
3	4	5	2

$3452 = 3000 + 400 + 50 + 2$
$= (3 \times 1000) + (4 \times 100) + (5 \times 10) + (2 \times 1)$ We use brackets for clarity.
$= (3 \times 10^3) + (4 \times 10^2) + (5 \times 10^1) + (2 \times 10^0)$

Example 3 Interpreting Numbers in the Media



Head-Smashed-In Buffalo Jump is a UNESCO World Heritage Site in Southern Alberta. This site covers 600 hectares and contains cultural remains used in the communal hunting of buffalo. Head-Smashed-In was first used for hunting bison at least 5700 years ago and perhaps as early as 10 000 years ago. It is estimated that close to sixty million Plains Bison roamed the prairies prior to the Europeans' arrival in Western Canada. Less than one hundred years later, fewer than 1000 animals remained.

Use powers of 10 to write each number in the above paragraph.

A Solution

$600 = 6 \times 100$
$= 6 \times 10^2$
$5700 = 5000 + 700$
$= (5 \times 1000) + (7 \times 100)$
$= (5 \times 10^3) + (7 \times 10^2)$
$10\ 000 = 1 \times 10^4$
$60\ 000\ 000 = 6 \times 10\ 000\ 000$
$= 6 \times 10^7$
$100 = 1 \times 10^2$
$1000 = 1 \times 10^3$

Discuss the ideas

1. In *Example 1*, why are 4^0 and $(-4)^0$ equal to 1, while -4^0 is equal to -1?

2. What is meant by "a power of 10"? Name 6 numbers that are powers of 10.

3. How would you use patterns to explain that $10^0 = 1$?

Practice

Check

4. Evaluate each power.
   a) 50^0 b) 9^0 c) 1^0 d) 17^0

5. Evaluate each power.
   a) $(-6)^0$ b) -11^0 c) -8^0 d) $(-24)^0$

6. Write each number as a power of 10.
   a) 1000 b) 100 000 c) 1 000 000 000
   d) ten thousand e) one hundred billion

Apply

7. Write 1 as a power in three different ways.

8. Evaluate each power of 10.
   a) 10^7 b) 10^2 c) 10^0
   d) 10^{10} e) 10^1 f) 10^6

9. Use powers of 10 to write each number.
   a) 6 000 000 000 b) 200 c) 51 415
   d) 60 702 008 e) 302 411 f) 2 000 008

10. Write each number in standard form.
    a) 7×10^7
    b) $(3 \times 10^4) + (9 \times 10^3) + (5 \times 10^1) + (7 \times 10^0)$
    c) $(8 \times 10^8) + (5 \times 10^5) + (2 \times 10^2)$
    d) $(9 \times 10^{10}) + (8 \times 10^9) + (1 \times 10^0)$
    e) 1×10^{15}
    f) $(4 \times 10^3) + (1 \times 10^0) + (9 \times 10^5) + (3 \times 10^1)$

11. The data below refer to trees in Vancouver. Use powers of 10 to write each number.
    - Street trees have an estimated value of over $500 million.
    - In the past decade, the Park Board has planted almost 40 000 new street trees.
    - Nearly 3 million ladybugs are released every year to help control aphids on street trees.
    - The most common street tree is the Japanese flowering cherry, with over 17 000 growing on city streets.
    - There are 130 000 trees lining the streets of Vancouver.
    - There are nearly 600 different types of trees.

12. **Assessment Focus** Choose a negative integer as the base of a power. Copy and complete the table below. Use patterns to explain why the power with exponent 0 is equal to 1.

Exponent	Power	Standard Form
5		
4		
3		
2		
1		
0		

13. In each pair, which number is greater? How do you know?

a) $(4 \times 10^3) + (6 \times 10^2) + (6 \times 10^1) + (7 \times 10^0)$ or 4327

b) $(2 \times 10^4) + (4 \times 10^3) + (2 \times 10^2) + (4 \times 10^1)$ or 2432

c) $(7 \times 10^7) + (7 \times 10^3)$ or 777 777

14.

- Worldwide, about one billion people lack access to safe drinking water.
- Glacier ice over 100 000 years old can be found at the base of many Canadian Arctic ice caps.
- Approximately 1000 kg of water is required to grow 1 kg of potatoes.
- Henderson Lake, British Columbia, has the greatest average annual precipitation in Canada of 6655 mm. That is more than 100 times as much as Eureka, in Nunavut, which has the least average annual precipitation of 64 mm.
- In November 2007, at the request of local First Nations, over 10 million hectares of the Mackenzie River Basin were protected from industrial development.

Using this information:

a) Identify the powers of 10 and write them using exponents.

b) Arrange the numbers in order from least to greatest.

c) Explain how writing powers of 10 using exponents can help you to order and compare numbers.

Take It Further

15. What are the meanings of the words trillion, quadrillion, and quintillion? Write these numbers as powers. What strategies did you use?

Reflect

Why is a power with exponent 0 equal to 1?

Math Link

Your World

The amount of data that an MP3 player can store is measured in gigabytes. For example, one MP3 player can store 2 GB (gigabytes) of songs. One song uses about 7000 KB (kilobytes) of space, where 1 GB = 2^{20} KB. About how many songs can the MP3 player hold?

2.3 Order of Operations with Powers

FOCUS

- Explain and apply the order of operations with exponents.

This was a skill-testing question in a competition:
$6 \times (3 + 2) - 10 \div 2$
Which answer is correct: 5, 10, 15, or 25?
How do you know?

Investigate

2

Use each of the digits 2, 3, 4, and 5 once to write an expression.

➤ The expression must have at least one power.
The base of the power can be a positive or negative integer.

➤ The expression can use any of:
addition, subtraction, multiplication, division, and brackets

Evaluate the expression.
Write and evaluate as many different expressions as you can.

Reflect & Share

Share your expressions with another pair of students.
Where does evaluating a power fit in the order of operations?
Why do you think this is?

Connect

To avoid getting different answers when we evaluate an expression, we use this order of operations:

- Evaluate the expression in brackets first.
- Evaluate the powers.
- Multiply and divide, in order, from left to right.
- Add and subtract, in order, from left to right.

Example 1 Adding and Subtracting with Powers

Evaluate.

a) $3^3 + 2^3$ **b)** $3 - 2^3$ **c)** $(3 + 2)^3$

A Solution

a) Evaluate the powers before adding.

$$\begin{aligned} 3^3 + 2^3 &= (3)(3)(3) + (2)(2)(2) \\ &= 27 + 8 \\ &= 35 \end{aligned}$$

b) Evaluate the power, then subtract.

$$\begin{aligned} 3 - 2^3 &= 3 - (2)(2)(2) \\ &= 3 - 8 \\ &= -5 \end{aligned}$$

c) Add first, since this operation is within the brackets. Then evaluate the power.

$$\begin{aligned} (3 + 2)^3 &= 5^3 \\ &= (5)(5)(5) \\ &= 125 \end{aligned}$$

When we need curved brackets for integers, we use square brackets to show the order of operations. When the numbers are too large to use mental math, we use a calculator.

Example 2 Multiplying and Dividing with Powers

Evaluate.

a) $[2 \times (-3)^3 - 6]^2$ **b)** $(18^2 + 5^0)^2 \div (-5)^3$

A Solution

a) Follow the order of operations.

Do the operations in brackets first: evaluate the power $(-3)^3$

$$\begin{aligned} [2 \times (-3)^3 - 6]^2 &= [2 \times (-27) - 6]^2 && \text{Then multiply: } 2 \times (-27) \\ &= [-54 - 6]^2 && \text{Then subtract: } -54 - 6 \\ &= (-60)^2 && \text{Then evaluate the power: } (-60)^2 \\ &= 3600 \end{aligned}$$

b) Use a calculator to evaluate $(18^2 + 5^0)^2 \div (-5)^3$.

For the first bracket:

Use mental math when you can: $5^0 = 1$

Evaluate $18^2 + 1$ to display 325.

Evaluate 325^2 to display 105 625.

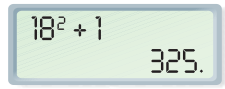
For the second bracket:

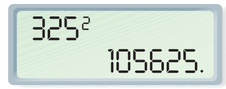
$(-5)^3$ is negative, so simply evaluate 5^3 to display 125.

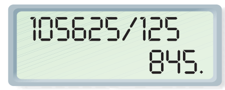
To evaluate $105\,625 \div (-125)$, the integers have opposite signs, so the quotient is negative.

Evaluate $105\,625 \div 125$ to display 845.

So, $(18^2 + 5^0)^2 \div (-5)^3 = -845$







Example 3 Solving Problems Using Powers

Lyn has a square swimming pool, 2 m deep with side length 4 m. The swimming pool is joined to a circular hot tub, 1 m deep with diameter 2 m. Lyn adds 690 g of chlorine to the pool and hot tub each week. This expression represents how much chlorine is present per 1 m^3 of water:

$$\frac{690}{2 \times 4^2 + \pi \times 1^3}$$

The suggested concentration of chlorine is 20 g/m^3 of water.
What is the concentration of chlorine in Lyn's pool and hot tub?
Is it close to the suggested concentration?

A Solution

Use a calculator. Since the denominator has a sum, draw brackets around it.
This ensures the entire denominator is divided into the numerator.

Key in the expression as it now appears: $\dfrac{690}{(2 \times 4^2 + \pi \times 1^3)} \doteq 19.634\,85$

The concentration is about 19.6 g/m^3. This is very close to the suggested concentration.

Discuss the ideas

1. Explain why the answers to $3^3 + 2^3$ and $(3 + 2)^3$ are different.

2. Use the meaning of a power to explain why powers are evaluated before multiplication and division.

Practice

Check

3. Evaluate.

a) $3^2 + 1$ b) $3^2 - 1$

c) $(3 + 1)^2$ d) $(3 - 1)^2$

e) $2^2 + 4$ f) $2^2 - 4$

g) $(2 + 4)^2$ h) $(2 - 4)^2$

i) $2 - 4^2$ j) $2^2 - 4^2$

4. Evaluate. Check using a calculator.

a) $2^3 \times 5$ b) 2×5^2

c) $(2 \times 5)^3$ d) $(2 \times 5)^2$

e) $(-10)^3 \div 5$ f) $(-10) \div 5^0$

g) $[(-10) \div 5]^3$ h) $[(-10) \div 5]^0$

5. Evaluate.

a) $2^3 + (-2)^3$ b) $(2 - 3)^3$

c) $2^3 - (-3)^3$ d) $(2 + 3)^3$

e) $2^3 \div (-1)^3$ f) $(2 \div 2)^3$

g) $2^3 \times (-2)^3$ h) $(2 \times 1)^3$

Apply

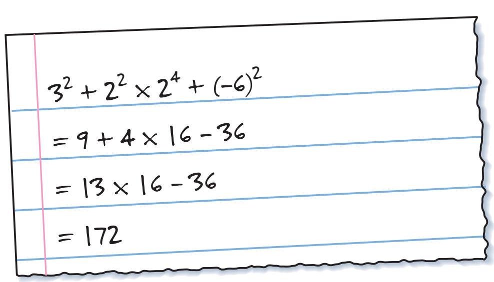
6. a) Evaluate. Record your work.

i) $4^2 + 4^3$ ii) $5^3 + 5^6$

b) Evaluate. Record your work.

i) $6^3 - 6^2$ ii) $6^3 - 6^5$

7. Identify, then correct, any errors in the student work below. Explain how you think the errors occurred.



8. State which operation you will do first, then evaluate.

a) $(7)(4) - (5)^2$ b) $6(2 - 5)^2$

c) $(-3)^2 + (4)(7)$ d) $(-6) + 4^0 \times (-2)$

e) $10^2 \div [10 \div (-2)]^2$ f) $[18 \div (-6)]^3 \times 2$

9. Sometimes it is helpful to use an acronym as a memory trick. Create an acronym to help you remember the order of operations. Share it with your classmates.

An acronym is a word formed from the first letters of other words.

10. Evaluate.

a) $(3 + 4)^2 \times (4 - 6)^3$

b) $(8 \div 2^2 + 1)^3 - 3^5$

c) $4^3 \div [8(6^0 - 2^1)]$

d) $9^2 \div [9 \div (-3)]^2$

e) $(2^2 \times 1^3)^2$

f) $(11^3 + 5^2)^0 + (4^2 - 2^4)$

11. Explain why the brackets are not necessary to evaluate this expression.
$(-4^3 \times 10) - (6 \div 2)$
Evaluate the expression, showing each step.

12. Winona is tiling her 3-m by 3-m kitchen floor. She bought stone tiles at $70/m^2. It costs $60/m^2 to install the tiles. Winona has a coupon for a 25% discount off the installation cost. This expression represents the cost, in dollars, to tile the floor:
$70 \times 3^2 + 60 \times 3^2 \times 0.75$
How much does it cost to tile the floor?

13. Evaluate this expression:
$2^3 + (3 \times 4)^2 - 6$
Change the position of the brackets. Evaluate the new expression. How many different answers can you get by changing only the position of the brackets?

14. Evaluate each pair of expressions. Why are some answers the same? Why are other answers different?

a) $3 + 5 \times 8$ and $5 \times 8 + 3$

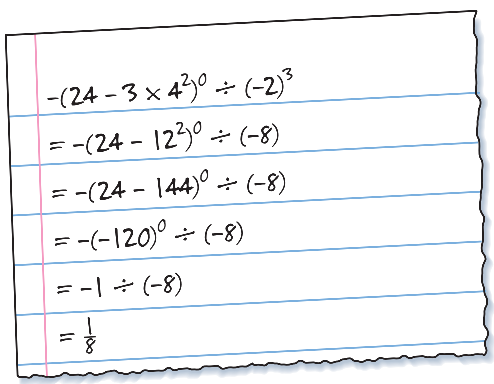
b) $3^2 + 2^2$ and $(3 + 2)^2$

c) $3^3 \times 2^3 - 5^2$ and $(3 \times 2)^3 - 5 \times 5$

d) $2^3 \times 3^2$ and $(2 \times 3)^5$

e) $5 \times 3 - 3^2 \times 4 + 20 \times 7$ and $5 \times (3 - 3^2) \times 4 + 20 \times 7$

15. This student got the correct answer, but she did not earn full marks. Find the mistake this student made. Explain how it is possible she got the correct answer. Write a more efficient solution for this problem.



16. Use a calculator to evaluate. Write the key strokes you used.

a) $(14 + 10)^2 \times (21 - 28)^3$

b) $(36 \div 2^2 + 11)^3 - 10^5$

c) $\dfrac{12^3}{36(12^0 - 13^1)}$

d) $\dfrac{81^2}{9^2 + (-9)^2}$

e) $(14^2 + 6^3)^2$

f) $(11^3 + 25^2)^0 + (27^2 - 33^4)$

17. Assessment Focus Predict which expression has a value closest to 0. Explain your strategy for predicting, then verify your prediction.

$(30 + 9 \times 11 \div 3)^0$

$(-3 \times 6) + 4^2$

$1 + (1 \div 1)^2 + 1^0$

18. Robbie, Marcia, and Nick got different answers when they evaluated this expression: $(-6)^2 - 2[(-8) \div 2]^2$

Robbie's answer was 68, Marcia's answer was 4, and Nick's answer was -68.

a) Who had the correct answer?

b) Show and explain how the other two students might have got their answers. Where did they go wrong?

19. A timber supplier manufactures and delivers wood chips. The chips are packaged in boxes that are cubes with edge length 25 cm. The cost of the chips is $14/m^3, and delivery costs $10 per 25 km. One customer orders 150 boxes of wood chips and she lives 130 km from the supplier. This expression represents the cost, in dollars:

$\dfrac{10 \times 130}{25} + 25^3 \div 10^6 \times 14 \times 150$

How much does the customer pay?

20. Copy each statement. Insert brackets to make each statement true.

a) $10 + 2 \times 3^2 - 2 = 106$

b) $10 + 2 \times 3^2 - 2 = 24$

c) $10 + 2 \times 3^2 - 2 = 84$

d) $10 + 2 \times 3^2 - 2 = 254$

21. Copy each statement. Insert brackets to make each statement true.

a) $20 \div 2 + 2 \times 2^2 + 6 = 26$

b) $20 \div 2 + 2 \times 2^2 + 6 = 30$

c) $20 \div 2 + 2 \times 2^2 + 6 = 8$

d) $20 \div 2 + 2 \times 2^2 + 6 = 120$

22. Blake answered the following skill-testing question to try to win a prize:

$5 \times 4^2 - (2^3 + 3^3) \div 5$

Blake's answer was 11.

Did Blake win the prize? Show your work.

23. Write an expression that includes integers, powers, brackets, and all four operations. Evaluate the expression. Ask a classmate to evaluate the expression. Did both of you follow the same order of operations? Is it possible to get the same answer if you follow a different order of operations? Explain.

Take It Further

24. Copy and complete each set of equations. Describe any patterns you see. Extend each pattern by 2 more rows.

a) $1^3 = 1^2$

$1^3 + 2^3 = 3^2$

$1^3 + 2^3 + 3^3 = 6^2$

$1^3 + 2^3 + 3^3 + 4^3 =$

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 =$

b) $3^2 - 1^2 = \square^3$

$6^2 - 3^2 = \square^3$

$10^2 - 6^2 = \square^3$

$15^2 - 10^2 = \square^3$

$21^2 - 15^2 = \square^3$

25. Choose two numbers between -5 and $+5$.

a) Square the numbers, then add the squares. Write this as an expression.

b) Add the numbers, then square the sum. Write this as an expression.

c) Compare the answers to parts a and b. What do you notice?

d) A student said, "The sum of the squares of two numbers is equal to the square of the sum of the numbers." Do you agree with this statement? Justify your answer.

26. Use four 4s and any operations, brackets, or powers to write an expression for each whole number from 1 to 9.

27. a) Write each product as a power of 2 and in standard form.

i) $2 \times 2 \times 2 \times 2$

ii) 2×2

iii) $2 \times 2 \times 2 \times 2 \times 2$

iv) $2 \times 2 \times 2$

b) Write each number as a sum, using only powers of 2.

For example: $27 = 16 + 8 + 2 + 1$

$= 2^4 + 2^3 + 2^1 + 2^0$

i) 28 ii) 12

iii) 25 iv) 31

v) 50 vi) 75

c) Repeat part b with a different base. Share your results with a classmate.

Reflect

Why is the order of operations important? Include examples in your explanation.

Mid-Unit Review

2.1

1. Write each power in standard form.

a) 14^2 **b)** 5^1 **c)** -8^3

d) $-(-4)^4$ **e)** $(-6)^3$ **f)** $(-2)^8$

2. Copy and complete this table.

	Power	Base	Exponent	Repeated Multiplication	Standard Form
a)	4^3				
b)	2^5				
c)	8^6				
d)		7	2		
e)				3 × 3 × 3 × 3	

3. a) Evaluate the first 8 powers of 7. Copy and complete this table.

Power of 7	Standard Form
7^1	
7^2	
7^3	
7^4	
7^5	
7^6	
7^7	
7^8	

b) What pattern do you see in the ones digits of the numbers in the second column?

c) Verify that the pattern continues by extending the table for as many powers of 7 as your calculator displays.

d) Use the pattern. Predict the ones digit of each power of 7. Explain your strategy.

i) 7^{12} **ii)** 7^{14}

iii) 7^{17} **iv)** 7^{22}

2.2

4. Write in standard form.

a) 10^6 **b)** 10^0 **c)** 10^8 **d)** 10^4

5. Write as a power of 10.

a) one billion **b)** one

c) 100 **d)** 100 000

6. Evaluate.

a) $(-5)^0$ **b)** 25^0 **c)** -6^0 **d)** 9^0

7. The area of land is measured in hectares (ha). One hectare is the area of a square with side length 100 m. Write the number of square metres in 1 ha as a power.

2.3

8. Evaluate. State which operation you do first.

a) $(-21 - 6)^2 + 14$

b) $6 \div (-2) + (2 \times 3)^2$

c) $[5 - (-4)]^3 - (21 \div 7)^4$

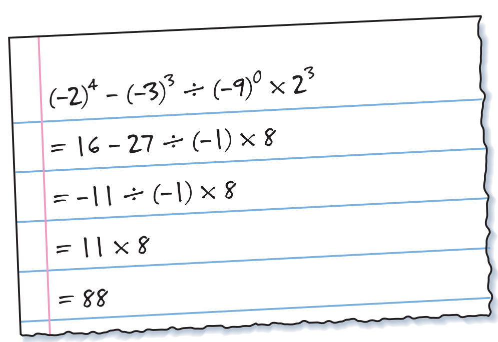
d) $[(6 - 21)^3 \times (2 + 2)^6]^0$

e) $(3 - 5)^5 \div (-4)$

f) $-30 - (7 - 4)^3$

9. Both Sophia and Victor evaluated this expression: $-2^4 \times 5 + 16 \div (-2)^3$
Sophia's answer was −82 and Victor's answer was 78. Who is correct? Find the likely error made by the other student.

10. Identify, then correct, any errors in the student work below. How do you think the errors occurred?



Start Where You Are

What Strategy Could I Try?

Suppose I have to evaluate this expression:

$$\frac{3^2(5^0 + 2 + 2^2)}{2(5 + 4^2)}$$

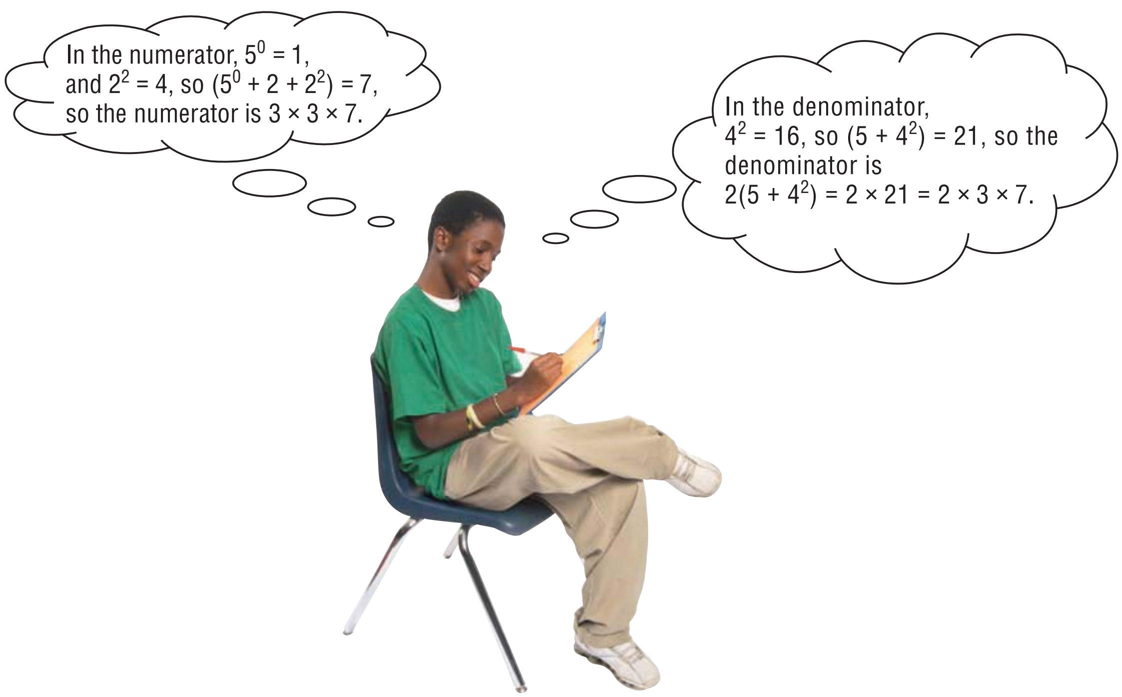
➤ What math tools could I use?

- mental math
- mental math, and paper and pencil
- a calculator

If I use only mental math, I might forget the numbers, so I write down the values of the numerator and denominator.

If I use mental math, and paper and pencil,

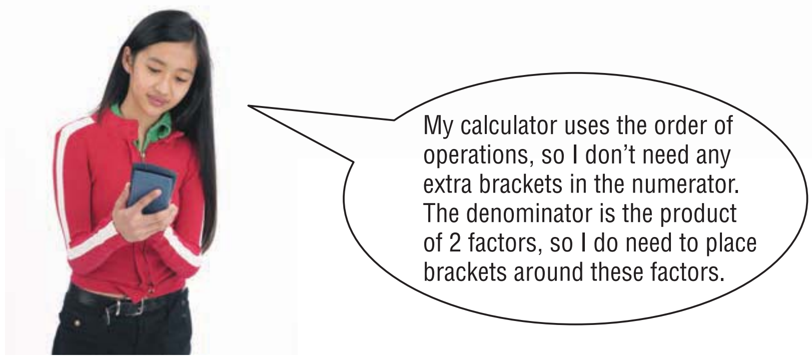
- I must use the order of operations.
- The fraction bar acts like a bracket, so I work on the numerator and denominator separately.
- I write down the values of the numerator and the denominator.
- I look for friendly numbers to help with the division.

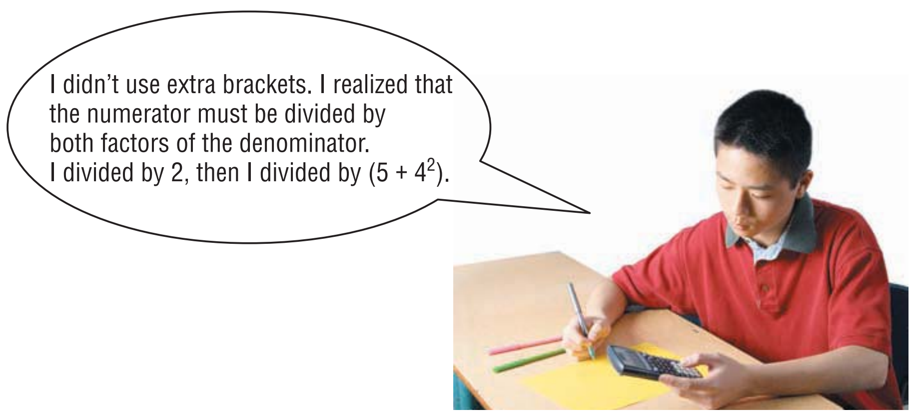


So, $\dfrac{3^2(5^0 + 2 + 2^2)}{2(5 + 4^2)} = \dfrac{3 \times 3 \times 7}{2 \times 3 \times 7} = \dfrac{3}{2}$

If I use a calculator,

- The fraction bar means divide the numerator by the denominator.
- My calculator uses the order of operations.
- Can I enter the expression as it is written?
- Do I need to add extra brackets, or change any operations?





Check

Use any strategies *you* know to evaluate these expressions.

1. a) $\dfrac{3^2 \times 6^2}{2^2 + 1}$ **b)** $\dfrac{3^2 \times 6^2}{2^3 \div 2^2}$ **c)** $\dfrac{3^2 + 6^2}{2^2 - 1}$

d) $\dfrac{3^2 - 6^2}{2^2 - 1}$ **e)** $\dfrac{6^2 \div 3^2}{2^2 \div 2}$

2. a) $\dfrac{3^4 - 2^2}{4^3 + 4^2 - 3^1}$ **b)** $\dfrac{4^2(3^4 \div 2^0)}{2^4(3^4 - 2^0)}$ **c)** $\dfrac{2^4(4^3 \div 2^2) - 4^0}{3(3^4 + 2^2)}$

GAME

Operation Target Practice

You will need

- two different-coloured number cubes labelled 1 to 6

Number of Players

- 3 or more

Goal of the Game

- To use the order of operations to write an expression for the target number

How to Play

1. Decide which number cube will represent the tens and which will represent the ones of a 2-digit number.
2. One player rolls the cubes and states the 2-digit target number formed.
3. Use three, four, or five operations.
   Each player writes an expression equal to the target number.
   A power counts as an operation; brackets do not.
4. Score 1 point if you were able to write an expression that equals the target number.
   Score another point if you wrote a correct expression that no one else wrote.
5. The next player repeats Step 2, then all of you repeat Steps 3 and 4.
6. The first player to get 10 points wins.

2.4 Exponent Laws I

FOCUS

- Understand and apply the exponent laws for products and quotients of powers.

When we multiply numbers, the order in which we multiply does not matter.
For example, $(2 \times 2) \times 2 = 2 \times (2 \times 2)$
So, we usually write the product without brackets: $2 \times 2 \times 2$

Investigate

You will need 3 number cubes: 2 of one colour, the other a different colour

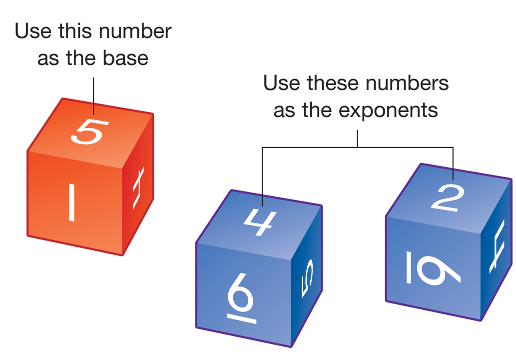
Two of you investigate multiplying powers. Make a table like this:

Product of Powers	Product as Repeated Multiplication	Product as a Power
$5^4 \times 5^2$	$(5 \times 5 \times 5 \times 5) \times (5 \times 5)$	$5^?$

Two of you investigate dividing powers. Make a table like this:

Quotient of Powers	Quotient as Repeated Multiplication	Quotient as a Power
$5^4 \div 5^2 = \frac{5^4}{5^2}$	$\frac{5 \times 5 \times 5 \times 5}{5 \times 5}$	$5^?$

Roll the cubes.
Use the numbers to create powers, as shown.



Record each quotient of powers with the greater exponent in the dividend (the numerator).

Express each power as repeated multiplication, and then as a single power.
Repeat the activity at least five times.

Reflect & Share

Describe the patterns in your table.
Share your patterns with the other pair in your group.
How are your patterns the same? How are they different?
Check your patterns with those of another group.
Use your patterns to describe a way to multiply two powers with the same base, and a way to divide two powers with the same base.

Connect

Patterns arise when we multiply and divide powers with the same base.

➤ To multiply $(-7)^3 \times (-7)^5$:

$$\begin{aligned}(-7)^3 \times (-7)^5 &= (-7)(-7)(-7) \times (-7)(-7)(-7)(-7)(-7)\\ &= (-7)(-7)(-7)(-7)(-7)(-7)(-7)(-7)\\ &= (-7)^8\end{aligned}$$

The base of the product is -7. The exponent is 8.
The sum of the exponents of the powers that were multiplied is $3 + 5 = 8$.
This relationship is true for the product of any two powers with the same base.
We use variables to represent the powers in the relationship:

Exponent Law for a Product of Powers

To multiply powers with the same base, add the exponents.

$a^m \times a^n = a^{m+n}$

The variable a is any integer, except 0.

The variables m and n are any whole numbers.

➤ To divide $8^7 \div 8^4$:

$$\begin{aligned}8^7 \div 8^4 &= \frac{8^7}{8^4}\\ &= \frac{8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8}{8 \times 8 \times 8 \times 8}\\ &= \frac{\cancel{8}^1 \times \cancel{8}^1 \times \cancel{8}^1 \times \cancel{8}^1 \times 8 \times 8 \times 8}{\cancel{8}_1 \times \cancel{8}_1 \times \cancel{8}_1 \times \cancel{8}_1}\\ &= \frac{8 \times 8 \times 8}{1}\\ &= 8 \times 8 \times 8\\ &= 8^3\end{aligned}$$

Divide the numerator and denominator of the fraction by their common factors: $8 \times 8 \times 8 \times 8$

So, $8^7 \div 8^4 = 8^3$
The base of the quotient is 8. The exponent is 3.
The difference of the exponents of the powers that were divided is $7 - 4 = 3$.
This relationship is true for the quotient of any two powers with the same base.

Exponent Law for a Quotient of Powers

To divide powers with the same base, subtract the exponents.

$a^m \div a^n = a^{m-n} \qquad m \geq n$

a is any integer, except 0; m and n are any whole numbers.

Example 1 Simplifying Products and Quotients with the Same Base

Write each expression as a power.

a) $6^5 \times 6^4$

b) $(-9)^{10} \div (-9)^6$

A Solution

a) The powers have the same base.
Use the exponent law for products: add the exponents.

$$6^5 \times 6^4 = 6^{(5+4)}$$
$$= 6^9$$

b) The powers have the same base.
Use the exponent law for quotients: subtract the exponents.

$$(-9)^{10} \div (-9)^6 = (-9)^{(10-6)}$$
$$= (-9)^4$$

Example 2 Evaluating Expressions Using Exponent Laws

Evaluate.

a) $(-2)^4 \times (-2)^7$

b) $3^2 \times 3^4 \div 3^3$

Solutions

Method 1

Simplify first using the exponent laws.

a) The bases are the same. Add exponents.

$$(-2)^4 \times (-2)^7 = (-2)^{(4+7)}$$
$$= (-2)^{11}$$
$$= -2048$$

b) All the bases are the same so add the exponents of the two powers that are multiplied. Then, subtract the exponent of the power that is divided.

$$3^2 \times 3^4 \div 3^3 = 3^{(2+4)} \div 3^3$$
$$= 3^6 \div 3^3$$
$$= 3^{(6-3)}$$
$$= 3^3$$
$$= 27$$

Method 2

Use the order of operations.

a) Evaluate each power first.
Then use a calculator.

$$(-2)^4 \times (-2)^7 = 16 \times (-128)$$
$$= -2048$$

b) Evaluate each power first.
Then use a calculator.

$$3^2 \times 3^4 \div 3^3 = 9 \times 81 \div 27$$

Multiply and divide in order from left to right.

$$3^2 \times 3^4 \div 3^3 = 27$$

Example 3 Using Exponent Laws and the Order of Operations

Evaluate.

a) $6^2 + 6^3 \times 6^2$

b) $(-10)^4[(-10)^6 \div (-10)^4] - 10^7$

A Solution

a) Multiply first. Add the exponents.

$$\begin{aligned} 6^2 + 6^3 \times 6^2 &= 6^2 + 6^{(3+2)} \\ &= 6^2 + 6^5 && \text{Evaluate each power.} \\ &= 36 + 7776 && \text{Then add.} \\ &= 7812 \end{aligned}$$

b) Evaluate the expression in the square brackets first.
Divide by subtracting the exponents.

$$\begin{aligned} (-10)^4[(-10)^6 \div (-10)^4] - 10^7 &= (-10)^4[(-10)^{(6-4)}] - 10^7 \\ &= (-10)^4(-10)^2 - 10^7 && \text{Multiply: add the exponents} \\ &= (-10)^{(4+2)} - 10^7 \\ &= (-10)^6 - 10^7 && \text{Evaluate each power.} \\ &= 1\,000\,000 - 10\,000\,000 && \text{Then subtract.} \\ &= -9\,000\,000 \end{aligned}$$

Discuss the ideas

1. Use your own words to explain how to:
   a) multiply two powers with the same base
   b) divide two powers with the same base

2. Do you think it makes sense to simplify an expression as much as possible before using a calculator? Explain.

3. When can you not add or subtract exponents to multiply or divide powers?

Practice

Check

4. Write each product as a single power.
   a) $5^5 \times 5^4$
   b) $10^2 \times 10^{11}$
   c) $(-3)^3 \times (-3)^3$
   d) $21^6 \times 21^4$
   e) $(-4)^1 \times (-4)^3$
   f) $6^{12} \times 6^3$
   g) $2^0 \times 2^4$
   h) $(-7)^3 \times (-7)^0$

5. Write each quotient as a power.
   a) $4^5 \div 4^3$
   b) $8^9 \div 8^6$
   c) $15^{10} \div 15^0$
   d) $(-6)^8 \div (-6)^3$
   e) $\frac{2^{12}}{2^{10}}$
   f) $\frac{(-10)^{12}}{(-10)^6}$
   g) $\frac{6^5}{6^1}$
   h) $\frac{(-1)^5}{(-1)^4}$

Apply

6. a) Evaluate.

i) $3^4 \div 3^4$ ii) $(-4)^6 \div (-4)^6$

iii) $\frac{5^8}{5^8}$ iv) $\frac{(-6)^3}{(-6)^3}$

b) Use the results of part a. Explain how the exponent law for the quotient of powers can be used to verify that a power with exponent 0 is 1.

7. a) Compare these products.

i) $3^4 \times 3^9$ ii) $3^9 \times 3^4$

b) Explain the results in part a.

8. Express as a single power.

a) $3^4 \times 3^9 \div 3^{11}$

b) $(-4)^3 \div (-4)^2 \times (-4)^{10}$

c) $6^0 \times 6^3 \div 6^2$

d) $\frac{4^3 \times 4^5}{4^2 \times 4^6}$ e) $\frac{(-3)^4 \times (-3)^4}{(-3)^4}$

9. a) Express as a single power, then evaluate.

i) $(-6)^1 \times (-6)^7 \div (-6)^7$

ii) $(-6)^7 \div (-6)^7 \times (-6)^1$

b) Explain why changing the order of the terms in the expressions in part a does not affect the answer.

10. Simplify, then evaluate.

a) $10^2 \times 10^2 + 10^4$ b) $10^3 \times 10^3 - 10^3$

c) $10^{11} - 10^3 \times 10^6$ d) $10^1 + 10^5 \times 10^2$

e) $10^6 \div 10^2 \times 10^2$ f) $10^9 \div 10^9$

g) $\frac{10^{12}}{10^6}$ h) $\frac{10^4 \times 10^3}{10^2}$

i) $\frac{10^{11}}{10^4 \times 10^2}$ j) $\frac{10^5}{10^3} + 10^2$

11. a) Evaluate: $2^6 - 2^2 \times 2^3$
Describe the steps you used.

b) Evaluate: $2^6 \times 2^2 - 2^3$
Describe the steps you used.

c) Were the steps for parts a and b different? Explain.

12. **Assessment Focus** An alfalfa field is a rectangle 10^4 m long and 10^3 m wide.

a) Write an expression for the area of the field, then evaluate the expression.

b) Write an expression for the perimeter of the field, then evaluate the expression.

c) i) Use the area in part a. Find all possible dimensions for a rectangular field with side lengths that are powers of 10.

ii) Find the perimeter of each field in part i.

d) Explain why the exponent laws are helpful for solving area problems, but not for perimeter problems.

13. Evaluate.

a) $2^3 \times 2^2 - 2^5 \times 2$

b) $3^2 \times 3 + 2^2 \times 2^4$

c) $4^2 - 3^0 \times 3 + 2^3$

d) $(-3)^6 \div (-3)^5 - (-3)^5 \div (-3)^3$

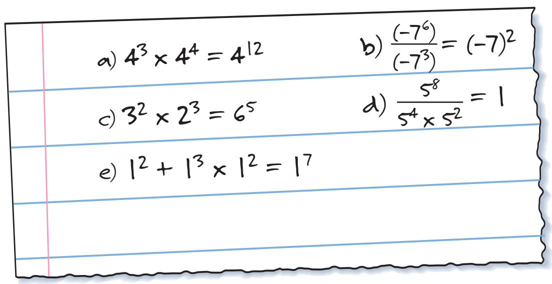
e) $(-2)^4[(-2)^5 \div (-2)^3] + (-2)^4$

f) $-2^4(2^6 \div 2^2) - 2^4$

g) $(-5)^3 \div (-5)^2 \times (-5)^0 + (-5)^2 \div (-5)$

14. Provide examples to show why the exponent laws for products and quotients cannot be applied when the powers have different bases.

15. Identify, then correct any errors in the student work below. Explain how you think the errors occurred.



16. Muguet uses a microscope to view bacteria. The bacteria are first magnified 10^2 times. This image is then magnified 10^1 times.

a) Use powers to write an expression for the total magnification.

b) How many times as large as the actual bacteria does the image appear?

17. a) Evaluate.

  i) $5^2 + 5^3$ **ii)** $5^2 \times 5^3$

b) In part a, explain why you could use an exponent law to simplify one expression, but not the other.

18. a) Evaluate.

  i) $4^3 - 4^2$ **ii)** $4^3 \div 4^2$

b) In part a, explain why you could use an exponent law to simplify one expression, but not the other.

19. Simplify, then evaluate only the expressions with a positive value. Explain how you know the sign of each answer without evaluating.

a) $(-2)^2 \times (-2)^3$

b) $(-2)^0 \times (-2)^5$

c) $(-2)^5 \div (-2)^3$

d) $(-2)^6 \div (-2)^6$

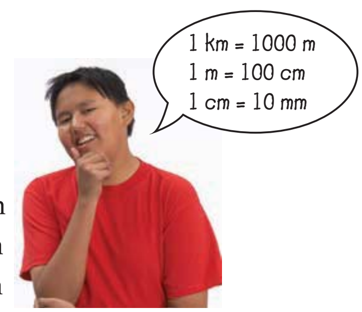
e) $\frac{(-2)^3 \times (-2)^4}{(-2)^3 \times (-2)^2}$

f) $\frac{(-2)^6}{(-2)^3 \times (-2)^2}$

Take It Further

20. Find two powers that have a product of 64. How many different pairs of powers can you find?

21. Write a product or quotient, then use the exponent laws to find the number of:



a) centimetres in 1 km

b) millimetres in 1 km

c) kilometres in 10^5 m

d) metres in 10^9 mm

22. Write a product or quotient, then use the exponent laws to find the number of:

a) square metres in 10^2 km^2

b) square metres in 10^6 cm^2

c) square millimetres in 10^6 cm^2

d) square centimetres in 1 km^2

23. Explain how the exponent laws help you to convert among units of measure.

Reflect

When can you use the exponent laws to evaluate an expression with powers? When can you *not* use these laws? Include examples in your explanation.

2.5 Exponent Laws II

FOCUS

- Understand and apply exponent laws for powers of: products; quotients; and powers.

A power indicates repeated multiplication.
What is the standard form of $(2^3)^2$? How did you find out?
$(2^3)^2$ is a **power of a power**.

The base of a power may be a product; for example, $(2 \times 3)^4$.
$(2 \times 3)^4$ is a **power of a product**.

Investigate

Copy and complete this table.
Choose your own power of a power to complete the 5th and 6th rows.
Choose your own power of a product to complete the 11th and 12th rows.

Power	As Repeated Multiplication	As a Product of Factors	As a Power	As a Product of Powers
$(2^4)^3$	$2^4 \times 2^4 \times 2^4$	(2)(2)(2)(2) × (2)(2)(2)(2) × (2)(2)(2)(2)	$2^?$	
$(3^2)^4$				
$[(-4)^3]^2$				
$[(-5)^3]^5$				
$(2 \times 5)^3$	$(2 \times 5) \times (2 \times 5) \times (2 \times 5)$	$2 \times 2 \times 2 \times 5 \times 5 \times 5$		$2^? \times 5^?$
$(3 \times 4)^2$				
$(4 \times 2)^5$				
$(5 \times 3)^4$				

Reflect & Share

What patterns do you see in the rows of the table?
Compare your patterns with those of another pair of classmates.
Use these patterns to record a rule for:

- writing the power of a power as a single power
- writing the power of a product as a product of two powers

How can you check your rules?

Connect

We can use the exponent laws from Lesson 2.4 to simplify powers written in other forms.

➤ Power of a power

We can raise a power to a power.
For example, 3^2 raised to the power 4 is written as $(3^2)^4$.
$(3^2)^4$ is a *power of a power*.
$(3^2)^4$ means $3^2 \times 3^2 \times 3^2 \times 3^2$.
So, $3^2 \times 3^2 \times 3^2 \times 3^2 = 3^{2+2+2+2}$ Using the exponent law for the product of powers
$= 3^8$
The exponent of 3^8 is the product of the exponents in $(3^2)^4$.
That is, $(3^2)^4 = 3^{2 \times 4}$
$= 3^8$
We can use this result to write an exponent law for the power of a power.

Exponent Law for a Power of a Power

To raise a power to a power, multiply the exponents.
$(a^m)^n = a^{mn}$
a is any integer, except 0.
m and *n* are any whole numbers.

mn means $m \times n$

➤ Power of a product

The base of a power may be a product; for example, $(3 \times 4)^5$.
$(3 \times 4)^5$ is a *power of a product*.
$(3 \times 4)^5$ means $(3 \times 4) \times (3 \times 4) \times (3 \times 4) \times (3 \times 4) \times (3 \times 4)$
So, $(3 \times 4) \times (3 \times 4) \times (3 \times 4) \times (3 \times 4) \times (3 \times 4)$
$= 3 \times 4 \times 3 \times 4 \times 3 \times 4 \times 3 \times 4 \times 3 \times 4$ Removing the brackets
$= (3 \times 3 \times 3 \times 3 \times 3) \times (4 \times 4 \times 4 \times 4 \times 4)$ Grouping equal factors
$= 3^5 \times 4^5$ Writing repeated multiplications as powers

We can use this result to write an exponent law for the power of a product.

Exponent Law for a Power of a Product

$(ab)^m = a^m b^m$

a and b are any integers, except 0.

m is any whole number.

➤ Power of a quotient

The base of a power may be a quotient; for example, $\left(\frac{5}{6}\right)^3$.

$\left(\frac{5}{6}\right)^3$ is a **power of a quotient**.

$\left(\frac{5}{6}\right)^3$ means $\left(\frac{5}{6}\right) \times \left(\frac{5}{6}\right) \times \left(\frac{5}{6}\right)$

So, $\left(\frac{5}{6}\right) \times \left(\frac{5}{6}\right) \times \left(\frac{5}{6}\right) = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$

$= \frac{5 \times 5 \times 5}{6 \times 6 \times 6}$ Multiplying the fractions

$= \frac{5^3}{6^3}$ Writing repeated multiplications as powers

We can use this result to write an exponent law for the power of a quotient.

Exponent Law for a Power of a Quotient

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $\quad b \neq 0$

a and b are any integers, except 0.

n is any whole number.

We can use these exponent laws to simplify or evaluate an expression.

Example 1 Simplifying a Power of a Power

Write as a power.

a) $[(-7)^3]^2$ **b)** $-(2^4)^5$ **c)** $(6^2)^7$

A Solution

Use the exponent law for a power of a power.

a) $[(-7)^3]^2 = (-7)^{3 \times 2}$
$= (-7)^6$

b) $-(2^4)^5 = -(2^{4 \times 5})$
$= -2^{20}$

c) $(6^2)^7 = 6^{2 \times 7}$
$= 6^{14}$

Example 2 Evaluating Powers of Products and Quotients

Evaluate.

a) $[(-7) \times 5]^2$ **b)** $[24 \div (-6)]^4$ **c)** $-(3 \times 2)^2$ **d)** $\left(\frac{78}{13}\right)^3$

Solutions

Method 1

a) Use the exponent law for a power of a product.

$$\begin{aligned}[(-7) \times 5]^2 &= (-7)^2 \times 5^2\\ &= 49 \times 25\\ &= 1225\end{aligned}$$

b) Use the exponent law for a power of a quotient. Write the quotient in fraction form.

$$\begin{aligned}[24 \div (-6)]^4 &= \left(\frac{24}{-6}\right)^4\\ &= \frac{24^4}{(-6)^4}\\ &= \frac{331\,776}{1296}\\ &= 256\end{aligned}$$

c) Use the exponent law for a power of a product.

$$\begin{aligned}-(3 \times 2)^2 &= -(3^2 \times 2^2)\\ &= -(9 \times 4)\\ &= -36\end{aligned}$$

d) Use the exponent law for a power of a quotient.

$$\begin{aligned}\left(\frac{78}{13}\right)^3 &= \frac{78^3}{13^3}\\ &= \frac{474\,552}{2197}\\ &= 216\end{aligned}$$

Method 2

Use the order of operations.

a) $$\begin{aligned}[(-7) \times 5]^2 &= (-35)^2\\ &= 1225\end{aligned}$$

b) $$\begin{aligned}[24 \div (-6)]^4 &= (-4)^4\\ &= 256\end{aligned}$$

c) $$\begin{aligned}-(3 \times 2)^2 &= -(6)^2\\ &= -6^2\\ &= -36\end{aligned}$$

d) $$\begin{aligned}\left(\frac{78}{13}\right)^3 &= 6^3\\ &= 216\end{aligned}$$

We can use the order of operations with the exponent laws when an expression involves the sum or difference of powers.

Example 3 Applying Exponent Laws and Order of Operations

Simplify, then evaluate each expression.

a) $(3^2 \times 3^3)^3 - (4^3 \times 4^2)^2$ **b)** $(6 \times 7)^2 + (3^8 \div 3^6)^3$ **c)** $[(-5)^3 + (-5)^4]^0$

A Solution

Use the exponent laws to simplify first, where appropriate.

a) In each set of brackets, the bases are the same, so use the exponent law for products.

$(3^2 \times 3^3)^3 - (4^3 \times 4^2)^2$

$= (3^{2+3})^3 - (4^{3+2})^2$ — Add the exponents in each set of brackets.

$= (3^5)^3 - (4^5)^2$ — Use the power of a power law.

$= 3^{5 \times 3} - 4^{5 \times 2}$ — Multiply the exponents.

$= 3^{15} - 4^{10}$ — Use a calculator.

$= 14\ 348\ 907 - 1\ 048\ 576$

$= 13\ 300\ 331$

b) Multiply in the first set of brackets. Use the exponent law for the quotient of powers in the second set of brackets.

$(6 \times 7)^2 + (3^8 \div 3^6)^3$

$= (42)^2 + (3^{8-6})^3$

$= 42^2 + (3^2)^3$ — Use the power of a power law.

$= 42^2 + 3^6$ — Use a calculator.

$= 1764 + 729$

$= 2493$

c) The expression is a power with exponent 0, so its value is 1.

$[(-5)^3 + (-5)^4]^0 = 1$

Discuss the ideas

1. Why do you add the exponents to simplify $3^2 \times 3^4$, but multiply the exponents to simplify the expression $(3^2)^4$?

2. a) What is the difference between a quotient of powers and a power of a quotient?

b) What is the difference between a product of powers and a power of a product?

3. In *Example 3*, is it easier to key the original expressions in a calculator or use the exponent laws to simplify first? Justify your answer.

Practice

Check

4. Write each expression as a product of powers.

a) $(6 \times 4)^3$ b) $(2 \times 5)^4$ c) $[(-2) \times 3]^5$
d) $(25 \times 4)^2$ e) $(11 \times 3)^1$ f) $[(-3) \times (-2)]^3$

5. Write each expression as a quotient of powers.

a) $(8 \div 5)^3$ b) $(21 \div 5)^4$ c) $[(-12) \div (-7)]^5$
d) $\left(\frac{10}{3}\right)^3$ e) $\left(\frac{1}{3}\right)^2$ f) $\left(\frac{27}{100}\right)^4$

6. Write as a power.

a) $(3^2)^4$ b) $(6^3)^3$ c) $(5^3)^1$
d) $(7^0)^6$ e) $-(8^2)^2$ f) $[(-3)^4]^2$

7. Simplify $(2^4)^2$ and $(2^2)^4$. What do you notice? Explain the results.

8. Write each expression as a product or quotient of powers.

a) $[3 \times (-5)]^3$ b) $-(2 \times 4)^5$
c) $\left(\frac{2}{3}\right)^4$ d) $\left(\frac{-7}{-2}\right)^2$
e) $-[(-10) \times 3]^3$ f) $(16 \div 9)^2$

Apply

9. Why is the value of $(-5^2)^3$ negative?

10. Simplify each expression, then evaluate it. For each expression, state the strategy you used and why.

a) $(3 \times 2)^3$ b) $[(-2) \times 4]^2$ c) $\left(\frac{9}{-3}\right)^3$
d) $\left(\frac{8}{2}\right)^2$ e) $(12^8)^0$ f) $[(-4)^2]^2$

11. Why is the value of $[(-2)^3]^4$ positive but the value of $[(-2)^3]^5$ is negative?

12. Compare the values of $-(4^2)^3$, $(-4^2)^3$, and $[(-4)^2]^3$.
What do you notice? Explain the results.

13. Assessment Focus For each expression below:

i) Evaluate it in two different ways:
- do the operation in brackets first
- use the exponent laws

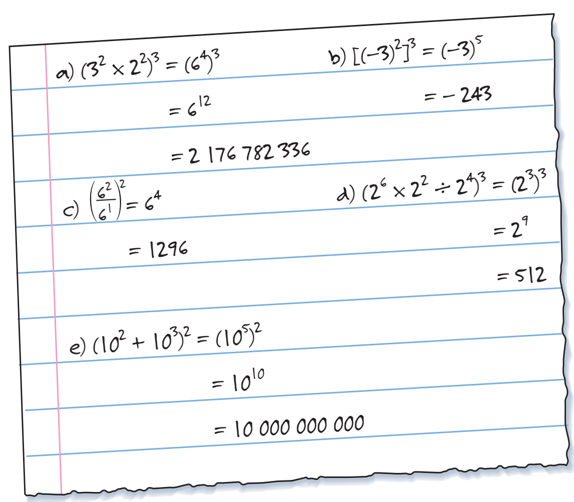
ii) Compare the results.
Which method do you prefer?
Was it always the same method each time? Explain.

a) $(4 \times 3)^3$ b) $[(-2) \times (-5)]^2$ c) $\left(\frac{6}{2}\right)^4$
d) $\left(\frac{14}{2}\right)^0$ e) $[(-5)^2]^2$ f) $(2^5)^3$

14. Simplify, then evaluate. Show your work.

a) $(3^2 \times 3^1)^2$ b) $(4^6 \div 4^4)^2$
c) $[(-2)^0 \times (-2)^3]^2$ d) $(10^6 \div 10^4)^3$
e) $(10^3)^2 \times (10^2)^3$ f) $(12^2)^4 \div (12^3)^2$
g) $(5^2)^6 \div (5^3)^4$ h) $[(-2)^2]^3 \times (-2)^3$

15. Find any errors in this student's work.
Copy the solution and correct the errors.



16. Simplify, then evaluate each expression.

a) $(4^2 \times 4^3)^2 - (5^4 \div 5^2)^2$

b) $(3^3 \div 3^2)^3 + (8^4 \times 8^3)^0$

c) $(2^3)^4 + (2^4 \div 2^3)^2$

d) $(6^2 \times 6^0)^3 + (2^6 \div 2^4)^3$

e) $(5^3 \times 5^3)^0 - (4^2)^2$

f) $(10^5 \div 10^2)^2 + (3^3 \div 3^1)^4$

17. Simplify, then evaluate each expression.

a) $[(-2)^3 \times (-2)^2]^2 - [(-3)^3 \div (-3)^2]^2$

b) $[(-2)^3 \div (-2)^2]^2 - [(-3)^3 \times (-3)^2]^2$

c) $[(-2)^3 \times (-2)^2]^2 + [(-3)^3 \div (-3)^2]^2$

d) $[(-2)^3 \div (-2)^2]^2 + [(-3)^3 \times (-3)^2]^2$

e) $[(-2)^3 \div (-2)^2]^2 - [(-3)^3 \div (-3)^2]^2$

f) $[(-2)^3 \times (-2)^2]^2 + [(-3)^3 \times (-3)^2]^2$

18. Use grid paper. For each expression below:

i) Draw a rectangle to represent the expression.

ii) Use the exponent laws to write the expression as a product of squares.

iii) Draw a rectangle to represent the new form of the expression.

iv) Compare the two rectangles for each expression.
How are the rectangles the same?
How are they different?
Use these rectangles to explain how the square of a product and the product of squares are related.

a) $(2 \times 3)^2$ b) $(2 \times 4)^2$

c) $(3 \times 4)^2$ d) $(1 \times 4)^2$

19. Simplify, then evaluate each expression.

a) $(2^3 \times 2^6)^2 - (3^7 \div 3^5)^4$

b) $(6 \times 8)^5 + (5^3)^2$

c) $[(-4)^3 \times (-4)^2]^2 + (4^3 \times 4^2)^2$

d) $[(-2)^4]^3 + [(-4)^3]^2 - [(-3)^2]^4$

e) $[(-3)^4]^2 \times [(-4)^0]^2 - [(-3)^3]^0$

f) $[(-5) \times (-4)]^3 + [(-6)^3]^2$
$- [(-3)^9 \div (-3)^8]^5$

Take It Further

20. a) Write 81:

i) as a power of 9

ii) as a power of a product

iii) as a power of 3

b) Write 64:

i) as a power of 8

ii) as a power of a product

iii) as a power of 2

c) Find other numbers for which you can follow steps similar to those in parts a and b.

21. a) List the powers of 2 from 2^0 to 2^{12} in standard form.

b) Use your list from part a to write each number in the expressions below as a power of 2. Evaluate each expression using the exponent laws and the list in part a.

i) 32×64 ii) $16 \times 8 \times 32$

iii) $1024 \div 128$ iv) $\frac{16 \times 256}{1024}$

v) $(8 \times 4)^3$ vi) $\left(\frac{256}{64}\right)^4$

Reflect

Design and create a poster that summarizes all the exponent laws you have learned. Provide an example of each law.

Study Guide

- A power represents repeated multiplication.

  $2^5 = 2 \times 2 \times 2 \times 2 \times 2$
  $= 32$
  $(-3)^4 = (-3)(-3)(-3)(-3)$
  $= 81$
  $-3^4 = -(3)(3)(3)(3)$
  $= -81$

- A power with an integer base, other than 0, and an exponent 0 is equal to 1.

  $2^0 = 1$
  $(-4)^0 = 1$
  $-4^0 = -1$

- To evaluate an expression, follow this order of operations:

  Evaluate inside brackets.
  Evaluate powers.
  Multiply and divide, in order, from left to right.
  Add and subtract, in order, from left to right.

Exponent Laws

m and n are whole numbers.
a and b are any integers, except 0.

- Product of Powers

  $a^m \times a^n = a^{m+n}$

- Quotient of Powers

  $a^m \div a^n = a^{m-n} \quad m \geq n$

- Power of a Power

  $(a^m)^n = a^{mn}$

- Power of a Product

  $(ab)^m = a^m b^m$

- Power of a Quotient

  $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad b \neq 0$

Review

2.1

1. Write as repeated multiplication, then in standard form.

a) 4^3 **b)** 7^2 **c)** $-(-2)^5$

d) -3^4 **e)** -1^8 **f)** $(-1)^8$

2. Use tiles and cubes to explain the difference between 2^2 and 2^3.

3. Write as a power, then in standard form.

a) $3 \times 3 \times 3 \times 3 \times 3 \times 3$

b) $(-8)(-8)(-8)$

c) $-(2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2)$

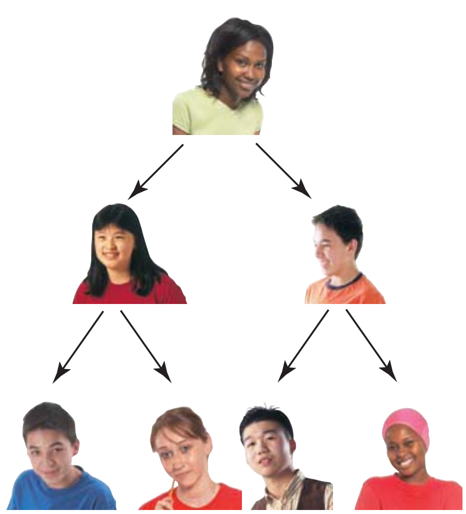
d) 12×12

e) $4 \times 4 \times 4 \times 4 \times 4$

f) $(-5)(-5)(-5)(-5)$

4. Explain the difference between 5^8 and 8^5.

5. A telephone tree is used to send messages.
The person at the top calls 2 people.
Each person calls 2 more people.
Suppose it takes 1 min to call someone.
A message is relayed until the bottom row of the tree has 256 people.
How long does this take?
How do you know?



6. a) Is the value of -4^2 different from the value of $(-4)^2$?
What purpose do the brackets serve?

b) Is the value of -2^3 different from the value of $(-2)^3$?
What purpose do the brackets serve?

7. a) Evaluate each expression.

i) -3^2 **ii)** $-(3)^2$ **iii)** $-(-3)^2$ **iv)** $(-3)^2$

b) For each expression in part a that includes brackets, explain what the brackets show.

2.2

8. Write as a power of 10.

a) one hundred million

b) $10 \times 10 \times 10 \times 10$

c) 1

d) 1 000 000 000

e) one thousand

9. Use powers of 10 to write each number.

a) 700 000 000

b) 345

c) 80 027

10. a) Copy and complete this table.

Power	Repeated Multiplication	Standard Form
3^5	3 × 3 × 3 × 3 × 3	243
3^4		
	3 × 3 × 3	
3^2		
		3

b) Describe the patterns in the table.

c) Extend the pattern to show why any number with an exponent of 0 is equal to 1.

11. **a)** The tallest tree in the world, Hyperion in California, is about 10^2 m tall. The highest mountain, Mount Everest, is about 10^4 m high. About how many times as high as the tree is the mountain?

b) Earth's diameter is about 10^7 m. The largest known star has a diameter of about 10^{12} m. About how many times as great as the diameter of Earth is the diameter of the largest known star?

12. Write each number in standard form.

a) $(4 \times 10^3) + (7 \times 10^2) + (2 \times 10^1) + (9 \times 10^0)$

b) $(3 \times 10^5) + (2 \times 10^2) + (8 \times 10^0)$

2.3 **13.** Evaluate.

a) $3^4 + 3^2$ **b)** $(-4)^2 + (-4)^3$

c) $10^3 - 10^2$ **d)** $(-5)^4 - (-5)^2$

14. Evaluate.

a) $2^3 + (5 - 2)^4$

b) $100 \div 2 + (4 + 1)^3$

c) $(6^2 + 7^2)^0 - (8^4 + 2^4)^0$

d) $3 \times 2^3 + 8 \div 4$

e) $(21 \div 7)^4 - 2^3$

f) $[(-4)^0 \times 10]^6 \div (15 - 10)^2$

15. Scientists grow bacteria. This table shows how the number of bacteria doubles every hour.

Time	Elapsed Time After Noon (h)	Number of Bacteria
noon	0	1000×2^0
1:00 P.M.	1	1000×2^1
2:00 P.M.	2	1000×2^2
3:00 P.M.	3	1000×2^3

a) Evaluate the expressions in the table to find the number of bacteria at each time.

i) noon **ii)** 1:00 P.M.

iii) 2:00 P.M. **iv)** 3:00 P.M.

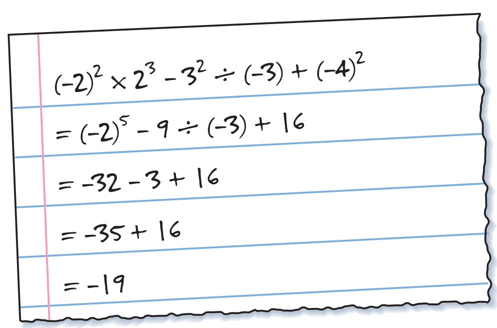
b) The pattern continues. Write an expression, then evaluate it, to find the number of bacteria at each time.

i) 4:00 P.M. **ii)** 6:00 P.M.

iii) 9:00 P.M. **iv)** midnight

16. Use a calculator to evaluate this expression:
$4^3 - (2 \times 3)^4 + 11$
Change the position of the brackets. Evaluate the new expression. How many different answers can you get by changing only the position of the brackets?

17. Identify, then correct, any errors in the student work below. Explain how you think the errors occurred.



2.4 **18.** Write each product as a power, then evaluate the power.

a) $5^3 \times 5^4$ b) $(-2)^3 \times (-2)^2$
c) $3^2 \times 3^3 \times 3^1$ d) $-10^4 \times 10^0$

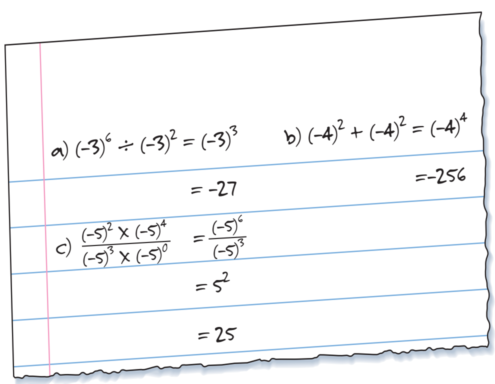
19. There are about 10^{11} galaxies in the universe. Each galaxy contains about 10^{11} stars. About how many stars are in the universe?

20. Write each quotient as a power, then evaluate the power.

a) $7^5 \div 7^3$ b) $(-10)^9 \div (-10)^3$
c) $\frac{8^4}{8^2}$ d) $-\frac{6^7}{6^4}$

21. a) Can you use the laws of exponents to simplify $6^3 \times 5^5$? Explain.
b) Can you use the laws of exponents to simplify $27^2 \div 9^2$? Explain.

22. Find and correct any errors in the student work below.
Explain how you think the errors occurred.



2.5 **23.** Write each expression as a product or quotient of powers, then evaluate it.

a) $(3 \times 5)^3$ b) $(12 \div 3)^5$
c) $[(-4) \times 2]^4$ d) $(63 \times 44)^0$
e) $\left(\frac{3}{2}\right)^5$ f) $\left(\frac{15}{2}\right)^2$

24. Write each expression as a power.

a) $(3^2)^3$ b) $(4^0)^6$
c) $[(-2)^3]^3$ d) $(5^5)^2$

25. For each expression below:
Evaluate it in two different ways:
i) do the operation in brackets first
ii) use the exponent laws
In each case, which method is more efficient? Explain why.

a) $(5 \times 3)^3$
b) $(3 \times 3)^4$
c) $(8 \div 2)^5$
d) $\left(\frac{9}{3}\right)^2$
e) $(2^3)^4$
f) $(6^2)^0$

26. Write each expression as a power, then evaluate.

a) $6^4 \times 6^3$
b) $(-11)^7 \div (-11)^5$
c) $\frac{3^4 \times 3^5}{3^3}$
d) $\frac{5^5}{5^3 \times 5^2}$
e) $\frac{(-4)^3 \times (-4)^6}{(-4)^2 \times (-4)^4}$
f) $\frac{10^6 \times 10^0}{10^3 \times 10^2}$

27. Simplify, then evaluate each expression.

a) $2^3 \times 2^2 - 2^0 + 2^4 \div 2^3$
b) $\frac{(-2)^3 \times (-2)^2}{(-2)^3 - (-2)^2}$
c) $12^2 \times 12^4 \div (-2)^4 - 12^0$
d) $\frac{(-12)^2 \times (-12)^4}{(-2)^4 - 12^0}$

Practice Test

1. Write as a product or quotient of powers.

   a) $(3 \times 4)^3$ b) $[(-5) \times 2]^4$ c) $\left(\frac{1}{4}\right)^4$ d) $-\left(\frac{9}{3}\right)^3$

2. Simplify.

   a) $-(2^3)^3$ b) $(6^2)^0$ c) $[(-5)^2]^3$ d) $-[(-3)^2]^4$

3. Simplify each expression, then evaluate it.

   a) $[(-3) \times (-2)]^4$ b) $\left(\frac{1}{2}\right)^5$ c) $(6^0)^4$ d) $[(-3)^2]^3$

4. Is the value of a power with a negative base always negative? Or, is it always positive? Or, is it sometimes negative and sometimes positive? Illustrate your answer with some examples.

5. A baseball diamond is a square with side length about 27 m. Is the area of the baseball diamond greater or less than 10^3 m^2? How do you know?

6. Explain why the brackets are not necessary in this expression:
   $(-3^5 \times 10) - (9 \div 3)$
   Evaluate the expression, showing each step.

7. Identify the correct answer for $(2^3 + 4)^2 \times (-10)^3 \div (5 + 5)^2$.

   a) -240 b) -1440 c) 1440 d) $-28\,825$

   Explain how each of the other incorrect answers could have been determined.

8. Evaluate only the expressions with a positive value. Explain how you know the sign of each expression before you evaluate it.

   a) $(-5)^3 \times (-5)^2 \div (-5)^1$ b) $[(-9)^6 - (-9)^3]^0$

   c) $\dfrac{(-1)^2 \times (-1)^4}{(-1)^3 \times (-1)^2}$ d) $(-4)^6 + (-4)^4 \times (-4)^0$

Unit Problem How Thick Is a Pile of Paper?

You will need a sheet of paper and a ruler.

➤ Fold the paper in half to form 2 layers. Fold it in half again.
Keep folding until you cannot make the next fold.

➤ Create a table to show how many layers of paper there are after each fold.

Number of Folds	Number of Layers
0	1
1	2

Complete the table for the number of folds you were able to make.

➤ Look for a pattern in the numbers of layers.
How can you express the pattern using powers?
Draw another column on your table to show the *Number of Layers as Powers.*
Suppose you could make 25 folds.
Use patterns in the table to predict a power for the number of layers after 25 folds. Evaluate the power.

➤ Measure the thickness of 100 sheets (200 pages) in your math textbook.
Use this measure to calculate the thickness of 1 sheet of paper in millimetres.
How high would the layers be if you could make 25 folds? Give your answer in as many different units as you can.
What do you know that is approximately this height or length?

Your work should show:

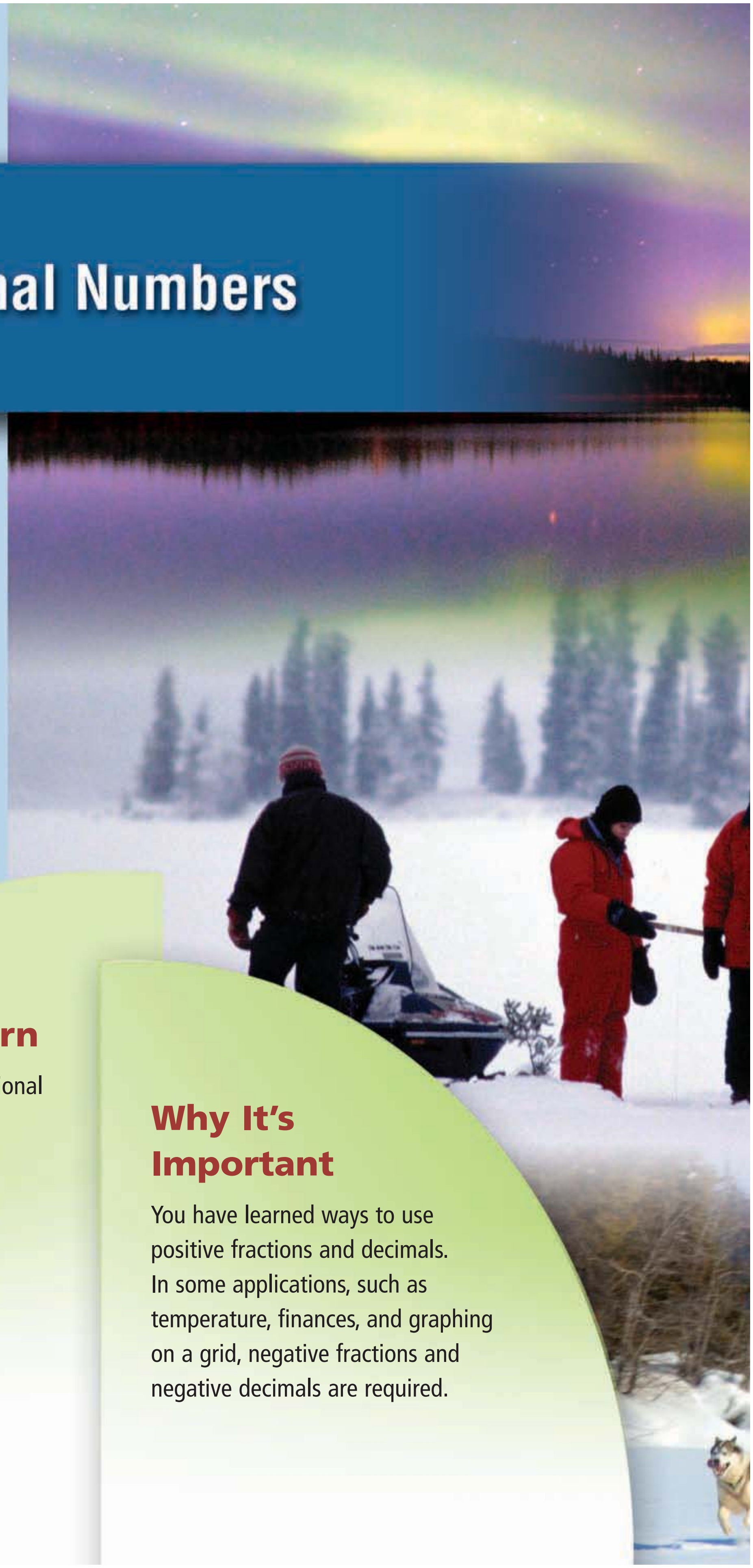
- a completed table showing the numbers of layers
- the calculations of the thickness of 1 layer and the height after 25 folds
- an example of something with the same height

Reflect on Your Learning

What have you learned about powers and their exponent laws?
What ways can you think of to remember the laws and how to use them?

UNIT 3

Rational Numbers



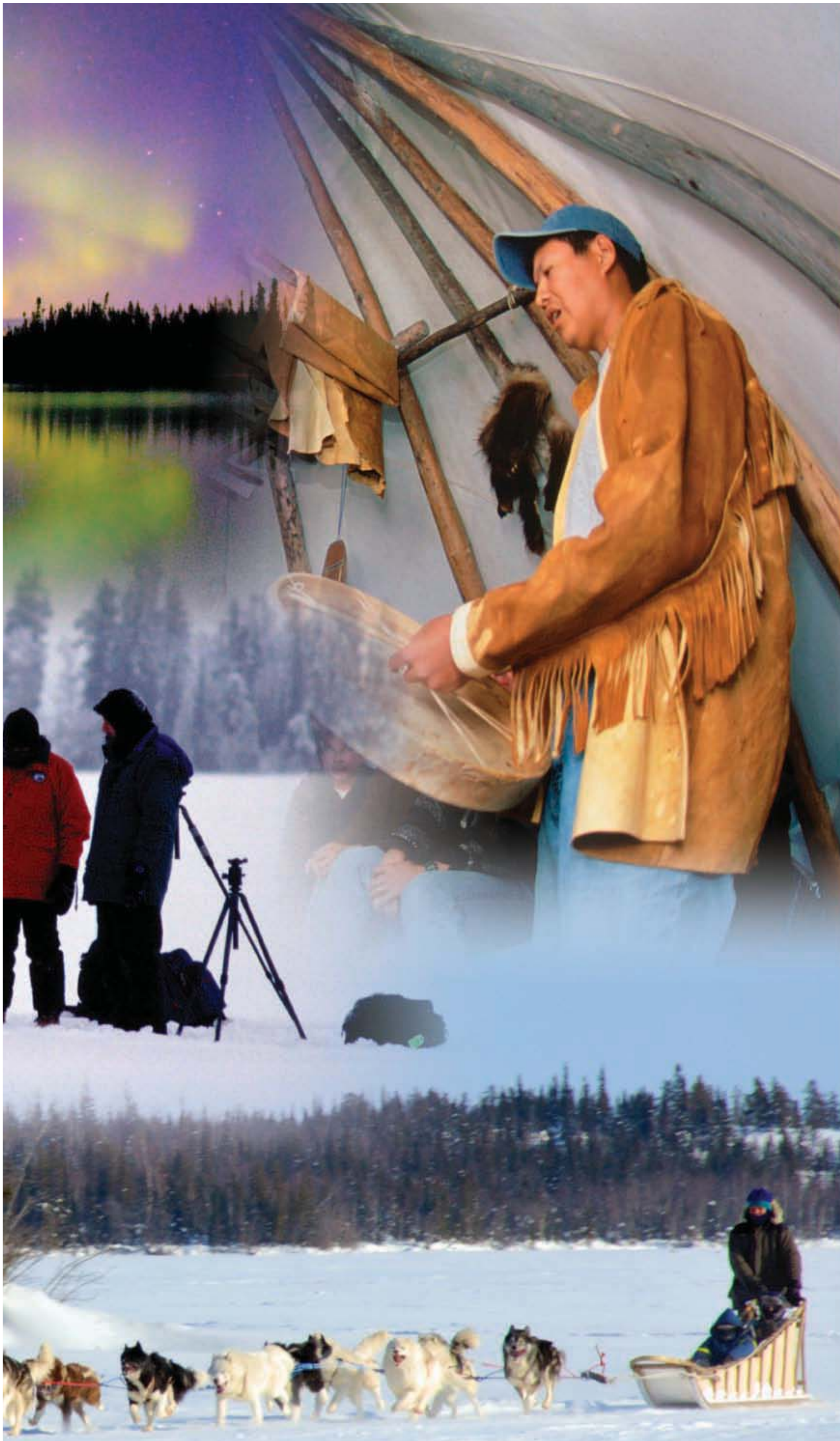
Suppose you are ice fishing on Blachford Lake, NWT. The temperature at midnight is –12°C. At 6 A.M. the next day, the temperature is –11°C. What must the temperature have been at some time during the night?

What You'll Learn

- Compare and order rational numbers.
- Solve problems by adding, subtracting, multiplying, and dividing rational numbers.
- Explain and apply the order of operations with rational numbers, with and without technology.

Why It's Important

You have learned ways to use positive fractions and decimals. In some applications, such as temperature, finances, and graphing on a grid, negative fractions and negative decimals are required.



Key Words

- rational number
- irrational number

3.1 What Is a Rational Number?

FOCUS
- Compare and order rational numbers.

The label on a package of frozen cranberries says that it must be stored at a temperature between −18°C and −22°C. Name some possible temperatures. How could these temperatures be shown on a number line?

Investigate

2

➤ Determine each quotient.

$\frac{12}{2}$ $\frac{-12}{2}$ $\frac{12}{-2}$

➤ Use what you know about integer division to determine each quotient.

$\frac{11}{2}$ $\frac{-11}{2}$ $\frac{11}{-2}$

$\frac{3}{5}$ $\frac{-3}{5}$ $\frac{3}{-5}$

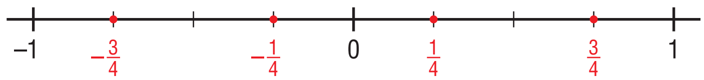
➤ On a number line, mark a point for each quotient.
How can you name the point another way?

Reflect & Share

Compare your strategies and answers with those of another pair of classmates.
Use integer division to explain what each fraction means.
How could you write each answer as a decimal?

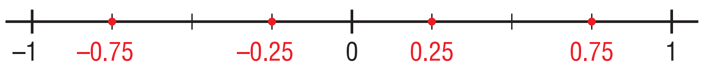
Connect

We extend a number line to the left of 0 to show negative integers.
We can also represent negative fractions on a number line.
$-\frac{3}{4}$ is the same distance to the left of 0 as $\frac{3}{4}$ is to the right of 0.



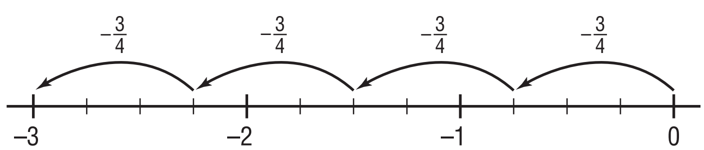
We use the same symbol to represent a negative number as we use for subtraction.

For every positive fraction, there is a corresponding negative fraction.
$-\frac{3}{4}$ and $\frac{3}{4}$ are opposites.
Any fraction can be written as a decimal; so, for every positive decimal there is a corresponding negative decimal.



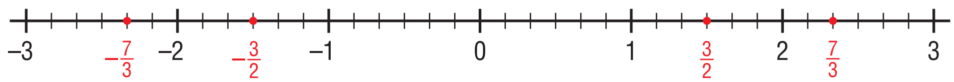
0.25 and -0.25 are opposites.

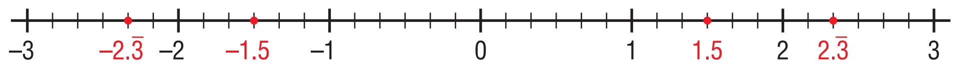
Any number that can be written as a fraction with an integer numerator and a non-zero integer denominator is a **rational number**; for example, $\frac{3}{4}, \frac{-3}{4}, \frac{3}{-4}$
To visualize $\frac{-3}{4}$, use a number line and think of $(-3) \div 4$.



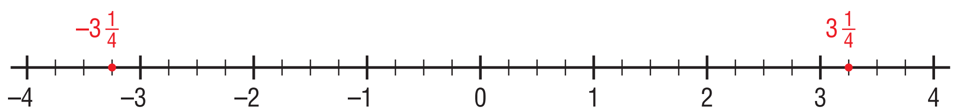
Each part is $-\frac{3}{4}$.
So, $\frac{-3}{4}$ is the same as $-\frac{3}{4}$.
The value of a fraction remains the same if its numerator and denominator are multiplied by the same non-zero number.
$\frac{3}{-4}$ can be written as $\frac{3}{-4} \times \frac{-1}{-1} = \frac{-3}{4}$
Since $\frac{3}{-4} = \frac{-3}{4}$ and $\frac{-3}{4} = -\frac{3}{4}$, then $\frac{3}{-4} = \frac{-3}{4} = -\frac{3}{4}$

A fraction can be written as a terminating or repeating decimal:





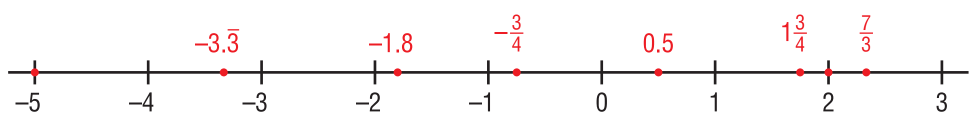
Any mixed number can be written as an improper fraction:
$3\frac{1}{4} = \frac{13}{4}$ and $-3\frac{1}{4} = -\frac{13}{4}$



So, mixed numbers are rational numbers.

Any integer can be written as a fraction with denominator 1;
for example, $-12 = \frac{-12}{1}$, so integers are rational numbers.

All these numbers are rational numbers:
$-\frac{3}{4}$, 0.5, -1.8, 0, -5, $\frac{7}{3}$, 2, $-3.\overline{3}$, $1\frac{3}{4}$



Definition of a Rational Number
A rational number is any number that can be written in the form $\frac{m}{n}$, where m and n are integers and $n \neq 0$.

Not all numbers can be written as fractions. For example, π and $\sqrt{2}$ are numbers that you have used in calculations but they cannot be written as fractions. These are **irrational numbers**.

Example 1 Writing a Rational Number between Two Given Numbers

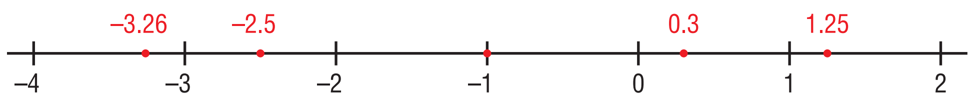
Write 3 rational numbers between each pair of numbers.

a) 1.25 and -3.26 **b)** -0.25 and -0.26 **c)** $-\frac{1}{2}$ and $\frac{1}{4}$ **d)** $-\frac{1}{2}$ and $-\frac{1}{4}$

A Solution

There are many rational numbers between any two given numbers.
Sketch or visualize a number line in each case.

a) 1.25 and -3.26
Label a number line with integers from -4 to 2.

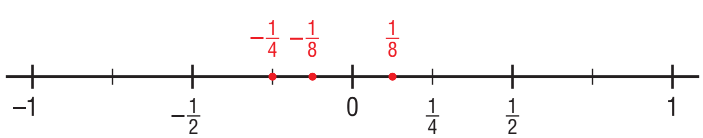


From the number line, 3 possible rational numbers are:
-2.5, -1, and 0.3

b) -0.25 and -0.26
Label a number line with these rational numbers.
Divide the line into 10 equal parts.

From the number line, 3 possible rational numbers are:
-0.252, -0.255, and -0.259

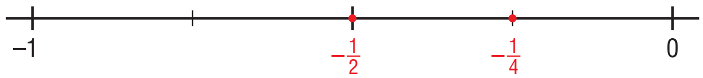
c) $-\frac{1}{2}$ and $\frac{1}{4}$
Label a number line from -1 to 1.
Divide the line into quarters.



From the number line, 3 possible rational numbers are:
$-\frac{1}{4}$, $-\frac{1}{8}$, and $\frac{1}{8}$

d) $-\frac{1}{2}$ and $-\frac{1}{4}$

Label a number line from -1 to 0. Divide the line into quarters.



Write equivalent fractions for $-\frac{1}{2}$ and $-\frac{1}{4}$ with denominators of 8 to identify fractions between the two numbers.

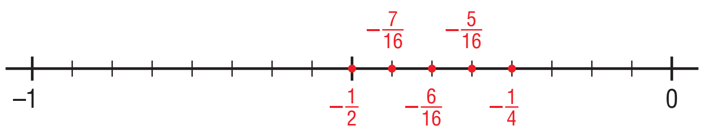
$-\frac{1}{2} = -\frac{2}{4} = -\frac{4}{8}$ $\qquad$ $-\frac{1}{4} = -\frac{2}{8}$

Between $-\frac{4}{8}$ and $-\frac{2}{8}$, there is only one fraction, $-\frac{3}{8}$, with denominator 8.

So, write equivalent fractions with denominator 16:

$-\frac{1}{2} = -\frac{2}{4} = -\frac{4}{8} = -\frac{8}{16}$ $\qquad$ $-\frac{1}{4} = -\frac{2}{8} = -\frac{4}{16}$

Divide the number line into sixteenths.



From the number line, 3 possible rational numbers are: $-\frac{5}{16}$, $-\frac{6}{16}$, and $-\frac{7}{16}$

Example 2 Ordering Rational Numbers in Decimal or Fraction Form

a) Use a number line. Order these numbers from least to greatest.

$0.35, 2.5, -0.6, 1.7, -3.2, -0.\overline{6}$

b) Order these numbers from greatest to least. Record the numbers on a number line.

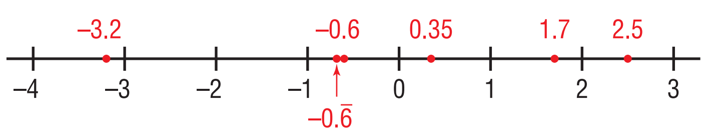
$-\frac{3}{8}, \frac{5}{9}, -\frac{10}{4}, -1\frac{1}{4}, \frac{7}{10}, \frac{8}{3}$

Solutions

a) $0.35, 2.5, -0.6, 1.7, -3.2, -0.\overline{6}$

Mark each number on a number line.

$-0.\overline{6} = -0.666\ 666\ldots$; so, $-0.\overline{6} < -0.6$

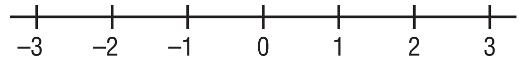


For least to greatest, read the numbers from left to right: $-3.2, -0.\overline{6}, -0.6, 0.35, 1.7, 2.5$

Method 1

b) $-\frac{3}{8}, \frac{5}{9}, -\frac{10}{4}, -1\frac{1}{4}, \frac{7}{10}, \frac{8}{3}$

Visualize a number line.



Consider the positive numbers: $\frac{5}{9}, \frac{7}{10}, \frac{8}{3}$

Only $\frac{8}{3}$ is greater than 1.

Both $\frac{5}{9}$ and $\frac{7}{10}$ are between 0 and 1.

To order $\frac{5}{9}$ and $\frac{7}{10}$, write them with a common denominator:

$9 \times 10 = 90$

$\frac{5}{9} = \frac{50}{90} \qquad \frac{7}{10} = \frac{63}{90}$

Since $\frac{63}{90} > \frac{50}{90}$, then $\frac{7}{10} > \frac{5}{9}$

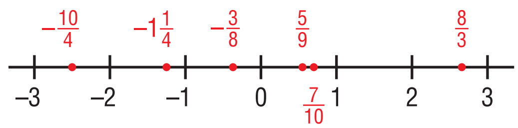
Consider the negative numbers: $-\frac{3}{8}, -\frac{10}{4}, -1\frac{1}{4}$

$-1\frac{1}{4}$ is the improper fraction $-\frac{5}{4}$, which is greater than $-\frac{10}{4}$.

$-\frac{3}{8}$ is greater than $-1\frac{1}{4}$.

From greatest to least, the numbers are:

$\frac{8}{3}, \frac{7}{10}, \frac{5}{9}, -\frac{3}{8}, -1\frac{1}{4}, -\frac{10}{4}$



Method 2

$-\frac{3}{8}, \frac{5}{9}, -\frac{10}{4}, -1\frac{1}{4}, \frac{7}{10}, \frac{8}{3}$

Write each number as a decimal.

Use a calculator when necessary.

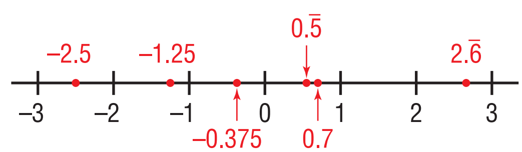
$-\frac{3}{8} = -0.375 \qquad \frac{5}{9} = 0.\overline{5}$

$-\frac{10}{4} = -2.5 \qquad -1\frac{1}{4} = -1.25$

$\frac{7}{10} = 0.7 \qquad \frac{8}{3} = 2.\overline{6}$

Mark each decimal on a number line.

Use the order of the decimals to order the fractions.



From greatest to least, the numbers are:

$\frac{8}{3}, \frac{7}{10}, \frac{5}{9}, -\frac{3}{8}, -1\frac{1}{4}, -\frac{10}{4}$

Example 3 Ordering Rational Numbers in Fraction and Decimal Form

Order these rational numbers from least to greatest.

$1.13, -\frac{10}{3}, -3.4, 2.\overline{7}, \frac{3}{7}, -2\frac{2}{5}$

Record the numbers on a number line.

A Solution

$1.13, -\frac{10}{3}, -3.4, 2.\overline{7}, \frac{3}{7}, -2\frac{2}{5}$

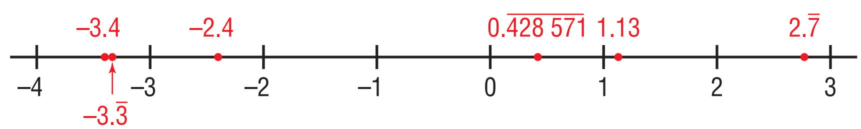
Write the fractions and mixed number as decimals.

$-\frac{10}{3} = -3.\overline{3}$

$\frac{3}{7} = 0.\overline{428\,571}$

$-2\frac{2}{5} = -2.4$

Mark each decimal on a number line.



For least to greatest, read the decimals from left to right.

The order is:

$-3.4, -\frac{10}{3}, -2\frac{2}{5}, \frac{3}{7}, 1.13, 2.\overline{7}$

Discuss the ideas

1. How can you use what you know about fractions and integers to explain what a rational number is?

2. How are positive fractions and their opposites related on a number line?

3. In the definition of a rational number as $\frac{m}{n}$, where m and n are integers, why is it important that $n \neq 0$?

4. Describe the numbers that are rational, but are not positive fractions or integers.

Practice

Check

5. Identify equal rational numbers in the list that follows.

$\frac{2}{3}$ $\quad \frac{-3}{2}$ $\quad \frac{-2}{3}$ $\quad -\frac{2}{3}$

$-\frac{3}{2}$ $\quad \frac{2}{-3}$ $\quad \frac{3}{-2}$ $\quad \frac{3}{2}$

6. For each rational number, write two fractions that represent the same number.

a) $\frac{7}{-9}$ **b)** $\frac{-5}{3}$ **c)** $-\frac{6}{11}$

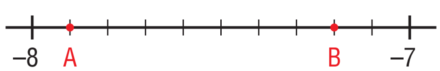
7. Write each rational number as a decimal.

a) $\frac{6}{5}$ **b)** $-\frac{6}{5}$

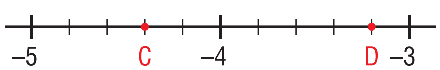
c) $\frac{9}{4}$ **d)** $-\frac{11}{6}$

8. Write the rational number represented by each letter on the number line, as a decimal.

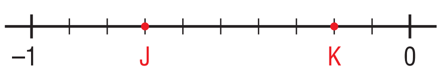
a)



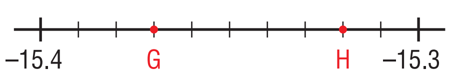
b)



c)



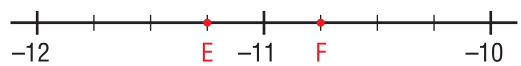
d)



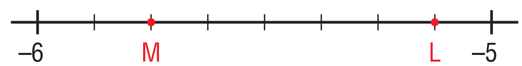
9. For each pair of rational numbers in question 8, identify the greater number.

10. Write the rational number represented by each letter on the number line, as a fraction.

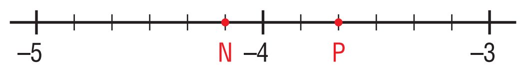
a)



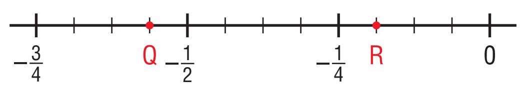
b)



c)



d)



11. For each pair of rational numbers in question 10, identify the lesser number.

Apply

12. Write 3 rational numbers between each pair of numbers. Sketch a number line to show all the rational numbers.

a) 3.7, 4.2

b) −1.5, 0

c) −4.5, −4

d) −5.6, −4.5

e) −5.6, 5.7

f) 5.6, −5.7

g) −5.6, −5.7

h) −2.98, −2.99

13. The thermostat on a freezer is set at $-18°C$. The compressor on the freezer turns on and cools down the freezer when the temperature rises to $-15.5°C$. The compressor turns off when the temperature drops to $-19.5°C$.

a) Sketch a thermometer and mark the 3 freezer temperatures.

b) A package of meat must remain below $-18°C$. Should this freezer be used? Explain.

14. Write 3 rational numbers between each pair of numbers. Sketch a number line to show all the rational numbers.

a) $\frac{5}{8}, \frac{13}{8}$ **b)** $\frac{17}{10}, -\frac{11}{5}$

c) $-\frac{15}{4}, -\frac{11}{3}$ **d)** $-\frac{1}{2}, -\frac{1}{8}$

e) $\frac{1}{6}$, 0.5 **f)** $-0.25, -\frac{1}{3}$

g) $-\frac{14}{5}, -3$ **h)** $5\frac{3}{5}, 5\frac{4}{5}$

15. Sketch a number line and mark each rational number on it:
$\frac{3}{5}, -\frac{5}{7}, -\frac{8}{3}, -\frac{19}{5}$

16. Which rational number is greater? Which strategies did you use to find out?

a) 2.34, 2.3 **b)** $-2.34, -2.3$

c) -1.4, 1.4 **d)** 3.96, -4.12

e) $-5.\overline{6}, -5.6$ **f)** $2.\overline{86}$, 2.866

17. Which rational number is less? Explain how you know.

a) $\frac{3}{4}, \frac{3}{5}$ **b)** $2\frac{1}{2}, -1\frac{7}{8}$

c) $-\frac{13}{10}, -\frac{13}{5}$ **d)** $-\frac{11}{3}, -\frac{5}{6}$

18. Which rational number is greater? How do you know?

a) $\frac{3}{4}, \frac{6}{7}$ **b)** $-\frac{3}{4}, -\frac{6}{7}$

c) $-\frac{6}{7}, -\frac{7}{6}$ **d)** $-\frac{9}{5}, \frac{5}{9}$

19. A student said, "When I compare two numbers, I know that the lesser number is closer to 0." Is this statement always true? Sometimes true? Never true? Explain.

20. **Assessment Focus**

a) Mark these rational numbers on a number line:
$1.4, -\frac{11}{8}, -3.6, 4\frac{1}{3}, 0.8, -\frac{17}{3}$

b) Which rational numbers in part a are less than -1? How do you know?

c) Which rational numbers in part a are greater than -2? How do you know?

d) Write one rational number between each pair of numbers on the number line.

21. Use <, >, or = to make each expression true. Justify your answers.

a) $-\frac{5}{7} \square -\frac{4}{7}$

b) $-\frac{5}{6} \square -\frac{5}{7}$

c) $-2.2 \square -\frac{11}{5}$

d) $-4.4\overline{6} \square -4.46$

22. Three hikers are returning to base camp after a mountain climbing expedition. Hiker A is 26.4 m above base camp, hiker B is 37.2 m below base camp, and hiker C is 15.7 m below base camp.

a) Represent each hiker's distance above or below base camp as a rational number.

b) Sketch and label a vertical number line to show the base camp and the positions of the hikers.

c) Which hiker is closest to base camp? Explain your reasoning.

d) Which hiker has the lowest altitude? How do you know?

23. Show each set of numbers on a number line. Order the numbers from least to greatest.

a) 1.5, −3.5, 4, 0, −2.5, 7.5

b) −1.7, 5.9, −3.2, −0.8, 1, 4.3

c) 1.2, 2.1, −2.01, −1.2, $1.\overline{2}$, −1.22

d) 5.44, −5.4, −5.04, $5.\overline{4}$, 5.04, −5.44

24. Show each set of numbers on a number line. Order the numbers from greatest to least.

a) $\frac{3}{8}, -\frac{3}{4}, -\frac{1}{2}, -\frac{5}{8}, \frac{1}{4}, 0$

b) $\frac{10}{9}, -\frac{5}{3}, \frac{7}{2}, -\frac{3}{2}, -\frac{7}{6}, \frac{17}{3}$

c) $-\frac{9}{5}, -\frac{17}{10}, -1\frac{1}{2}, \frac{16}{4}, -\frac{11}{4}, \frac{21}{5}$

d) $-\frac{11}{2}, \frac{10}{3}, 2\frac{1}{4}, -\frac{8}{6}, \frac{7}{12}, -\frac{6}{4}$

25. Show each set of numbers on a number line. Order the numbers from least to greatest.

a) $3.8, \frac{3}{8}, -1.5, \frac{5}{3}, -2.3, -\frac{3}{2}$

b) $0.3, -0.\overline{3}, \frac{1}{3}, -0.3, 0.33, -3$

Take It Further

26. Use the definition of a rational number to show that each of the following numbers is rational.

a) 3 **b)** −2 **c)** −0.5 **d)** −7.45

27. Which of the following numbers do you think are rational numbers? Explain why.

a) $4.\overline{21}$ **b)** −3.121 121 112 111 12…

c) 2.78 **d)** −2.122 222 22…

Reflect

What is a rational number? List 3 rational numbers in decimal form and 3 rational numbers in fraction form. Show the numbers on a number line.

Start Where You Are

How Can I Learn from Others?

Three students discuss the answers to these questions:

1. Evaluate: $\frac{5}{6}+\frac{3}{4}$

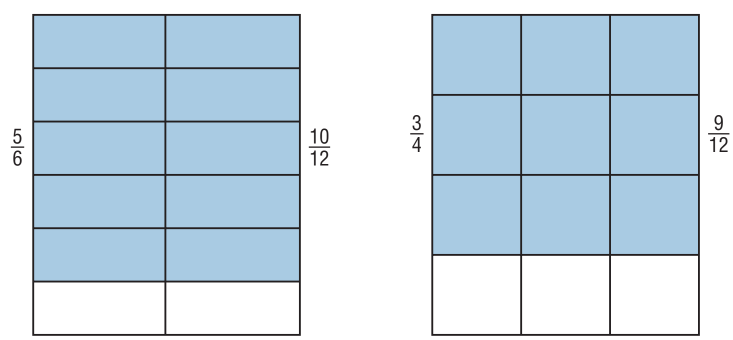
2. Evaluate: $3-5$

1. Evaluate: $\frac{5}{6}+\frac{3}{4}$

Dan said: The sum is $\frac{8}{10}$, which simplifies to $\frac{4}{5}$.
Jesse said: Dan must be wrong; the answer has to be greater than 1.
Philippe said: The answer has to be greater than $\frac{5}{6}$, and $\frac{4}{5}$ is less than $\frac{5}{6}$.

To help Dan, Jesse explained how he knew his answer was wrong:
I use benchmarks and estimate. Both $\frac{5}{6}$ and $\frac{3}{4}$ are greater than $\frac{1}{2}$, so their sum has to be greater than $\frac{1}{2}+\frac{1}{2}=1$.

Philippe explained his strategy for adding:
I know I can add the same types of fractions. For $\frac{5}{6}$ and $\frac{3}{4}$ to be the same type, I write them as equivalent fractions with the same denominator. Then I add the numerators.



Then, $\frac{5}{6}+\frac{3}{4}=\frac{10}{12}+\frac{9}{12}$

$=\frac{10+9}{12}$

$=\frac{19}{12}$, or $1\frac{7}{12}$

2. Evaluate: $3 - 5$

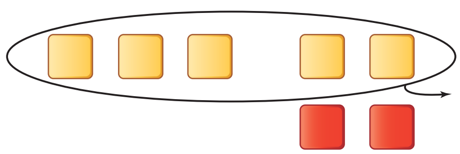
Philippe said: There is no answer because 5 is greater than 3.

Jesse said: I just switch the numbers around and calculate $5 - 3 = 2$.

Dan said: No, you can't change the order of the numbers — subtraction is not commutative. You have to think about integers.

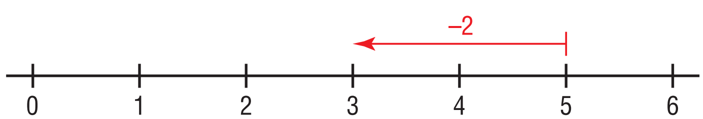
To help Philippe and Jesse, Dan explained two strategies:

➤ I can visualize coloured tiles, and add zero pairs.



$3 - 5 = -2$

➤ I can also use a number line. The difference between 2 numbers is the distance between 2 points on the number line.



$3 - 5 = -2$

Check

1. Evaluate.

a) $\frac{2}{3} + \frac{5}{2}$ **b)** $\frac{9}{8} + \frac{7}{4}$ **c)** $\frac{9}{10} + \frac{3}{5}$ **d)** $\frac{8}{3} + \frac{11}{4}$

e) $\frac{7}{2} - \frac{4}{5}$ **f)** $\frac{11}{6} - \frac{4}{3}$ **g)** $\frac{13}{4} - \frac{7}{5}$ **h)** $\frac{17}{3} - \frac{17}{6}$

2. Evaluate.

a) $7 - 3$ **b)** $3 - 7$ **c)** $-3 - 7$ **d)** $-3 - (-7)$

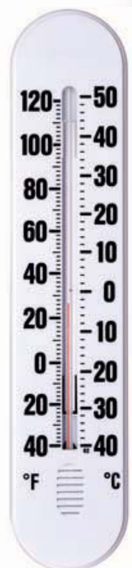
e) $-5 + 4$ **f)** $-6 - (-3)$ **g)** $8 - (-10)$ **h)** $-8 - 10$

3.2 Adding Rational Numbers

FOCUS

- Solve problems that require adding rational numbers.

At 6 A.M., the temperature was −3°C. By 10 A.M., the temperature had risen by 6°C. How can you use a number line to determine the temperature at 10 A.M.?



Investigate

Use what you know about adding integers and adding fractions to determine each sum.
Draw a number line to illustrate each sum.

$3 + 7$	$\frac{3}{8} + \frac{7}{8}$	$1\frac{3}{8} + 2\frac{7}{8}$
$-3 + 7$	$-\frac{3}{8} + \frac{7}{8}$	$-1\frac{3}{8} + 2\frac{7}{8}$
$-3 + (-7)$	$-\frac{3}{8} + \left(-\frac{7}{8}\right)$	$-1\frac{3}{8} + \left(-2\frac{7}{8}\right)$
$3 + (-7)$	$\frac{3}{8} + \left(-\frac{7}{8}\right)$	$1\frac{3}{8} + \left(-2\frac{7}{8}\right)$

Reflect & Share

Compare your strategies with those of another pair of students.
How did the first sum in each line help you determine the other sums?
How could you check your answers?
How are the strategies for adding rational numbers similar to those for adding integers and adding fractions?
Check your ideas by adding other rational numbers.

Connect

To add rational numbers in fraction form, recall how to add fractions and add integers.

➤ To add $\frac{2}{3} + \frac{1}{5}$, use a common denominator.

So, $\frac{2}{3} + \frac{1}{5} = \frac{10}{15} + \frac{3}{15}$ Write the numerators as a sum of integers.

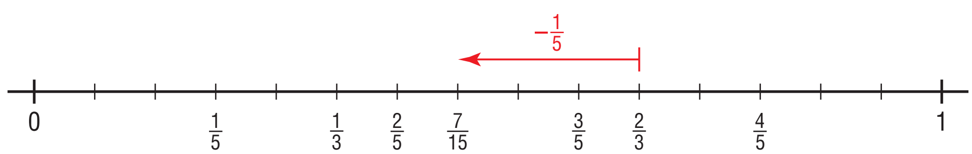
$= \frac{10 + 3}{15}$

$= \frac{13}{15}$

➤ Here are 2 strategies to add $\frac{2}{3} + \left(-\frac{1}{5}\right)$:

- Visualize a number line.

  To add $-\frac{1}{5}$, start at $\frac{2}{3}$ then move $\frac{1}{5}$ to the left.



$\frac{2}{3} + \left(-\frac{1}{5}\right) = \frac{7}{15}$

- Use equivalent fractions.

$\frac{2}{3} + \left(-\frac{1}{5}\right) = \frac{2}{3} + \left(\frac{-1}{5}\right)$

$= \frac{10}{15} + \left(\frac{-3}{15}\right)$ Add the integers in the numerator.

$= \frac{10 - 3}{15}$

$= \frac{7}{15}$

A number line is useful when the denominators of the fractions are compatible or when we want to estimate a sum.

Example 1 Adding Rational Numbers in Fraction and Mixed Number Form

Evaluate.

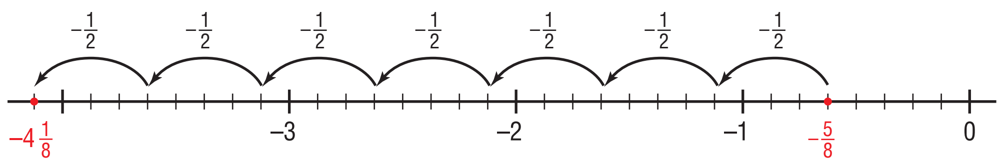
a) $-\frac{5}{8} + \left(-\frac{7}{2}\right)$ **b)** $-\frac{1}{4} + 2\frac{1}{6}$

A Solution

a) $-\frac{5}{8} + \left(-\frac{7}{2}\right)$

The denominators are compatible, so use a number line.

To add $-\frac{7}{2}$, start at $-\frac{5}{8}$ then move $\frac{7}{2}$ to the left.



$-\frac{5}{8} + \left(-\frac{7}{2}\right) = -4\frac{1}{8}$

b) $-\frac{1}{4} + 2\frac{1}{6}$

Write the mixed number $2\frac{1}{6}$ as the improper fraction $\frac{13}{6}$,

then write the fractions with a common denominator of 12.

$$\text{Then, } -\frac{1}{4} + 2\frac{1}{6} = \frac{-3}{12} + \frac{26}{12}$$
$$= \frac{-3 + 26}{12}$$
$$= \frac{23}{12}\text{, or } 1\frac{11}{12}$$

Example 2 Adding Rational Numbers in Mixed Number Form

Evaluate.

$-3\frac{1}{3} + 2\frac{5}{6}$

Solutions

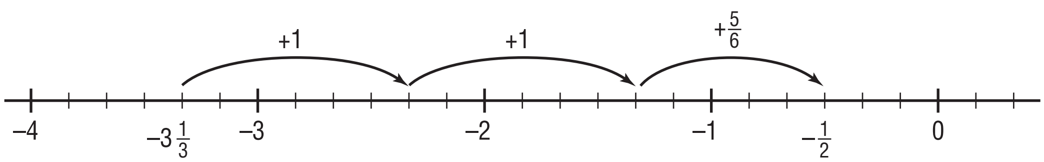
Method 1

$-3\frac{1}{3} + 2\frac{5}{6}$

Draw a number line.

Use a common denominator of 6, and divide the line into sixths.

Start at $-3\frac{1}{3}$ and move $2\frac{5}{6}$ to the right.



From the number line, $-3\frac{1}{3} + 2\frac{5}{6} = -\frac{1}{2}$

Method 2

$-3\frac{1}{3} + 2\frac{5}{6}$

Use equivalent fractions with denominator 6.

$-3\frac{1}{3} = -\frac{10}{3} = \frac{-20}{6}$

$$\begin{aligned} -3\frac{1}{3} + 2\frac{5}{6} &= \frac{-20}{6} + \frac{17}{6} \\ &= \frac{-20 + 17}{6} \\ &= \frac{-3}{6} \\ &= -\frac{1}{2} \end{aligned}$$

Method 3

$-3\frac{1}{3} + 2\frac{5}{6}$

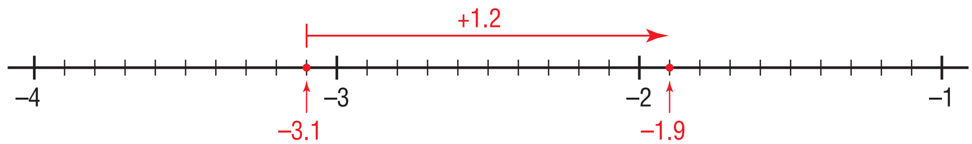
Add the whole numbers and add the fractions.

$$\begin{aligned} -3\frac{1}{3} + 2\frac{5}{6} &= -3 + \left(-\frac{1}{3}\right) + 2 + \frac{5}{6} \\ &= [-3 + 2] + \left[\left(-\frac{1}{3}\right) + \frac{5}{6}\right] \\ &= -1 + \left(\frac{-2}{6}\right) + \frac{5}{6} \\ &= -1 + \frac{3}{6} \\ &= -1 + \frac{1}{2} \\ &= -\frac{1}{2} \end{aligned}$$

You can use what you know about adding integers and adding decimals to add rational numbers in decimal form.

Visualize a number line.

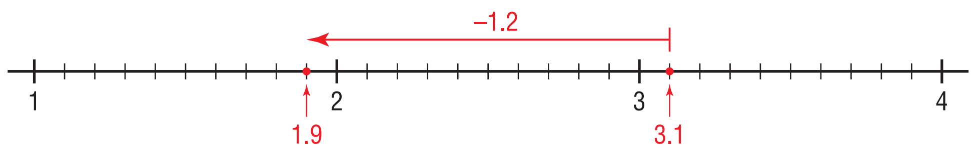
- $-3.1 + 1.2$



$-3.1 + 1.2 = -1.9$ This is an addition statement.

- $3.1 + (-1.2)$

Remember that when we add a negative number, we move to the left.



$3.1 + (-1.2) = 1.9$

Example 3 Solving a Problem by Adding Rational Numbers

At the beginning of June, the Frosty Snow Blower Company was \$235.46 in debt. By the end of August, the company had increased its debt by \$156.71.

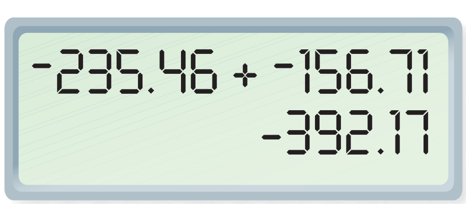
a) Use a rational number to represent each amount.

b) Calculate how much the company owed at the end of August.

A Solution

a) A debt of \$235.46 can be represented as -235.46.
A debt of \$156.71 can be represented as -156.71.

b) At the end of August, the company owed:
$-235.46 + (-156.71)$
Use a calculator.



$-235.46 + (-156.71) = -392.17$
At the end of August, the company owed \$392.17.

Discuss the ideas

1. How can you use what you know about representing the sum of 2 integers on a number line to add 2 rational numbers?

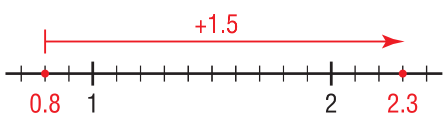
2. How can you use what you know about adding integers and adding fractions to add 2 rational numbers in fraction form?

Practice

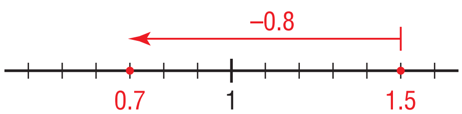
Check

3. Write the addition statement that each number line represents.

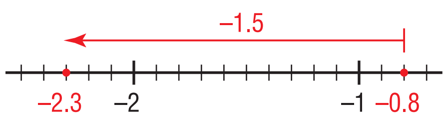
a)



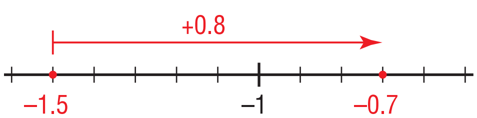
b)



c)

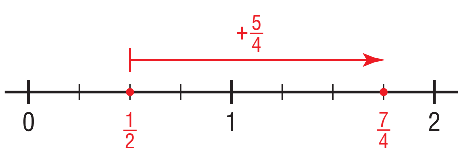


d)

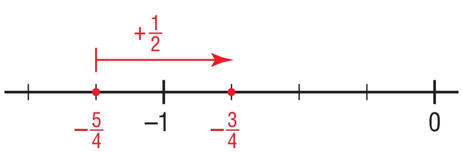


4. Write the addition statement that each number line represents.

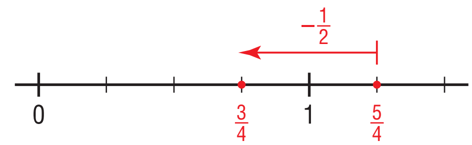
a)



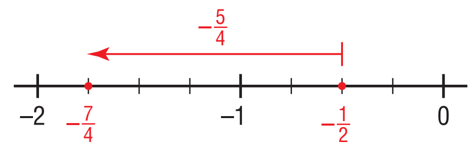
b)



c)



d)



5. Determine each sum.

a) i) $3 + 2$ **ii)** $3.8 + 2.4$

b) i) $-3 + (-2)$ **ii)** $-3.8 + (-2.4)$

c) i) $-3 + 2$ **ii)** $-3.8 + 2.4$

d) i) $3 + (-2)$ **ii)** $3.8 + (-2.4)$

6. Which of the following expressions have the same sum as $-2.3 + (-1.9)$? Use a number line to explain how you know.

a) $2.3 + 1.9$

b) $(-2.3) + 1.9$

c) $-1.9 + (-2.3)$

d) $(-2.3) + (-1.9)$

7. Determine each sum.

a) i) $9 + 3$ **ii)** $\frac{9}{2} + \frac{3}{2}$

b) i) $-9 + (-3)$ **ii)** $-\frac{9}{2} + \left(-\frac{3}{2}\right)$

c) i) $-9 + 3$ **ii)** $-\frac{9}{2} + \frac{3}{2}$

d) i) $9 + (-3)$ **ii)** $\frac{9}{2} + \left(-\frac{3}{2}\right)$

8. Which of the following expressions have the same sum as $-\frac{3}{4} + \frac{7}{8}$? Use a number line to show how you know.

a) $-\frac{3}{4} + \left(-\frac{7}{8}\right)$

b) $-\frac{7}{8} + \frac{3}{4}$

c) $\frac{7}{8} + \left(-\frac{3}{4}\right)$

d) $\frac{7}{8} + \frac{3}{4}$

Apply

9. Use integers to estimate each sum. Then, determine each sum.

a) $-5.6 + 3.2$

b) $7.95 + (-4.51)$

c) $-0.325 + (-32.5)$

d) $-123.5 + 27.45$

e) $82.001 + 100.28$

f) $-17.84 + (-0.098)$

10. Is it possible to add 2 rational numbers and get a sum that is less than both the numbers you added? Explain your reasoning.

11. Determine each sum.

a) $-\frac{2}{3} + \frac{1}{2}$

b) $\frac{4}{5} + \left(-\frac{1}{3}\right)$

c) $-\frac{11}{4} + \left(-\frac{6}{5}\right)$

d) $\frac{13}{5} + \frac{9}{2}$

e) $-2\frac{1}{3} + \left(-1\frac{3}{4}\right)$

f) $\frac{9}{5} + \left(-\frac{17}{6}\right)$

g) $-3\frac{3}{4} + 4\frac{5}{8}$

h) $1\frac{5}{6} + \left(-5\frac{2}{3}\right)$

i) $-3\frac{1}{4} + \left(-2\frac{1}{6}\right)$

j) $2\frac{3}{5} + \left(-1\frac{7}{8}\right)$

12. **Assessment Focus** What can you say about the sign of the sum of 2 rational numbers in each case? Include examples and explain your reasoning.

a) Both rational numbers are positive.

b) Both rational numbers are negative.

c) One rational number is positive and one rational number is negative.

13. Zoe owes her mother \$36.25, then Zoe borrows another \$25.35.

a) Write each amount as a rational number.

b) Use the numbers in part a.

i) Write an expression for the amount Zoe owes.

ii) How much does Zoe owe?

c) Zoe pays back \$14.75.

i) Write an expression for the amount Zoe now owes.

ii) How much does Zoe now owe?

14. Estimate whether each sum is greater than or less than 0. Explain how you know. Calculate to check your prediction.

a) $-0.61 + 0.23$

b) $12.94 + (-12.56)$

c) $-\frac{7}{3} + \left(\frac{17}{5}\right)$

d) $\frac{7}{4} + \left(-\frac{6}{5}\right)$

15. On Tuesday, December 23rd, the lowest temperature in Winnipeg was −13.4°C. By noon the next day, the temperature had increased by 5.7°C.

a) What was the temperature at noon?

b) On Wednesday, December 24th, the lowest temperature was 3.7°C less than the lowest the previous day. What was the lowest temperature on Wednesday?

c) Sketch a thermometer to show these changes in temperature.

16. For each pair of expressions below, how can you tell which sum is greater without adding? Explain your reasoning. Determine each sum to check.
   a) i) $-9.23 + 3.46$ ii) $9.23 + (-3.46)$
   b) i) $-\frac{2}{3} + \left(-\frac{3}{4}\right)$ ii) $-\frac{2}{3} + \frac{3}{4}$

17. In January, Keith earned $45.50 babysitting and $22.25 shovelling snow. He spent $15.77 on a CD, and $33.10 on a computer game.
   a) Write each amount above as a rational number. Justify your choice of sign for each number.
   b) Write an addition statement that represents Keith's balance at the end of January.
   c) What is Keith's balance?

18. The table shows the money earned and spent by Lucille in the first six months of running her new business, Lucille's Café.

Item	Income	Expense
New tables and chairs		$545.50
New stove		$978.44
Profit on food	$2115.70	
Repair of roof		$888.00
Profit on coffee	$2570.40	
Salary of part-time cook		$2540.20

Did Lucille's business make a profit in the first six months? Use rational numbers in your explanation.

19. Use a calculator to help determine a rational number that makes each sentence true.
   a) $5.6 + \square \leq 9.1$
   b) $11.8 + (-\square) \leq 23.4$
   c) $-7.2 + \square \geq 7.2$
   d) $-7.2 + \square \leq 7.2$

Take It Further

20. Determine the missing rational number in each addition statement. What strategies did you use?
   a) $-\frac{3}{4} + \square = \frac{7}{8}$
   b) $\square + \frac{4}{5} = -\frac{2}{3}$
   c) $\square + \left(-\frac{5}{2}\right) = 3\frac{1}{8}$
   d) $\frac{7}{3} + \square = -\frac{5}{4}$

21. Determine the range of numbers that makes this sentence true.
   Explain your reasoning.
   $7.9 + \square \leq 11.2$

22. Use any four of the rational numbers:
   $-1, -2, -3, -4, 1, 2, 3, 4$
   in the boxes below to make an expression with the greatest sum less than 0.
   Explain how you know you have determined the greatest sum less than 0.

   $\frac{\square}{\square} + \frac{\square}{\square}$

Reflect

Before you add 2 rational numbers, how can you tell if their sum will be positive or negative? Include both fraction and decimal examples in your explanation.

3.3 Subtracting Rational Numbers

FOCUS

- Solve problems that require subtracting rational numbers.

Canada's national debt was $559 billion in 1999. By 2008, this debt had been reduced to $467 billion.
How would you write each amount as a rational number?
How could you use a number line to calculate the difference in debt?

Investigate

2

Here is part of a stock market report from February 5, 2008, for some Canadian companies.

Company	Stock price at the end of the day ($)	Stock price at the start of the day ($)
Bombardier	4.670	4.710
Canadian National Railway	50.630	51.330
Canadian Tire Corporation	64.840	65.970
Potash Corporation of Saskatchewan	144.580	144.15

For each stock:

- ➤ Determine: (price at the end of the day) − (price at the start of the day)
- ➤ What does it mean when this difference in prices is positive? Is negative?
- ➤ Sketch a number line to show each subtraction.
- ➤ Use rational numbers to write a subtraction statement.

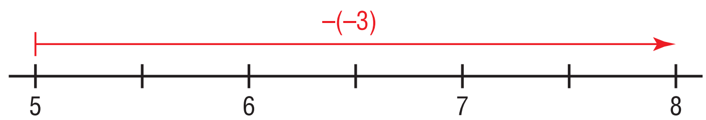
Reflect & Share

Compare your strategies and answers with those of another pair of students.
How did you use what you know about subtracting integers to subtract rational numbers?
How could you check your answers?

Connect

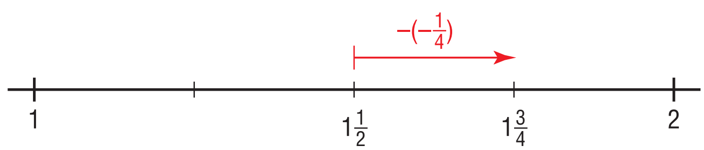
To subtract 2 rational numbers, we use a strategy similar to that for subtracting integers.

- For $5 - (-3)$, add the opposite: $5 + (+3)$
  Start at 5 then move 3 to the right.



$5 - (-3) = 8$

- For $1\frac{1}{2} - \left(-\frac{1}{4}\right)$, add the opposite: $1\frac{1}{2} + \left(+\frac{1}{4}\right)$
  Start at $1\frac{1}{2}$ then move $\frac{1}{4}$ to the right.



$1\frac{1}{2} - \left(-\frac{1}{4}\right) = 1\frac{3}{4}$

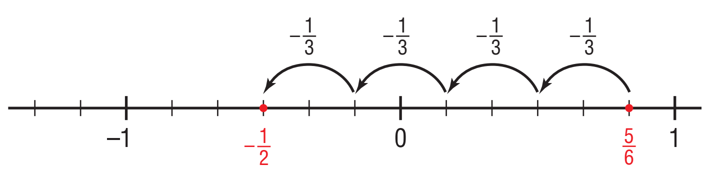
Example 1 Subtracting Rational Numbers in Fraction and Mixed Number Form

a) $\frac{5}{6} - \frac{4}{3}$ **b)** $-\frac{5}{4} - \left(-3\frac{1}{5}\right)$

A Solution

a) $\frac{5}{6} - \frac{4}{3}$

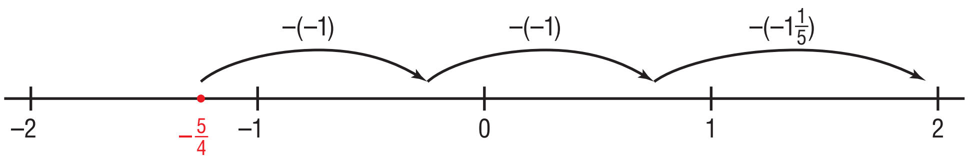
On a number line, start at $\frac{5}{6}$ then move $\frac{4}{3}$ to the left.



$\frac{5}{6} - \frac{4}{3} = -\frac{1}{2}$

b) $-\frac{5}{4} - \left(-3\frac{1}{5}\right)$

Visualize a number line to estimate the difference.



The difference is a little less than 2.

Use equivalent fractions to calculate the difference.

Write $-3\frac{1}{5}$ as the improper fraction $-\frac{16}{5}$.

$$\text{So, } -\frac{5}{4} - \left(-3\frac{1}{5}\right) = -\frac{5}{4} - \left(-\frac{16}{5}\right)$$

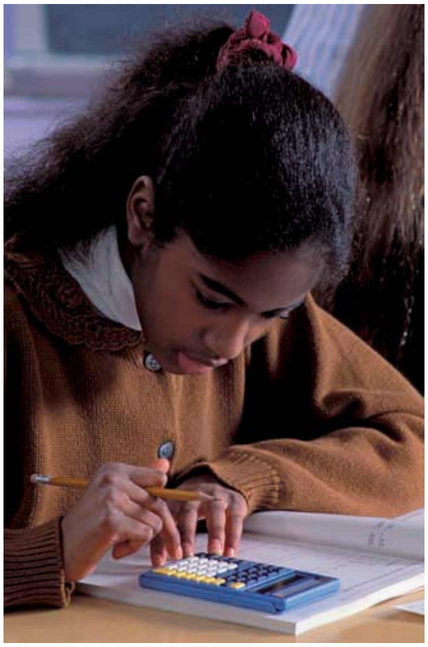
$$= -\frac{25}{20} - \left(-\frac{64}{20}\right)$$

$$= \frac{-25 - (-64)}{20}$$

$$= \frac{-25 + 64}{20}$$

$$= \frac{39}{20}, \text{ or } 1\frac{19}{20}$$

To subtract the integers in the numerator, add the opposite.



Example 2 Subtracting Rational Numbers in Mixed Number Form

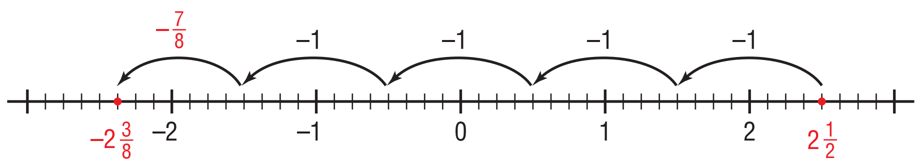
Evaluate. $2\frac{1}{2} - 4\frac{7}{8}$

Solutions

Method 1

$2\frac{1}{2} - 4\frac{7}{8}$

On a number line, start at $2\frac{1}{2}$ then move $4\frac{7}{8}$ to the left.



$2\frac{1}{2} - 4\frac{7}{8} = -2\frac{3}{8}$

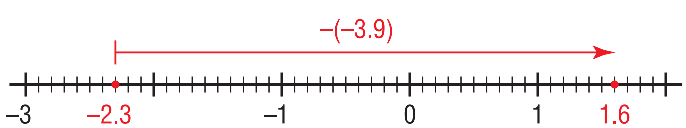
Method 2

$2\frac{1}{2} - 4\frac{7}{8}$

Write each mixed number as an improper fraction.

$$\begin{aligned} 2\tfrac{1}{2} - 4\tfrac{7}{8} &= \frac{5}{2} - \frac{39}{8} \quad \text{Use equivalent fractions.} \\ &= \frac{20}{8} - \frac{39}{8} \\ &= \frac{20 - 39}{8} \\ &= \frac{-19}{8} \\ &= -\frac{19}{8}, \text{ or } -2\tfrac{3}{8} \end{aligned}$$

We can use a number line to subtract rational numbers in decimal form.
To subtract $-2.3 - (-3.9)$, add the opposite: $-2.3 + (+3.9)$



$$\begin{aligned} -2.3 - (-3.9) &= -2.3 + (+3.9) \\ &= -2.3 + 3.9 \\ &= 1.6 \end{aligned}$$

Example 3 Solving a Problem by Subtracting Rational Numbers

A diver jumps off a cliff that is 14.7 m above sea level. After hitting the water, he plunges 3.8 m below the surface of the water before returning to the surface.

a) Use rational numbers to represent the difference in heights from the top of the cliff to the bottom of his dive. Sketch a number line.

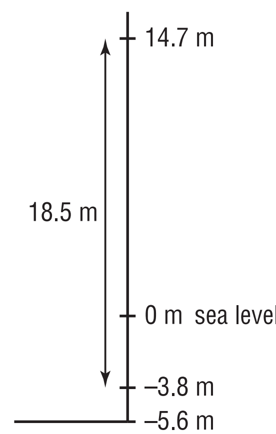
b) The water is 5.6 m deep. What is the distance from the ocean floor to the bottom of the dive?

A Solution

a) A distance measured above the water can be considered as positive.
A distance measured below the water can be considered as negative.
The diver travels 14.7 m above the water and −3.8 m below the water.
The difference in heights is: 14.7 − (−3.8)
From the number line:

$14.7 - (-3.8) = 14.7 + 3.8$ Adding the opposite decimal
$= 18.5$

The diver travelled 18.5 m.



b) The diver travelled −3.8 m below the surface.
The ocean floor is −5.6 m below the surface.
The difference in heights is: −5.6 − (−3.8)

$-5.6 - (-3.8) = -5.6 + 3.8$
$= -1.8$

The distance from the bottom of the ocean floor to the bottom of the dive is 1.8 m.

Discuss the ideas

1. When you use a number line to subtract 2 rational numbers, how do you know in which direction to move?

2. How can you use what you know about subtracting integers and subtracting fractions to subtract 2 rational numbers in fraction form?

Practice

Check

3. Determine each difference.
   a) i) $5 - 3$ ii) $5.1 - 3.3$
   b) i) $-5 - 3$ ii) $-5.1 - 3.3$
   c) i) $-3 - (-5)$ ii) $-3.3 - (-5.1)$
   d) i) $3 - 5$ ii) $3.3 - 5.1$

4. Which of the following expressions have the same answer as $-7.2 - 1.8$? How do you know?
   a) $7.2 - 1.8$ b) $-7.2 - (-1.8)$
   c) $1.8 - (-7.2)$ d) $-1.8 - 7.2$

5. Determine each difference.
   a) i) $11 - 2$ ii) $\frac{11}{5} - \frac{2}{5}$
   b) i) $-11 - 2$ ii) $-\frac{11}{5} - \frac{2}{5}$
   c) i) $11 - (-2)$ ii) $\frac{11}{5} - \left(-\frac{2}{5}\right)$
   d) i) $2 - (-11)$ ii) $\frac{2}{5} - \left(-\frac{11}{5}\right)$

Apply

6. Which of the following expressions have the same answer as $-\frac{3}{10} - \frac{9}{5}$? How do you know?
   a) $-\frac{3}{10} - \left(-\frac{9}{5}\right)$ b) $\frac{3}{10} - \frac{9}{5}$
   c) $-\frac{9}{5} - \frac{3}{10}$ d) $\frac{9}{5} - \frac{3}{10}$

7. Use integers to estimate each difference. Then, determine each difference.
   a) $10.8 - 3.5$ b) $-37.23 - 48.54$
   c) $50.06 - (-14.67)$ d) $64.19 - 95.76$
   e) $-28.31 - 9.72$ f) $70.59 - (-81.25)$

8. On January 25th, 2008, the lowest temperature in Iqaluit, Nunavut, was −28.5°C. On the same day, the lowest temperature in Inuvik, Northwest Territories, was −33.1°C.
   a) What is the difference in these temperatures?
   b) Why are there two possible answers to part a?

9. Determine each difference.
   a) $\frac{17}{3} - \frac{19}{2}$ b) $-\frac{13}{5} - \frac{7}{3}$ c) $1\frac{5}{6} - 6\frac{3}{4}$
   d) $-\frac{19}{6} - \frac{7}{8}$ e) $\frac{15}{4} - \frac{5}{12}$ f) $-2\frac{1}{8} - \left(-4\frac{1}{3}\right)$

10. **Assessment Focus** Is it possible to subtract 2 rational numbers and get a difference that is greater than both the numbers you subtracted? Explain your reasoning. Include examples of rational numbers in decimal form and in fraction form.

11. In Asia, the lowest point on land is the shore of the Dead Sea, which is 417.5 m below sea level. The highest point is the peak of Mount Everest, which 8844.43 m above sea level.

a) Write each measurement above as a rational number.

b) Write a subtraction statement that represents the distance between the highest point and the lowest point. What is this distance?

12. Predict whether each answer is positive or negative. Explain how you know. Calculate to check your prediction.

a) $-3.86 - 41.12$ b) $1.32 - (-5.79)$

c) $-\frac{5}{4} - \left(-\frac{7}{2}\right)$ d) $-\frac{23}{5} - \frac{5}{3}$

13. Evaluate each expression.

a) $\frac{3}{5} - \left(-\frac{1}{2}\right) + \frac{2}{3}$

b) $-2.34 + 8.6 + (-5.71)$

c) $-\frac{16}{5} - \left(-\frac{14}{3}\right) + \frac{13}{4}$

d) $23.5 + (-12.61) - 3.2$

14. Determine a rational number that makes each statement true. Use a calculator to check your answer.

a) $-1.2 - \square \leq 3.7$

b) $4.3 - \square \geq 8.9$

c) $\square - 2.9 \geq 5.3$

d) $\square - 7.2 \leq -10.9$

Take It Further

15. Determine the missing number in each subtraction statement.

a) $\square - 28.4 = 37.3$ b) $\frac{9}{10} - \square = \frac{3}{5}$

c) $\square - 0.05 = -2.08$ d) $\frac{11}{6} - \square = -\frac{7}{3}$

e) $-1.25 - \square = 3.75$ f) $-3\frac{1}{2} - \square = 5\frac{1}{4}$

16. Find two pairs of rational numbers that make each equation true.

a) $-7.4 + \square - \square = -10.9$

b) $\square - (-12.8) + \square = -1.1$

c) $-21.6 - \square - \square = -15.4$

17. Determine the range of numbers that makes each sentence true. Explain your thinking.

a) $-11.8 - \square \leq 5.7$

b) $6.3 - \square \geq 9.4$

Reflect

How is subtracting 2 rational numbers similar to adding 2 rational numbers? How is it different? Include examples of rational numbers in your explanation.

Mid-Unit Review

3.1

1. a) Sketch a number line. On the line, place each rational number below.
$-1.3, 2\frac{3}{4}, 1.51, -\frac{8}{5}, -\frac{9}{3}$

b) Which numbers in part a are less than -1.5? Explain how you know.

2. Order the following rational numbers from least to greatest. Place each number on a number line to support your answer.
$-\frac{6}{5}, 1.2, -1.1, -\frac{1}{4}, 0.2, -1\frac{3}{8}$

3. Replace each □ with < or >.
How could you check your answers?

a) $-\frac{2}{3}$ □ $-\frac{3}{4}$ **b)** $-\frac{8}{3}$ □ $-\frac{9}{4}$

c) -2.5 □ 0.5 **d)** $-\frac{4}{5}$ □ -0.9

4. Identify a rational number between each pair of numbers. Sketch a number line to illustrate each answer.

a) $1.2, 1.4$ **b)** $-\frac{3}{4}, \frac{5}{8}$

c) $0.4, \frac{1}{3}$ **d)** $-1.05, -\frac{9}{10}$

3.2

5. a) How can you determine the sign of the sum of two numbers before you add them?

b) Determine the sign of each sum, then check by using a calculator.

i) $2.35 + 3.47$

ii) $-5.783 + (-0.247)$

iii) $-\frac{2}{3} + \left(-1\frac{1}{8}\right)$

iv) $-5.27 + 6.58$

v) $-\frac{17}{5} + \frac{4}{9}$

vi) $0.085 + (-0.125)$

6. Determine each sum.

a) $8.37 + 0.58$ **b)** $-21.25 + (-36.57)$

c) $-157.4 + 32.7$ **d)** $\frac{5}{8} + \left(-\frac{1}{9}\right)$

e) $-8\frac{1}{4} + 5\frac{1}{5}$ **f)** $-\frac{5}{3} + \left(-\frac{23}{7}\right)$

3.3

7. The temperature of a freezer changed from $-16.1°C$ to $-14.7°C$.

a) i) By how much did the temperature change?

ii) Is this an increase or a decrease in temperature? Explain how you know.

b) By how much does the temperature need to change again before it is at $-3.8°C$?

8. Determine each difference.

a) $40.25 - 63.10$ **b)** $-112.2 - (-14.8)$

c) $\frac{2}{5} - \frac{9}{10}$ **d)** $-4\frac{4}{9} - 3\frac{5}{6}$

e) $-1.8 - 4.3$ **f)** $\frac{23}{8} - \left(-\frac{7}{2}\right)$

9. The lowest point on land in North America is Death Valley at 86 m below sea level. The highest point is the peak of Mt. McKinley at 6193.7 m above sea level. How can you use rational numbers to calculate the distance between these two points?

10. a) How can you determine the sign of the difference of two numbers before you subtract them?

b) Determine the sign of each difference, then check by using a calculator.

i) $62.4 - 53.7$ ii) $-0.54 - 1.98$

iii) $\frac{1}{12} - \frac{9}{10}$ iv) $5\frac{2}{3} - \left(-7\frac{1}{2}\right)$

GAME

Closest to Zero

You will need

- a deck of 52 playing cards
- a calculator (optional)

Number of Players

- 2 to 4

Goal of the Game

- To add or subtract fractions to get an answer that is close to 0

How to Play

An Ace is worth 1, a Jack is worth 11, a Queen is worth 12, and a King is worth 13.
All the red cards are negative and the black cards are positive.

1. For each round, each player is dealt 4 cards.
2. Each player organizes her 4 cards to create 2 proper fractions – two cards are the numerators and two are the denominators.
3. Each player chooses to add or subtract her 2 fractions. This is then the value of that player's hand.
4. The winner of the round is the person whose hand has a value closest to 0. The winner gets 1 point.
   If a player has a hand whose value *is* 0, then that person wins the round and gets 2 points.
   Players record their points.
5. The cards are shuffled and play continues with the next round.
   The first player to get 10 points is the winner.

Play the game a few times.
What strategies do you have for winning a round?

3.4 Multiplying Rational Numbers

FOCUS

- Solve problems that require multiplying rational numbers.

What strategies do you use:

- to multiply two integers such as $(-9) \times 8$?
- to multiply two fractions such as $\frac{3}{4} \times \frac{5}{2}$?

Investigate

2

➤ Use what you know about multiplying integers and multiplying fractions to predict each product of rational numbers.

6×8	5×7	$\frac{6}{5} \times \frac{8}{7}$		
$(-7) \times 9$	4×2	$\left(-\frac{7}{4}\right) \times \frac{9}{2}$	$\left(\frac{-7}{4}\right) \times \frac{9}{2}$	$\left(\frac{7}{-4}\right) \times \frac{9}{2}$
$(-8) \times (-6)$	3×5	$\left(-\frac{8}{3}\right) \times \left(-\frac{6}{5}\right)$	$\left(\frac{-8}{3}\right) \times \left(\frac{-6}{5}\right)$	$\left(\frac{8}{-3}\right) \times \left(\frac{6}{-5}\right)$
$9 \times (-3)$	2×10	$\frac{9}{2} \times \left(-\frac{3}{10}\right)$	$\frac{9}{2} \times \left(\frac{-3}{10}\right)$	$\frac{9}{2} \times \left(\frac{3}{-10}\right)$

➤ Use a calculator to check your predictions.

➤ Choose 2 different rational numbers in fraction form. Determine their product. Check with a calculator.

Reflect & Share

Share your answers with another pair of students.
Explain to each other how you found the products.
How did the first 2 products in each line help you determine the next products?

Connect

To multiply 2 rational numbers, use the properties for determining the sign of the product of 2 integers to predict the sign of the product of the rational numbers. Then:

- If the rational numbers are in fraction form:
  Use the procedures for multiplying 2 fractions to determine the numerical value of the product.
- If the rational numbers are in decimal form:
  Use the procedures for multiplying 2 decimals to determine the numerical value of the product.

For example,

- When two rational numbers have the same sign, their product is positive.
  $\left(-\frac{3}{2}\right) \times \left(-\frac{1}{5}\right) = \frac{3}{10}$ and $\frac{3}{2} \times \frac{1}{5} = \frac{3}{10}$
  $(-1.5) \times (-1.8) = 2.7$ and $1.5 \times 1.8 = 2.7$
- When two rational numbers have opposite signs, their product is negative.
  $\frac{3}{2} \times \left(-\frac{1}{5}\right) = -\frac{3}{10}$ and $\left(-\frac{3}{2}\right) \times \frac{1}{5} = -\frac{3}{10}$
  $(-1.5) \times 1.8 = -2.7$ and $1.5 \times (-1.8) = -2.7$

When we use brackets to write a product statement, we do not need the multiplication sign. For the rational numbers above, we can write
$\frac{3}{2} \times \left(-\frac{1}{5}\right)$ as $\left(\frac{3}{2}\right)\left(-\frac{1}{5}\right)$, and $(-1.5) \times 1.8$ as $(-1.5)(1.8)$.

Example 1 Multiplying Rational Numbers in Fraction or Mixed Number Form

Determine each product.

a) $\left(-\frac{11}{7}\right)\left(-\frac{21}{44}\right)$ **b)** $\left(2\frac{2}{3}\right)\left(-1\frac{3}{4}\right)$

A Solution

a) $\left(-\frac{11}{7}\right)\left(-\frac{21}{44}\right)$

Predict the sign of the product: since the fractions have the same sign, their product is positive.

Simplify the fractions before multiplying.

$$\left(-\frac{11}{7}\right)\left(-\frac{21}{44}\right) = \left(-\frac{\overset{1}{\cancel{11}}}{\underset{1}{\cancel{7}}}\right)\left(-\frac{\overset{3}{\cancel{21}}}{\underset{4}{\cancel{44}}}\right)$$
$$= \frac{1 \times 3}{1 \times 4}$$
$$= \frac{3}{4}$$

Look for common factors in the numerators and denominators:
11 and 44 have the common factor 11.
21 and 7 have the common factor 7.
Divide numerator and denominator by their common factors.
Then multiply the numerators and multiply the denominators.

So, $\left(-\frac{11}{7}\right)\left(-\frac{21}{44}\right) = \frac{3}{4}$

b) $\left(2\frac{2}{3}\right)\left(-1\frac{3}{4}\right)$

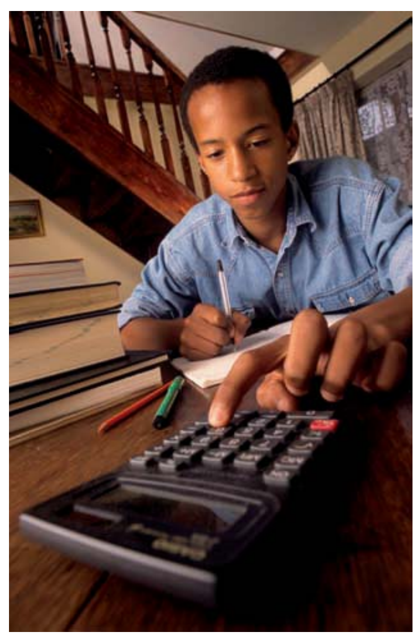
Since the fractions have opposite signs, their product is negative.

Write the mixed numbers as fractions.

$$\left(\frac{8}{3}\right)\left(-\frac{7}{4}\right) = \left(\frac{\overset{2}{\cancel{8}}}{3}\right)\left(-\frac{7}{\underset{1}{\cancel{4}}}\right)$$ Dividing numerator and denominator by their common factor 4

$$= \frac{(2)(-7)}{(3)(1)}$$
$$= -\frac{14}{3}$$
$$= -4\frac{2}{3}$$

So, $\left(2\frac{2}{3}\right)\left(-1\frac{3}{4}\right) = -4\frac{2}{3}$



Example 2 Multiplying Rational Numbers to Solve Problems

On February 5, 2008, the price of a share in CIBC changed by $-\$1.640$.
A person owns 35 shares. By how much did those shares change in value that day?

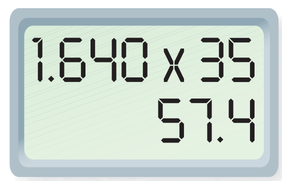
Solutions

Method 1

Change in value:
$-\$1.640 \times 35$
Since the rational numbers have opposite signs, their product is negative.
To determine the product: $(-1.64)(35)$, multiply integers, then estimate to place the decimal point.
$(-164)(35) = -5740$
Estimate to place the decimal point:
Since -1.64 is close to -2, then
$(-1.64)(35)$ is close to $(-2)(35) = -70$.
So, place the decimal point after the 7 in -5740.
$-\$1.640 \times 35 = -\$57.40$
The shares lost \$57.40 that day.

Method 2

Change in value:
$-\$1.640 \times 35$
Use a calculator.
Key in 1.640×35 to display: 57.4



Since the rational numbers have opposite signs, their product is negative, so we do not need to enter the negative sign.

$-\$1.640 \times 35 = -\$57.40$
The shares lost \$57.40 that day.

Example 3 Multiplying Rational Numbers in Decimal Form

Determine each product.

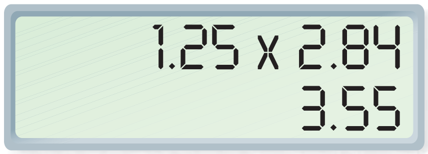
a) $(0.8)(-2.4)$ **b)** $(-1.25)(-2.84)$

A Solution

a) $(0.8)(-2.4)$
Since the rational numbers have opposite signs, their product is negative.
Use mental math to determine the product:
$(8)(-24) = -192$
Estimate to place the decimal point:
Since 0.8 is close to 1 and -2.4 is close to -2, then
$(0.8)(-2.4)$ is close to $(1)(-2) = -2$.
So, place the decimal point after the 1 in -192.
Then, $(0.8)(-2.4) = -1.92$

b) $(-1.25)(-2.84)$

When there are more than 2 digits in both numbers being multiplied, use a calculator.
The rational numbers have the same sign, so their product is positive.
Key in 1.25×2.84 to display: 3.55



$(-1.25)(-2.84) = 3.55$

Discuss the ideas

1. Why does it help to predict the sign of a product before you multiply 2 rational numbers?

2. Why does it make sense that the rules for signs when you multiply integers must apply when you multiply rational numbers?

Practice

Check

3. Predict which products are greater than 0, then multiply to determine each product. Explain the strategy you used to predict.

a) $3 \times (-5.2)$

b) $2.6 \times (-4)$

c) $(-1.3) \times 5$

d) $(-0.9) \times (-7.1)$

4. Predict which products are less than 0, then multiply to determine each product. Explain the strategy you used to predict.

a) $(-3) \times \frac{2}{3}$ **b)** $\left(-\frac{1}{4}\right) \times (-5)$

c) $\left(\frac{4}{5}\right) \times (-2)$ **d)** $\left(-\frac{1}{2}\right) \times \frac{7}{8}$

5. Determine each product. Estimate to place the decimal point.

a) $(-0.64)(0.2)$ **b)** $(-0.5)(-5.71)$

c) $(-4.13)(-0.8)$ **d)** $(0.7)(8.5)$

6. Which of the following expressions have the same product as $\left(-\frac{3}{4}\right)\left(\frac{5}{2}\right)$?
Explain how you know.

a) $\left(\frac{5}{2}\right)\left(-\frac{3}{4}\right)$ **b)** $\left(\frac{3}{4}\right)\left(-\frac{5}{2}\right)$ **c)** $\left(-\frac{3}{2}\right)\left(\frac{5}{4}\right)$

d) $\left(\frac{3}{4}\right)\left(\frac{5}{2}\right)$ **e)** $\left(\frac{3}{2}\right)\left(-\frac{5}{4}\right)$ **f)** $\left(-\frac{3}{4}\right)\left(-\frac{5}{2}\right)$

7. Determine each product.

a) $\left(-\frac{1}{3}\right)\left(\frac{2}{5}\right)$ **b)** $\left(\frac{1}{4}\right)\left(-\frac{3}{5}\right)$

c) $\left(\frac{4}{5}\right)\left(\frac{1}{2}\right)$ **d)** $\left(-\frac{5}{6}\right)\left(-\frac{2}{3}\right)$

Apply

8. Suppose each rational number below is multiplied by -2.5.
Which products are greater than 10?
How can you find out by estimating?
Evaluate only those products that are greater than 10.

a) -5.1 b) 3.5 c) -4.4
d) -3.6 e) -5 f) 5

9. On February 5, 2008:

a) The price of a share in Petro-Canada changed by $-\$0.80$.
A person owns 120 shares. By how much did the shares change in value that day?

b) The price of a share in Research in Motion changed by $-\$2.10$.
A person owns 50 shares. By how much did the shares change in value that day?

c) The price of a share in Shoppers Drug Mart changed by $0.23.
A person owns 65 shares. By how much did the shares change in value that day?

10. A diver descends at an average speed of 10.4 m/min. Use rational numbers to write her depth after 3.6 min.

11. Determine each product.

a) $(-1.23)(2.8)$ b) $(-23.7)(-1.2)$
c) $(15.2)(15.2)$ d) $(-20.1)(-5.2)$

12. Determine each product.

a) $\left(\frac{5}{4}\right)\left(-\frac{16}{5}\right)$ b) $\left(-\frac{2}{3}\right)\left(-\frac{5}{6}\right)$
c) $\left(-2\frac{8}{9}\right)\left(5\frac{1}{8}\right)$ d) $\left(-4\frac{2}{5}\right)\left(-\frac{5}{3}\right)$

13. Assessment Focus

a) Multiply: $(-26)(-4)$

b) Use your answer to part a to determine each product.

i) $(-2.6)(-0.4)$
ii) $(-0.26)(0.4)$
iii) $(260)(-0.04)$
iv) $(-0.026)(-4)$

c) Why did you not have to multiply to determine each product in part b?

d) Write 3 more products you could determine using your answer to part a.

14. A courier company has a bank account balance of $45 567.87. The company must repaint all its 25 delivery trucks at an average cost of $3457.25 per truck.

a) Write a multiplication statement with rational numbers to determine the cost of painting the trucks.

b) What is the bank account balance after the bill for painting has been paid? Explain your result.

15. Predict the sign of each product, then calculate the product.

a) $(-2.0)(-0.5)(3.1)$ b) $\left(\frac{5}{6}\right)\left(-\frac{4}{7}\right)\left(\frac{3}{2}\right)$

Take It Further

16. Determine the missing number in each product statement.
What strategies did you use?

a) $-3.25 \times \square = 15.275$ b) $-\frac{5}{4} \times \square = -\frac{35}{8}$

c) $\square \times 0.045 = -0.018$ d) $\square \times 3\frac{3}{4} = 5\frac{1}{4}$

17. A positive rational number is multiplied by a negative rational number. Is it possible that the product is closer to 0 than either of the numbers being multiplied? Explain.

18. Karen used her calculator to evaluate $-\frac{89}{91} \times \frac{31}{86}$. She reported the product as about $-0.352\ 542\ 806$.

a) How did Karen know that the value is approximate?

b) What is the exact answer?

Reflect

Rational numbers can be in fraction form or decimal form.
Which form do you prefer to multiply? Explain your choice.
Include examples in your explanation.

Math Link

History

When the New York Stock Exchange began in 1792, it modelled its system on the Spanish one.
The Spanish dollar was divided into eight parts, so when a stock increased or decreased in value, the change was represented in eighths. A decrease was represented by a negative fraction, while an increase was represented by a positive fraction.
In 2000, the New York Stock Exchange moved to the current system that shows the change in value of a stock as a decimal.
Here are the changes in values of 5 different stocks on a particular day:
$1\frac{5}{8}, -\frac{3}{16}, \frac{3}{4}, -1\frac{7}{16}, -1\frac{1}{2}$
Arrange the fractions from least to greatest.
Write each fraction as a decimal to show the change in dollars.

3.5 Dividing Rational Numbers

FOCUS

- Solve problems that require dividing rational numbers.

Marcel has $2\frac{1}{4}$ cups of juice.
He pours $\frac{3}{4}$ of a cup of juice into each glass.
How many glasses can Marcel fill?
Write a division statement to describe this situation.

Investigate

2

➤ The 3rd, 4th, and 5th terms of a number pattern are: +27, −18, +12

- To get the next term, you divide the term before it by one of these rational numbers: $\frac{2}{3}, \frac{3}{2}, -\frac{2}{3}, -\frac{3}{2}$
  Which number is correct? How do you know?
- Determine the 6th and 7th terms of the pattern.
  Describe your strategy and show your work.
- Determine the 1st and 2nd terms of the pattern.
  Describe your strategy and show your work.

➤ Choose a different rational number and a different 1st term.
Calculate the first 5 terms of your pattern.

Reflect & Share

Trade patterns with another pair of classmates.
Write the pattern rule and determine the next 3 terms in your classmates' pattern.
How did you use what you know about rational numbers to identify and extend the pattern?

Connect

To divide 2 rational numbers, use the properties for determining the sign of the quotient of 2 integers to predict the sign of the quotient of the rational numbers. Then:

➤ If the rational numbers are in fraction form:
Use the procedures for dividing 2 fractions to determine the numerical value of the quotient.

➤ If the rational numbers are in decimal form:
Use the procedures for dividing 2 decimals to determine the numerical value of the quotient.

For example,

- When two rational numbers have the same sign, their quotient is positive.
$\left(-\frac{3}{2}\right) \div \left(-\frac{1}{5}\right) = \frac{15}{2}$ and $\frac{4}{5} \div \frac{3}{2} = \frac{8}{15}$
$(-3.9) \div (-1.5) = 2.6$ and $9.9 \div 4.5 = 2.2$
- When two rational numbers have opposite signs, their quotient is negative.
$\frac{3}{2} \div \left(-\frac{1}{5}\right) = -\frac{15}{2}$ and $\left(-\frac{4}{5}\right) \div \frac{3}{2} = -\frac{8}{15}$
$(-3.9) \div 1.5 = -2.6$ and $9.9 \div (-4.5) = -2.2$

Example 1 Dividing Rational Numbers in Fraction or Mixed Number Form

Determine the sign of each quotient, then divide.

a) $\left(-\frac{5}{8}\right) \div \frac{3}{4}$ **b)** $\left(-4\frac{1}{5}\right) \div \left(-3\frac{1}{3}\right)$

A Solution

a) $\left(-\frac{5}{8}\right) \div \frac{3}{4}$

The fractions have opposite signs, so their quotient is negative.

Use the strategy of dividing fractions with a common denominator.

Write each fraction with a common denominator of 8.

$$\left(-\frac{5}{8}\right) \div \frac{3}{4} = \left(-\frac{5}{8}\right) \div \frac{6}{8}$$
$$= -\frac{5}{6}$$

Since the denominators are the same, divide the numerators.

So, $\left(-\frac{5}{8}\right) \div \frac{3}{4} = -\frac{5}{6}$

b) $\left(-4\frac{1}{5}\right) \div \left(-3\frac{1}{3}\right)$

The mixed numbers have the same sign, so their quotient is positive.

Write each mixed number as an improper fraction: $\left(-\frac{21}{5}\right) \div \left(-\frac{10}{3}\right)$

Use the strategy of multiplying the dividend by the reciprocal of the divisor.

$$\begin{aligned}\left(-4\frac{1}{5}\right) \div \left(-3\frac{1}{3}\right) &= \left(-\frac{21}{5}\right) \div \left(-\frac{10}{3}\right) \\ &= \left(-\frac{21}{5}\right) \div \left(-\frac{10}{3}\right) \\ &= \frac{63}{50} \\ &= 1\frac{13}{50}\end{aligned}$$

So, $\left(-4\frac{1}{5}\right) \div \left(-3\frac{1}{3}\right) = 1\frac{13}{50}$

> To get the reciprocal of a fraction, interchange the numerator and denominator. So, the reciprocal of $-\frac{10}{3}$ is $-\frac{3}{10}$.

You could use a calculator to divide fractions and mixed numbers. You do not need to input the negative signs if you determine the sign of the quotient first.

When you divide decimals, the quotient may be a terminating or repeating decimal. If you divide using pencil and paper, and the quotient appears to be a repeating decimal, continue to divide until you can identify which digits repeat.

Example 2 Dividing Rational Numbers in Decimal Form

Divide.

a) $(-1.38) \div 0.6$ **b)** $(-0.25) \div (-0.3)$

A Solution

a) $(-1.38) \div 0.6$

Since the dividend and divisor have opposite signs, their quotient is negative.

Estimate first; use compatible numbers.

-1.38 is close to -1, and 0.6 is close to 0.5.

$(-1) \div 0.5$ is -2.

So, $(-1.38) \div 0.6$ is about -2.

Divide integers:
$(-138) \div 6 = -23$
The estimate is -2, so place the decimal point in the quotient between the 2 and the 3.
So, $(-1.38) \div 0.6 = -2.3$

b) $(-0.25) \div (-0.3)$
Since the dividend and divisor have the same sign, their quotient is positive.
Determine the numerical value of the quotient:
$0.25 \div 0.3 = 0.833\,333\ldots$
$= 0.8\overline{3}$
So, $(-0.25) \div (-0.3) = 0.8\overline{3}$

When the divisor is a decimal with more than 1 digit, we use a calculator to divide.

Example 3 Solving Problems Involving Rational Numbers

Determine the missing number in each division statement.

a) $\square \div (-2.6) = 9.62$ **b)** $\left(-\frac{5}{8}\right) \div \square = -\frac{15}{56}$

A Solution

a) $\square \div (-2.6) = 9.62$
Division is the inverse of multiplication.
Any division statement can be written as an equivalent multiplication statement.
Estimate.
Think: $\square \div (-3) = 9$
We know that $(-27) \div (-3) = 9$, or as a multiplication statement: $(-27) = (-3) \times 9$
Rewrite the given statement the same way:
$\square \div (-2.6) = 9.62$ can be written as $\square = (-2.6) \times 9.62$
Use a calculator: $\square = -25.012$
The missing number is -25.012.

b) $\left(-\frac{5}{8}\right) \div \square = -\frac{15}{56}$

$\left(-\frac{5}{8}\right) \div \square = -\frac{15}{56}$ can be written as $\square = \left(-\frac{5}{8}\right) \div \left(-\frac{15}{56}\right)$

The quotient is positive.

Use the strategy of multiplying by the reciprocal to determine the numerical value of the quotient.

$$\square = \frac{5}{8} \div \frac{15}{56}$$

$$= \frac{5}{8} \times \frac{56}{15}$$ Simplify by dividing by common factors in the numerator and denominator.

$$= \frac{{}^{1}\cancel{5}}{{}_{1}\cancel{8}} \times \frac{\cancel{56}^{7}}{\cancel{15}_{3}}$$

$$= \frac{1}{1} \times \frac{7}{3}$$

$$= \frac{7}{3}$$

The missing number is $\frac{7}{3}$.

Solve a simpler problem.
Think: $6 \div \square = 2$
We know $6 \div 3 = 2$,
so we write the related statement:
$3 = 6 \div 2$

Discuss the ideas

1. How can you use what you know about dividing integers and dividing fractions to divide 2 rational numbers in fraction form?

2. How can you use what you know about dividing integers and dividing decimals to divide 2 rational numbers in decimal form?

Practice

Check

3. Predict the sign of each quotient, then calculate the quotient.

a) $(-1.5) \div 3$ b) $2.8 \div (-2)$

c) $(-8.4) \div (-4)$ d) $1.6 \div (-8)$

e) $(-14.4) \div (-6)$ f) $(-6.3) \div 7$

4. Predict the sign of each quotient, then calculate the quotient.

a) $\frac{1}{2} \div \left(-\frac{3}{4}\right)$ b) $\left(-\frac{2}{5}\right) \div \frac{3}{10}$

c) $\left(-\frac{7}{6}\right) \div \left(-\frac{8}{3}\right)$ d) $\frac{1}{4} \div \frac{11}{3}$

e) $\frac{5}{2} \div \left(-\frac{2}{3}\right)$ f) $\left(-\frac{9}{5}\right) \div \left(-\frac{11}{4}\right)$

5. Which of the following expressions have the same answer as $\left(-\frac{1}{3}\right) \div \left(-\frac{3}{4}\right)$?

a) $\left(-\frac{1}{3}\right) \times \left(-\frac{3}{4}\right)$ b) $\left(-\frac{3}{4}\right) \div \left(-\frac{1}{3}\right)$

c) $\left(-\frac{1}{3}\right) \times \left(-\frac{4}{3}\right)$ d) $\left(-\frac{4}{3}\right) \times \left(-\frac{1}{3}\right)$

e) $\frac{1}{3} \div \frac{3}{4}$ f) $\frac{4}{3} \times \frac{1}{3}$

Apply

6. At a sea port, the effect of the tide changed the water level by −5.6 m in 3.5 h.
What was the mean change in water level per hour?

7. Determine each quotient without a calculator. Estimate to place the decimal point in the quotient.
a) $0.32 \div 0.4$
b) $(-1.17) \div 0.8$
c) $0.25 \div (-0.6)$
d) $(-1.02) \div (-0.2)$
e) $3.76 \div (-0.3)$
f) $3.15 \div 0.9$

8. On a winter's day, the temperature at 6 P.M. was 0°C. Suppose the temperature decreased by 2.5°C each hour until it was −12.5°C. How long did it take to reach this temperature? How do you know?

9. Use a calculator to determine each quotient.
a) $20.736 \div (-1.8)$
b) $(-27.94) \div 1.2$
c) $(-84.41) \div (-2.3)$
d) $23.04 \div 4.8$
e) $76.63 \div (-7.5)$
f) $(-0.1081) \div 0.45$

10. Assessment Focus Suppose each rational number below is divided by −0.5.
Predict which quotients are greater than −10.
Explain the strategies you used to predict.
Then evaluate only those quotients that are greater than −10.
a) −20.5 **b)** 18.8 **c)** 10.7 **d)** 0.6

11. To pay for a skiing holiday in Whistler, Paige borrowed $1450.50 from her parents.
She pays back $30.75 each week.
a) How many weeks will it be until Paige is no longer in debt? Justify your answer.
b) How did you use rational numbers to calculate the answer in part a?

12. Determine each quotient.
a) $\frac{5}{4} \div \left(-\frac{7}{6}\right)$ **b)** $\frac{3}{10} \div \frac{12}{5}$
c) $\left(-\frac{3}{4}\right) \div \left(-1\frac{1}{8}\right)$ **d)** $\left(-4\frac{3}{5}\right) \div \frac{3}{4}$
e) $3\frac{2}{3} \div \left(-2\frac{1}{4}\right)$ **f)** $3\frac{4}{9} \div 6\frac{1}{3}$

13. A thermometer on a freezer is set at −5.5°C.
Each time the freezer door is opened, the temperature increases by 0.3°C.
Suppose there is a power outage.
How many times can the door be opened before the temperature of the freezer increases to 5°C? Justify your solution.

14. On one day in January, the temperature changed by −15.4°C in 5.5 h.
What was the mean change in temperature per hour?

15. A person has 54 shares in WestJet Airlines. On February 6, 2008, these shares lost \$17.28 in value.
What was the change in value of 1 share? How do you know?

16. Suppose each rational number below was divided by $-\frac{2}{3}$. Predict which quotients would be less than $-\frac{1}{2}$. Explain the strategy you used to predict.

a) $-\frac{2}{3}$ **b)** $\frac{1}{3}$

c) $\frac{5}{6}$ **d)** $\frac{1}{4}$

17. Determine the missing number in each division statement.

a) $\square \div 1.25 = -3.6$

b) $\square \div \left(-\frac{3}{4}\right) = \frac{7}{8}$

c) $(-0.5875) \div \square = -0.25$

d) $\frac{68}{15} \div \square = -\frac{4}{5}$

18. Replace each □ with a rational number to make each equation true. Explain the strategy you used.

a) $(-0.3) \times \square = 0.78$

b) $0.8 \times \square = -5.52$

c) $(-1.26) \div \square = 0.2$

d) $\square \div (-1.1) = 3.26$

Take It Further

19. Alex and Ellice run in opposite directions from school to their homes.
Ellice runs 1.3 km to her home in 7.8 min.
Alex runs 630 m to his home in 4.2 min.

a) Write division statements using positive and negative rational numbers to represent each student's average speed in metres per minute.
What do the positive and negative numbers represent?

b) Who runs at the greater average speed?

20. Write 6 division statements that have a quotient between $-\frac{3}{4}$ and $-\frac{1}{4}$.

21. Which expression below has the greatest value? How can you find out without calculating every answer?

a) $-\frac{1}{2} + \left(-\frac{2}{3}\right)$ **b)** $-\frac{1}{2} - \left(-\frac{2}{3}\right)$

c) $\left(-\frac{1}{2}\right) \times \left(-\frac{2}{3}\right)$ **d)** $\left(-\frac{1}{2}\right) \div \left(-\frac{2}{3}\right)$

Reflect

How is dividing rational numbers similar to multiplying them?
Include examples of fractions and decimals in your explanation.

3.6 Order of Operations with Rational Numbers

FOCUS

- Explain and apply the order of operations with rational numbers.

Two students were asked to evaluate: $(-8) - 2(24 \div (-8))^2$
Here are their calculations.

$$\begin{aligned}&(-8) - 2(24 \div (-8))^2\\&= (-10)(24 \div (-8))^2\\&= (-10)(-3)^2\\&= (-10)(9)\\&= -90\end{aligned}$$

$$\begin{aligned}&(-8) - 2(24 \div (-8))^2\\&= (-8) - 2(-3)^2\\&= (-8) - (-6)^2\\&= -8 - 36\\&= -44\end{aligned}$$

Why did both these students get incorrect answers?
What is the correct answer?

Investigate

2

Use a calculator when you need to.
Use any operations or brackets with these rational numbers: −2.1, −0.5, 3.4, 0.9

➤ Write an expression and determine its value.
➤ Try to find an expression with a greater value. What strategies did you use to do this?
➤ Repeat this activity several times. Which expression has the greatest value?

Reflect & Share

Compare your expression with the greatest value with that of another pair of students. Are the expressions the same? If not, whose expression has the greater value? Share your strategies.
Work together to find the expression with the least value.

Connect

In Lesson 3.1, you learned that integers and fractions are rational numbers. So, the order of operations for all rational numbers is the same as that for integers and fractions:

- Do the operations in brackets first.
- Do any work with exponents.
- Multiply and divide, in order, from left to right.
- Add and subtract, in order, from left to right.

Example 1 Using the Order of Operations with Decimals

Evaluate.

a) $(-0.8) + 1.2 \div (-0.3) \times 1.5$ **b)** $(-3.2) - 0.9 \div [0.7 - (-1.2)]^2$

A Solution

a)

$(-0.8) + 1.2 \div (-0.3) \times 1.5$	Divide first: $1.2 \div (-0.3) = -4$
$= (-0.8) + (-4) \times 1.5$	Then multiply: $(-4) \times 1.5 = -6$
$= (-0.8) + (-6)$	Then add.
$= -6.8$	

b)

$(-3.2) - 0.9 \div [0.7 - (-1.2)]^2$	Subtract in the brackets first: add the opposite.
$= (-3.2) - 0.9 \div [0.7 + 1.2]^2$	Add: $0.7 + 1.2 = 1.9$
$= (-3.2) - 0.9 \div [1.9]^2$	Use a calculator to evaluate the power: $[1.9]^2 = 3.61$
$= (-3.2) - 0.9 \div 3.61$	Then divide: $0.9 \div 3.61 \doteq 0.249\,307\,479$
$\doteq -3.2 - 0.249\,307\,479$	
$\doteq -3.449\,307\,479$	

Since the answer does not terminate or appear to repeat, round the answer to the nearest tenth because the numbers in the question are given in that form.

So, $(-3.2) - 0.9 \div [0.7 - (-1.2)]^2 \doteq -3.4$

After we substitute rational numbers for variables in a formula, we simplify the numerical expression using the order of operations.
When you evaluate with decimals, use a calculator when the divisor has more than 1 digit and when the number you multiply by has more than 2 digits.

Example 2 Solving Problems Using the Order of Operations

To convert a temperature in degrees Fahrenheit to degrees Celsius, we use the formula:

$$C = \frac{F - 32}{1.8}$$

In Fort Simpson, Northwest Territories, the mean temperature in December is -9.4°F. What is this temperature in degrees Celsius?

A Solution

Substitute $F = -9.4$ into the formula:

$$C = \frac{F - 32}{1.8}$$
$$= \frac{-9.4 - 32}{1.8}$$

The fraction bar indicates division, but also acts like brackets.

That is, the expression means $C = (-9.4 - 32) \div 1.8$

So, simplify the numerator first, then divide.

$$C = \frac{-9.4 - 32}{1.8} \qquad \text{Subtract.}$$
$$= \frac{-41.4}{1.8} \qquad \text{Divide.}$$
$$= -23$$

The mean temperature in December is $-23°C$.

Example 3 Using the Order of Operations with Fractions

Evaluate.

$$\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) - \left(-\frac{2}{3}\right) \div \left[\frac{1}{3} + \left(-\frac{3}{12}\right)\right]$$

A Solution

$$\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) - \left(-\frac{2}{3}\right) \div \left[\frac{1}{3} + \left(-\frac{3}{12}\right)\right] \qquad \text{Add in the square brackets first.}$$
$$= \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) - \left(-\frac{2}{3}\right) \div \left[\frac{1}{3} - \frac{3}{12}\right] \qquad \text{Use a common denominator of 12.}$$
$$= \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) - \left(-\frac{2}{3}\right) \div \left[\frac{4}{12} - \frac{3}{12}\right]$$
$$= \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) - \left(-\frac{2}{3}\right) \div \left(\frac{1}{12}\right) \qquad \text{Multiply next.}$$
$$= \frac{1}{4} - \left(-\frac{2}{3}\right) \div \left(\frac{1}{12}\right) \qquad \text{Then divide: multiply by the reciprocal of the divisor.}$$
$$= \frac{1}{4} - \left(-\frac{2}{\cancel{3}_1}\right) \times \left(\frac{\cancel{12}^4}{1}\right)$$
$$= \frac{1}{4} - \left(-\frac{2}{1}\right) \times \left(\frac{4}{1}\right)$$
$$= \frac{1}{4} - (-8)$$
$$= \frac{1}{4} + 8$$
$$= 8\frac{1}{4}$$

Discuss the ideas

1. What does a fraction bar indicate?

2. As the number of operations increases and the expressions become more complex, it is easy to make mistakes.
   What can you do to prevent yourself making mistakes?

Practice

Check

3. Evaluate. Do not use a calculator.
   a) $2.3 - (-1.6) \times (0.8)$
   b) $(-14.8) \times 0.9 - 3.1$
   c) $(-12.8) \div (-0.2) + 4.5 \div 0.5$
   d) $(-4.8) \times (-0.4 + 0.6)^2$

4. Evaluate. Do not use a calculator.
   a) $\frac{1}{2} + \left(-\frac{3}{4}\right) \times \frac{1}{3}$
   b) $\left(-\frac{5}{4}\right) \div \left(-\frac{1}{4} + \frac{3}{2}\right)\left(-\frac{1}{4} + \frac{3}{2}\right)$
   c) $\left(-\frac{7}{10}\right) \div \left(-\frac{2}{5}\right) - \left(-\frac{1}{4}\right) \times \frac{1}{2}$
   d) $\frac{6}{5} \times \left(-\frac{2}{3} + \frac{8}{3}\right)^2 - \frac{5}{12}$

Apply

5. a) Use a calculator to evaluate the expression below. Key in the expression as it is written.
   $-2.8 - 1.4 \times 4.5$
   b) Does the calculator follow the order of operations or does it perform operations from left to right? How did you find out?

6. Estimate which expression has the greatest value. Then use a calculator to evaluate each expression to verify your prediction.
   a) $9.1 - 3.5 \times (4.2)^2$ b) $(9.1 - 3.5) \times (4.2)^2$
   c) $9.1 - (3.5 \times 4.2)^2$ d) $9.1[(-3.5) \times (4.2)^2]$

7. Evaluate.
   a) $\left(-\frac{2}{3}\right) \div \frac{1}{4} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3}$
   b) $\left(-\frac{2}{3}\right) \div \left[\frac{1}{4} + \left(-\frac{1}{2}\right)\right] \times \frac{1}{3}$
   c) $\left(-\frac{2}{3}\right) \div \left[\frac{1}{4} - \left(-\frac{1}{2}\right)\right] \times \frac{1}{3}$
   d) $\left(-\frac{2}{3}\right) \div \left[\frac{1}{4} + \left(-\frac{1}{2}\right) \times \frac{1}{3}\right]$

8. Find the errors in each solution.
   Write the correct solution.

a) $(-3.7) \times (-2.8 + 1.5) - 4.8 \div (-1.2)$
$= (-3.7) \times (1.3) - 4.8 \div (-1.2)$
$= -4.81 - 4.8 \div (-1.2)$
$= -9.61 \div (-1.2)$
$= 8.008\overline{3}$

b) $-\frac{3}{8} - \frac{4}{5} \times \frac{3}{10} \div \left(-\frac{4}{5}\right)$
$= -\frac{15}{40} - \frac{32}{40} \times \frac{3}{10} \div \left(-\frac{4}{5}\right)$
$= -\frac{47}{40} \times \frac{3}{10} \div \left(-\frac{4}{5}\right)$
$= -\frac{141}{400} \div \left(-\frac{4}{5}\right)$
$= -\frac{141}{400} \times \left(-\frac{5}{4}\right)$
$= \frac{(-141) \times (-5)}{400 \times 4}$
$= \frac{705}{1600}$

9. A family moves from Chicago to Saskatoon. A company that rents moving trucks uses this formula, $C = 1.15[21.95d + 0.035(k - 120)]$, to determine the cost, including tax, of renting a truck for d days and k kilometres, when $k > 120$. The distance from Chicago to Saskatoon is 2400 km and the family travels for 4 days.
What is the cost to rent the truck?

10. A can of soup is a cylinder with radius 3.5 cm and height 11.5 cm.

Use the formula:
Surface area $= 2\pi r^2 + 2\pi r \times$ height,
where r is the radius of the can

a) Determine the area of tin needed to make the can, to the nearest square centimetre.

b) Explain how you used the order of operations in part a.

11. a) Use this formula to convert each Fahrenheit temperature below to Celsius:
$C = \frac{F - 32}{1.8}$

i) 0°F **ii)** −40°F **iii)** −53°F

b) Here is another way to write the formula in part a: $C = \frac{5}{9}(F - 32)$
Use this formula to convert each Fahrenheit temperature below to Celsius:

i) 50°F **ii)** −13°F **iii)** 32°F

c) Which formula in parts a and b was easier to use? Explain your choice.

12. Evaluate. State the order in which you carried out the operations.

a) $\left(-4\frac{1}{2}\right) + \left(-\frac{2}{3}\right) \times 2\frac{3}{4}$

b) $\left(-3\frac{2}{5}\right) \times \left(-1\frac{5}{6}\right) + \frac{3}{10}$

c) $(-3) \div \left(-\frac{4}{5}\right) + \left(-\frac{5}{12}\right) \times 1\frac{1}{2}$

d) $\left(1\frac{5}{8}\right) - \left(-2\frac{3}{4} + 2\right)\left(-2\frac{3}{4} + 2\right)$

13. Use a calculator to evaluate.
Write the answers to the nearest hundredth where necessary.

a) $2.3 + (-11.2) \div (-0.2) - 3.7$

b) $(-3.4) \times 0.7 - (-1.8)(-1.8)$

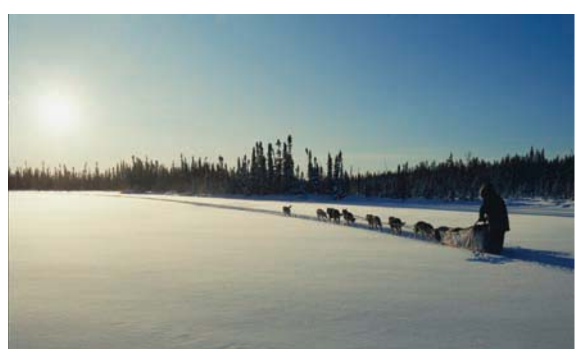
c) $\frac{0.67 - 4.2 \div (-0.2)}{(-7.3 + 8.6)^2}$

d) $\frac{8.9 \times (-3.1 + 22.7)^2 + 4.7}{(-9.6) \div 0.04 - 0.4}$

14. On one day in Black Lake, Saskatchewan, the maximum temperature was −8.1°C and the minimum temperature was −16.7°C.

a) What was the mean temperature that day?

b) How did you use the order of operations in part a?



15. **Assessment Focus** Use these numbers to make 4 fractions: 2, −3, 4, −5, 6, −8, 10, −12

a) Use the 4 fractions to write an expression using 3 different operations and brackets. Evaluate the expression.

b) Use the same 4 fractions a different way or use 4 different fractions. Write an expression whose value is as close to 0 as possible. Show your work.

16. The following maximum temperatures were recorded for one week in Abbotsford, BC: −3.1°C, −4.5°C, −6.2°C, −1.2°C, 1.5°C, 2.3°C, 4.1°C

a) Predict whether the mean maximum temperature for the week is above or below 0°C.

b) Calculate the mean maximum temperature for the week.

17. A student's solution to a problem, to the nearest hundredth, is shown below. The solution is incorrect. Identify the errors. Provide a correct solution.

$(-8.2)^2 \div (-0.3) - 2.9 \times (-5.7)$

$= 67.24 \div (-0.3) - 2.9 \times (-5.7)$

$= 67.24 \div (-0.3) - 16.53$

$= 67.24 \div (-16.83)$

$\doteq 4.00$

18. A student evaluated the following expression and the answer was 50.39 to the nearest hundredth. Another student evaluated the expression and the answer was 1.63 to the nearest hundredth.

$$\frac{23.7 - (-5.6) \div 0.7 + 6.8}{(-3) \times (-6.7) + 3.5}$$

a) Which answer is correct?

b) What mistake did one student likely make?

Take It Further

19. In question 11, you used these two versions of a formula to convert Fahrenheit temperatures to Celsius:

$C = \frac{F - 32}{1.8}$ and $C = \frac{5}{9}(F - 32)$

Explain how to get one version of the formula from the other.

20. In Flin Flon, Manitoba, the mean of the maximum and minimum temperatures on one day was −12.8°C. The maximum temperature was −11.5°C.
What was the minimum temperature?

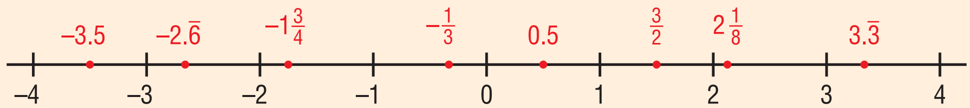
21. Insert brackets in the expression below so the statement is correct.
Is it possible to insert brackets and get a positive answer?
Explain your thinking.
$-3.8 + 9.1 \times -2.5 - 0.5 = -31.1$

Reflect

When you use the order of operations with rational numbers, do you prefer to work with the numbers in decimal form or fraction form? Explain your choice.

Study Guide

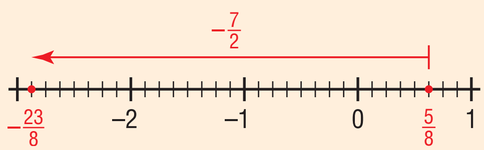
A rational number is any number that can be written in the form $\frac{m}{n}$, where m and n are integers and $n \neq 0$.
This number line illustrates some different forms of rational numbers:

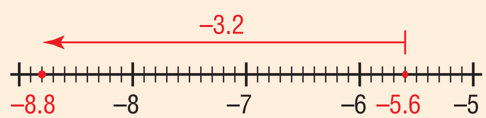


From least to greatest: $-3.5, -2.\overline{6}, -1\frac{3}{4}, -\frac{1}{3}, 0.5, \frac{3}{2}, 2\frac{1}{8}, 3.\overline{3}$

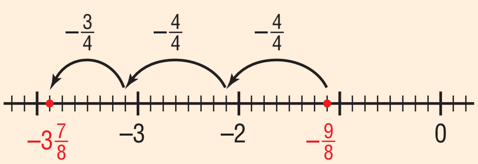
To operate with rational numbers, apply what you know about operating with fractions, decimals, and integers.

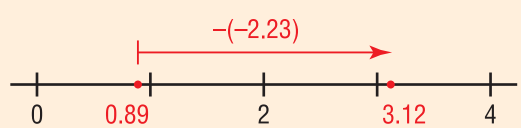
- To add rational numbers, visualize a number line.
  $\frac{5}{8} + \left(-\frac{7}{2}\right) = -\frac{23}{8}$ $(-5.6) + (-3.2) = -8.8$





- To subtract rational numbers, visualize a number line.
  $-\frac{9}{8} - \frac{11}{4} = -\frac{31}{8}$ $0.89 - (-2.23) = 3.12$





- To multiply rational numbers, determine the sign of the product first.
  $\left(\frac{3}{4}\right)\left(-\frac{5}{2}\right) = -\frac{15}{8}$ and $(-4.13)(-0.8) = 3.304$
- To divide rational numbers, determine the sign of the quotient first.
  $\left(-\frac{3}{10}\right) \div \left(-\frac{12}{5}\right) = \frac{1}{8}$ and $76.63 \div (-7.5) = -10.217\overline{3}$

The order of operations with rational numbers is the same as the order for whole numbers, fractions, and integers:

- Do the operations in brackets first.
- Then evaluate the exponents.
- Then divide and multiply, in order, from left to right.
- Then add and subtract, in order, from left to right.

Review

3.1

1. Which of the following rational numbers are between -2.5 and $-\frac{11}{3}$? How do you know?
   a) -3.4 b) $-\frac{9}{4}$ c) $-\frac{19}{6}$ d) -4.2

2. Order the following rational numbers from least to greatest. Show them on a number line.
   $3.12, -\frac{4}{3}, 0.9, -\frac{1}{2}, -0.4$

3. Write 3 rational numbers between each pair of numbers. Sketch number lines to show all the rational numbers.
   a) $-3.5, -3.1$ b) $\frac{1}{5}, \frac{7}{10}$
   c) $0.8, 0.9$ d) $-\frac{5}{2}, -\frac{3}{2}$

4. On one day, the prices of 5 stocks changed by the following amounts in dollars:
   $-0.09, -0.51, +0.95, +0.54, -2.00$
   Order the amounts from the greatest loss to the greatest gain.

3.2

5. Determine each sum.
   a) $-1.2 + (-0.3)$
   b) $134.89 + (-56.45)$
   c) $-23.6 - 4.57$
   d) $48.05 + 0.003$

6. A technician checked the temperature of a freezer and found that it was -15.7°C. She noted that the temperature had dropped 7.8°C from the day before.
   a) What was the temperature the day before?
   b) Show both temperatures on a vertical number line.

7. Determine each sum.
   a) $\frac{3}{4} + \frac{7}{8}$ b) $-1\frac{1}{2} + 3\frac{1}{3}$
   c) $-4\frac{5}{6} + \left(-1\frac{5}{12}\right)$ d) $\frac{11}{9} + \left(-\frac{17}{6}\right)$

3.3

8. Determine each difference.
   a) $-3.4 - (-4.8)$
   b) $-71.91 - 11.23$
   c) $90.74 - 100.38$
   d) $63.2 - 80.02$

9. At the end of a day, the price of a stock was \$21.60. During the day, the price of the stock had changed by $-\$0.75$. What was the price of the stock at the beginning of the day? How do you know?

10. Determine each difference.
    a) $\frac{4}{3} - \frac{11}{6}$ b) $-\frac{5}{8} - \left(-\frac{7}{5}\right)$
    c) $3\frac{5}{7} - \left(-6\frac{9}{10}\right)$ d) $-\frac{23}{4} - \frac{23}{3}$

3.4

11. Predict which expressions have a value between -1 and 1. Calculate each product to check.
    a) $(-1.4) \times (-0.8)$ b) $25.6 \times (-0.05)$
    c) $\left(-\frac{3}{5}\right)\left(\frac{4}{3}\right)$ d) $\left(-\frac{5}{6}\right)\left(-\frac{2}{3}\right)$

12. The temperature in Richmond, BC, at 4:00 P.M. was 2°C. The temperature drops 1.3°C each hour. What will the temperature be at 11:00 P.M.? Justify your answer.

13. Write 3 multiplication statements that have the same product as $\left(-\frac{4}{9}\right)\left(\frac{7}{5}\right)$. How can you check your answers?

14. Determine each product.

a) $3.5 \times (-0.3)$ **b)** $(-4.1)(2.3)$

c) $\left(-\frac{4}{7}\right)\left(-\frac{2}{3}\right)$ **d)** $1\frac{3}{5} \times \left(-2\frac{1}{2}\right)$

15. A mountain climber descends from base camp at an average speed of 5.9 m/h. How far below base camp will the climber be after 3.75 h? Use a vertical number line with the base camp at 0 to illustrate the climber's descent.

3.5

16. Predict which expressions have a value between -1 and 1. Calculate each quotient to check.

a) $(-2.2) \div 0.4$ **b)** $10.6 \div (-9.2)$

c) $\frac{9}{10} \div \left(-\frac{3}{2}\right)$ **d)** $\left(-\frac{5}{12}\right) \div \left(-\frac{5}{4}\right)$

17. Write 3 division statements that have the same quotient as $\frac{3}{8} \div \left(-\frac{5}{11}\right)$.

18. Replace each □ with a rational number to make each equation true. Explain the strategy you used.

a) $(-0.2) \times \square = 0.75$

b) $0.9 \times \square = -7.47$

c) $(-0.624) \div \square = -0.4$

19. Determine each quotient.

a) $8.4 \div (-1.2)$ **b)** $(-20.6) \div (-0.9)$

c) $\left(-\frac{9}{11}\right) \div \left(\frac{7}{5}\right)$ **d)** $\left(-1\frac{2}{3}\right) \div 3\frac{1}{2}$

3.6

20. a) Evaluate each expression. Do not use a calculator.

i) $-3.5 + 6.2 \times (-0.2)$

ii) $(-3.5 + 6.2) \times (-0.2)$

b) Are the answers in part a different? Explain.

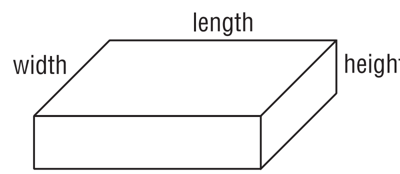
21. Predict whether the value of each expression below is positive or negative. Explain how you predicted. Evaluate to check your prediction.

a) $-\frac{3}{5} + \left[\frac{1}{3} \times \left(-\frac{3}{4}\right)\right]$

b) $\left(-\frac{3}{5} + \frac{1}{3}\right) \times \left(-\frac{3}{4}\right)$

c) $-\left(-\frac{3}{5} + \frac{1}{3}\right) \times \left(-\frac{3}{4}\right)$

22. A formula for the surface area of a right rectangular prism is:
2(length × width + length × height + width × height)



a) Determine the surface area of a right rectangular prism with length 25.3 cm, width 15.2 cm, and height 9.7 cm.

b) Explain how you used the order of operations in part a.

23. Evaluate each expression. Show your work to illustrate the order of operations.

a) $-1.2 \div (0.6) - [6.3 + (-3.4)]$

b) $-\frac{5}{12} + \left(\frac{4}{3}\right)\left(\frac{4}{3}\right)$

c) $-\frac{4}{5} \div \left[\frac{1}{2} + \left(-\frac{1}{6}\right)\left(-\frac{1}{6}\right) \times \frac{1}{4}\right]$

d) $\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right) \div \frac{2}{9} - \left(-\frac{4}{5}\right)$

e) $-1\frac{3}{7} \times \frac{1}{2} + \left(-3\frac{1}{7}\right)$

f) $0.2 - (-1.2) \times 0.5 \div (-0.1)$

g) $(-0.2 + 0.9)^2 + 9.8 \div (-0.7)$

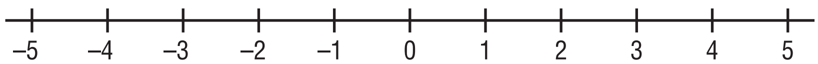
Practice Test

1. a) Identify a rational number between -0.5 and -0.6.

b) How do you know the number you identified in part a is a rational number?

2. a) Write the following rational numbers on a copy of the number line below:

$0.6, -0.\overline{3}, -2.5, 3.\overline{6}, 4\frac{1}{2}, -1\frac{3}{10}, -\frac{23}{5}, \frac{11}{3}$



b) List the numbers in part a from greatest to least.

3. Evaluate.

a) $-7.4 - (-6.1)$ **b)** $\frac{4}{5} + \left(-\frac{3}{10}\right)$ **c)** $(-3.2)(-0.5)$ **d)** $\left(-\frac{3}{4}\right) \div \frac{1}{3}$

4. Sarah has a balance of $-\$2.34$ in her account.
Each time she makes a withdrawal, she is charged \$1.20.

a) What does "a balance of $-\$2.34$" mean?

b) Sarah makes three more withdrawals of \$20.50 each.
What is her balance now?
How can you use rational numbers to calculate it?

c) Sarah's overdraft limit is \$500.00. How many more \$20.50 withdrawals can she make? Justify your answer.

5. Evaluate. How could you check your answers?

a) $(-56.8)(-14.5)$ **b)** $\left(-3\frac{1}{3}\right)\left(-2\frac{3}{10}\right)$

c) $\left(-4\frac{2}{5}\right) \div \left(-1\frac{5}{7}\right)$ **d)** $45.8 \div (-12.2)$

6. a) A student evaluated the expression below and got the answer 1.
What is the correct answer? How do you know?

$\frac{1}{2} + \left(-\frac{3}{4}\right) \div \left(-\frac{1}{4}\right)$

b) What might the student have done wrong to get the answer 1?

7. Evaluate. Use a calculator when you need to.

a) $-3.1 + 4.5 \times (-2.9) - 7.2 \div (-3)$

b) $(-9.7) \times (-1.2) + 5.4^2 \div (-3.6)$

Unit Problem — Investigating Temperature Data

The table shows the monthly lowest temperature, in degrees Celsius, in Edmonton.

Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
−14.7	−12.8	−7.2	0.5	5.9	10.1	12.3	10.6	6.1	−0.8	−8.3	−13.4

1. Determine the mean monthly lowest temperature in Edmonton for that year.

a) How did you use the order of operations in your calculation?

b) Which months have temperatures below the mean? How do you know?

Here are the monthly highest temperatures in Edmonton for the same year.

Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
−5.8	−2.9	2.1	10.6	17.2	20.7	23.4	22.2	16.7	9.8	1.1	−4.3

2. Determine the mean monthly highest temperature in Edmonton for that year.

3. Choose three months. For each month, calculate the difference between:

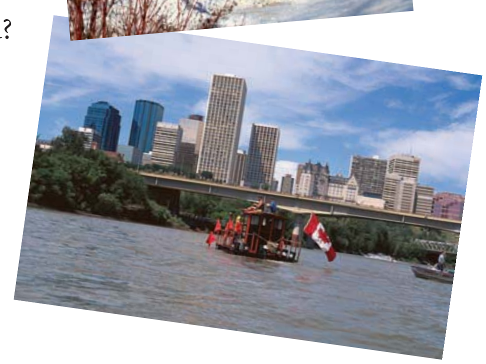
a) the mean monthly highest temperature and the monthly highest temperature

b) the mean monthly lowest temperature and the monthly lowest temperature

4. Some climatologists predict that, by the end of the century, due to global warming, the mean temperature will increase by between 1.4°C and 11°C.

a) How might the mean monthly highest temperature be affected? In what range could this temperature lie in 2100?

b) Repeat part a for the mean monthly lowest temperature.



Your work should show:

- your calculations for the mean lowest and highest temperatures
- your calculations of the differences in temperatures
- the possible ranges for temperatures in the year 2100

Reflect on Your Learning

What is the most important thing you have learned about rational numbers? Explain why it is important.

Cumulative Review Units 1–3

1 **1.** Determine the value of each square root.

a) $\sqrt{\frac{1}{25}}$ b) $\sqrt{\frac{225}{169}}$ c) $\sqrt{\frac{9}{121}}$

d) $\sqrt{1.44}$ e) $\sqrt{0.16}$ f) $\sqrt{3.24}$

2. Determine the side length of a square with each area below. Explain your strategy.

a) $64\ cm^2$

b) $1.21\ m^2$

c) $72.25\ mm^2$

3. Calculate the number whose square root is:

a) 0.7 b) 1.6 c) 0.006

d) $\frac{12}{17}$ e) $\frac{1}{3}$ f) $\frac{2}{13}$

4. Which decimals and fractions are perfect squares? Explain your reasoning.

a) $\frac{7}{63}$ b) $\frac{12}{27}$ c) $\frac{4}{18}$

d) 0.016 e) 4.9 f) 0.121

5. A square garden has area $6.76\ m^2$.

a) What is the side length of the garden?

b) One side of the garden is against a house. How much fencing is needed to enclose the garden? How do you know?

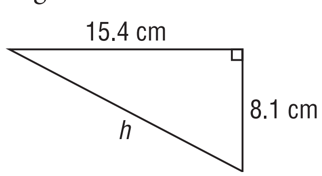
6. Determine 2 decimals that have square roots from 12 to 13.

7. Use any strategy you wish to estimate the value of each square root.

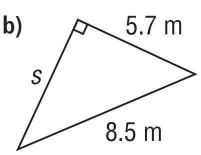
a) $\sqrt{\frac{1}{35}}$ b) $\sqrt{\frac{65}{4}}$ c) $\sqrt{0.8}$ d) $\sqrt{0.11}$

8. Determine the unknown length in each triangle to the nearest tenth.

a)



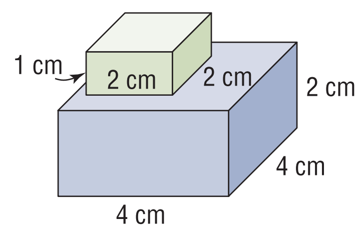
b)



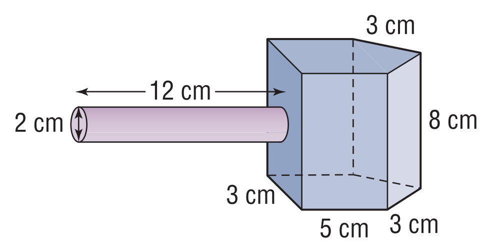
9. This object is built with 1-cm cubes. Determine its surface area.

10. Determine the surface area of each composite object. Give the answers to the nearest whole number. Explain how you accounted for the overlap in each case.

a)



b)



2 **11.** Write each product as a power, then evaluate.

a) $4 \times 4 \times 4$

b) $6 \times 6 \times 6 \times 6$

c) $(-3)(-3)(-3)(-3)(-3)(-3)(-3)$

d) $-(-2)(-2)(-2)(-2)(-2)(-2)(-2)$

e) $-(10 \times 10 \times 10 \times 10 \times 10)$

f) $-(1)(1)(1)(1)(1)(1)(1)(1)(1)(1)(1)(1)$

12. Predict the sign of each answer, then evaluate.

a) $-(-3)^4$ b) $(-5)^6$ c) -4^3

d) $(-7)^2$ e) -7^0 f) $(-10)^0$

13. Write each number using powers of 10.

a) 800 b) 52 000

c) 1760 d) 7 000 004

14. Evaluate.

a) $[3 \times (-2)^3 - 4]^2$

b) $(-7 + 5)^2 - [4 + (-1)^3]^2$

c) $9^2 \div (-3)^3 + 5^2 - 2^5$

d) $(3^7 - 2^{11})^1 \div (4^7 + 3^8)^0$

e) $(-4)^2 - 3^3 + (-2)^4 - 1^5$

f) $[8^4 \div (-4)^6 \times 2^0]^{10}$

15. Express as a single power.

a) $6^5 \times 6^{11} \div 6^8$

b) $(-3)^6 \div (-3)^2 \times (-3)^4$

c) $\dfrac{(-5)^6 \times (-5)^9}{(-5)^7 \times (-5)^5}$

d) $\dfrac{2^8}{2^2} \times \dfrac{2^{12}}{2^4}$

16. Evaluate.

a) $7^2 - 4^3 \times 4^0 + 3^2$

b) $(-2)^8 \div (-2)^4 - (-2)^7 \div (-2)^5$

c) $-5^2(5^4 \div 5) - 5^3$

d) $\dfrac{8^{13} \times 8^{14}}{8^{15} \times 8^9}$

17. A wheat field is 10 000 m wide. The area of the field is 10^8 m^2.

a) Use the exponent laws to determine the length of the field.

b) What is the perimeter of the field? Did you use any exponent laws to calculate the perimeter? Explain.

18. Simplify, then evaluate each expression.

a) $(6^2)^8 \div (6^4)^2$

b) $(7^4 \div 7^2)^3 + (3^5 \div 3^2)^3$

c) $[(-2)^5 \div (-2)^4]^3 - [(-5)^2 \times (-5)^3]^0$

d) $(4 \times 9)^4 + (3^5)^2$

e) $[(-4)^3]^2 - [(-2)^4]^3 - [(-3)^2]^4$

f) $[9 \div (-3)]^2 \times 3^4$

3

19. Show each set of numbers on a number line. Order the numbers from least to greatest.

a) $-1.9, -3.3, 4.8, -2.8, 1.2, -3.\overline{3}$

b) $\frac{19}{5}, -\frac{13}{4}, \frac{3}{4}, -2\frac{1}{2}, -\frac{13}{10}, -\frac{2}{5}$

c) $1.1, \frac{4}{3}, -\frac{1}{3}, -1.01, 1\frac{3}{8}, -0.11$

d) $\frac{2}{9}, -0.2, 0.25, -\frac{1}{6}, -0.\overline{1}, \frac{1}{8}$

20. Determine each sum or difference.

a) $17.4 + (-15.96)$ b) $-8.38 + (-1.927)$

c) $-4.5 - (-13.67)$ d) $13.28 - 19.71$

e) $-\frac{3}{4} + \frac{2}{3}$ f) $1\frac{5}{8} + \left(-6\frac{1}{3}\right)$

g) $-\frac{17}{4} - \frac{11}{3}$ h) $3\frac{5}{6} - \left(-2\frac{2}{3}\right)$

21. The changes in value of a stock were recorded in the table below.

Day	Change in Value ($)
Monday	–0.450
Tuesday	0.327
Wednesday	–0.065

The price of the stock by the end of the day on Wednesday was $85.460. Use rational numbers to calculate the price of the stock on Monday morning.

22. Determine each product or quotient.

a) $(-14.6)(2.5)$ b) $(-12.8)(-12.8)$

c) $(-8.64) \div (-2.7)$ d) $4.592 \div (-0.82)$

e) $\left(\frac{9}{5}\right)\left(6\frac{1}{3}\right)$ f) $\left(-8\frac{3}{4}\right)\left(2\frac{2}{15}\right)$

g) $\left(-\frac{5}{12}\right) \div \left(-8\frac{1}{3}\right)$ h) $\left(-3\frac{1}{5}\right) \div 2\frac{2}{3}$

23. Evaluate.

a) $\left(-\frac{7}{8}\right) - \frac{1}{5} \div \left(-\frac{3}{10}\right) - \frac{1}{4}$

b) $(-2.1)(18.5) - 6.8 \div 4$

c) $\left(-7\frac{1}{3}\right)\left(\frac{6}{55}\right) + 1\frac{1}{2} \div \left(-\frac{2}{7}\right)$

d) $2\frac{1}{4} - \left(-3\frac{7}{8} + 5\right)\left(\frac{4}{9} - 3\right)$

UNIT 4

Linear Relations

How do you think music sales have changed over the past 10 years? 20 years?
In what format do you buy the music you listen to?
In what format did your parents buy the music they listened to as students?
Why might record companies be interested in keeping track of these data?

What You'll Learn

- Use expressions and equations to generalize patterns.
- Verify a pattern by using substitution.
- Graph and analyze linear relations.
- Interpolate and extrapolate to solve problems.

Why It's Important

Patterns and relationships are an important part of math. We can model many real-world situations with a linear relation, and use the relation to make predictions and solve problems. For example, the total cost of a pizza is a fixed cost for a particular size, plus a cost that depends on the number of toppings added.