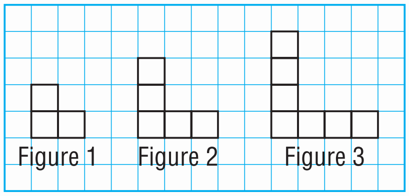
Key Words

- dependent variable
- independent variable
- relation
- linear relation
- interpolation
- extrapolation

Start Where You Are

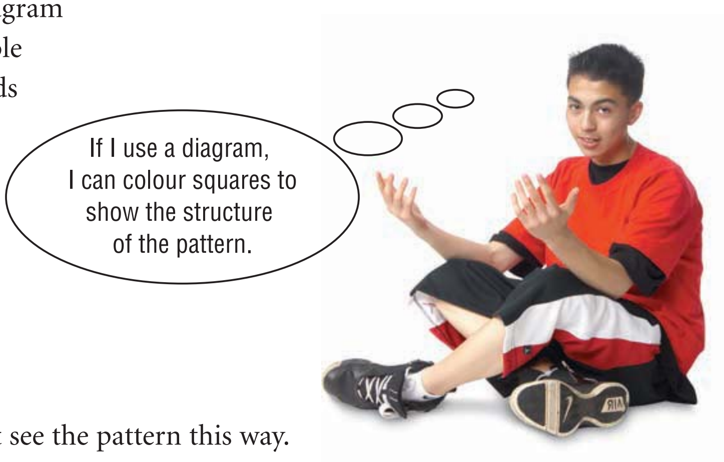
How Can I Explain My Thinking?

The pattern of figures below continues. Suppose I have to determine a rule for the number of squares in any figure n in this pattern.



What tools can I use to explain my thinking?

- a diagram
- a table
- words



I might see the pattern this way.

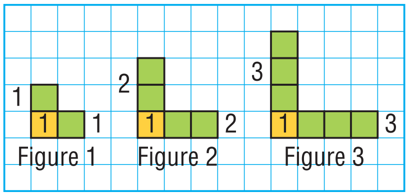


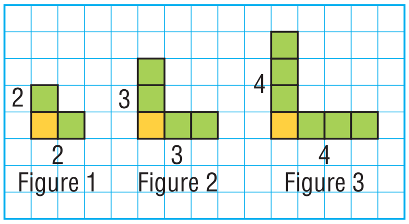
Figure 1: $1 + 1 + 1 = 1 + 2(1)$

Figure 2: $1 + 2 + 2 = 1 + 2(2)$

Figure 3: $1 + 3 + 3 = 1 + 2(3)$

Figure n: $1 + n + n = 1 + 2(n)$

I might also see the pattern this way.



I am counting the yellow square twice, so I will have to subtract 1.

Figure 1: $2 + 2 - 1 = (1 + 1) + (1 + 1) - 1$

Figure 2: $3 + 3 - 1 = (2 + 1) + (2 + 1) - 1$

Figure 3: $4 + 4 - 1 = (3 + 1) + (3 + 1) - 1$

Figure n: $(n + 1) + (n + 1) - 1$

I might also see the pattern a third way.

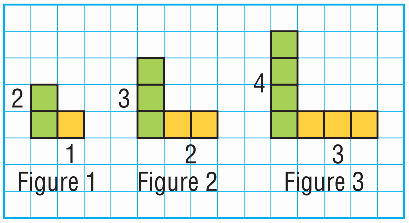


Figure 1: $2 + 1 = (1 + 1) + 1$

Figure 2: $3 + 2 = (2 + 1) + 2$

Figure 3: $4 + 3 = (3 + 1) + 3$

Figure n: $(n + 1) + n$

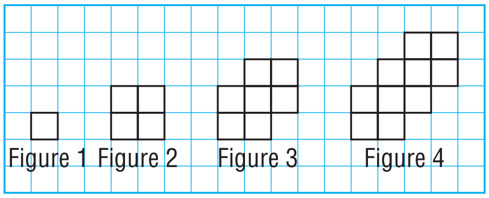
Figure Number	Number of Squares	Pattern
1	3	$2(1) + 1$
2	5	$2(2) + 1$
3	7	$2(3) + 1$
n		$2n + 1$

With each figure, I add 2 extra squares.
The numbers of squares are odd numbers.
Each number is 1 more than a multiple of 2.
So, figure n has $2n + 1$ squares.

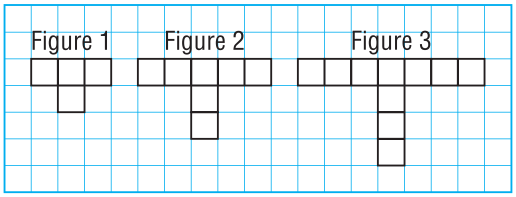
Check

Use the tools *you* find most helpful to determine a rule for the number of squares in figure n of each pattern.

1.



2.



4.1 Writing Equations to Describe Patterns

FOCUS

- Use equations to describe and solve problems involving patterns.

Here is a pattern made from square tiles.

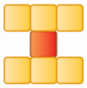


Figure 1

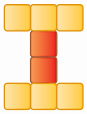


Figure 2

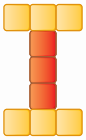


Figure 3

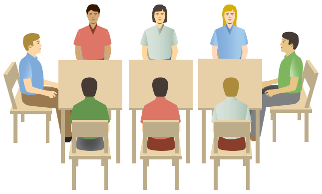
What stays the same in each figure? What changes?
How can we determine the number of square tiles in any figure in the pattern?

Investigate

A banquet hall has small square tables that seat 1 person on each side.
The tables can be pushed together to form longer tables.


1 table


2 tables


3 tables

The pattern continues.

➤ Sketch the next 2 table arrangements in the pattern.
What stays the same in each arrangement? What changes?

➤ What different strategies can you use to determine the number of people at 6 tables? At 10 tables? At 25 tables?

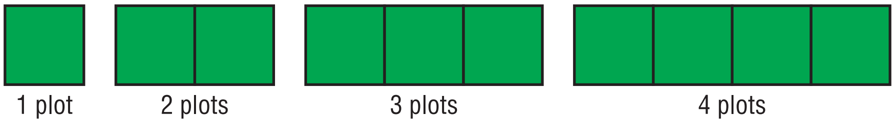
Reflect & Share

Compare your strategies and results with those of your classmates.
If you used different strategies, explain your strategies.
If you did not write an equation, work together to determine an equation that relates the number of people to the number of tables.
Use the equation to determine:

- the number of people at 30 tables
- the number of tables needed to seat 30 people

Connect

A landscape designer uses wooden boards as edging for the plots in a herb garden.



The number of boards, b, is *related* to the number of plots, p.
This relationship can be represented in different ways:

- using pictures
- using a table of values
- using an equation

Here are 2 ways to determine the equation.

➤ Determine a pattern in the number of boards.

	Number of Plots, p	Number of Boards, b	
	1	4	
+1	2	7	+3
+1	3	10	+3
+1	4	13	+3

As the number of plots increases by 1, the number of boards increases by 3. Repeated addition of 3 is the same as multiplication by 3. This suggests that the number of boards may be 3 times the number of plots. So, the equation $b = 3p$ may represent this relationship.

Check whether the equation $b = 3p$ is correct.
When $p = 1$,

$$\begin{aligned} b &= 3(1) \\ &= 3 \end{aligned}$$

This is 1 less than the number 4 in the table.
So, we add 1 to $3p$ to describe the number of boards correctly.

The terms $3p + 1$ form an *expression* that represents the number of boards for any number of plots p.
An equation is: $b = 3p + 1$

Number of Plots, p	Number of Boards, b
1	$3(1) + 1 = 4$
2	$3(2) + 1 = 7$
3	$3(3) + 1 = 10$
4	$3(4) + 1 = 13$

We verify the equation by substituting values of p and b from the table.
For example, check by substituting $p = 4$ and $b = 13$ in $b = 3p + 1$.

Left side: $b = 13$ Right side: $3p + 1 = 3(4) + 1$

$$\begin{aligned} &= 12 + 1 \\ &= 13 \end{aligned}$$

Since the left side equals the right side, the equation is verified.

➤ Determine a pattern in the figures that represent the garden.

Number of Plots, *p*	Pattern in the Number of Boards	Number of Boards, *b*
1	1 + 3	1 + 1(3)
2	1 + 3 + 3	1 + 2(3)
3	1 + 3 + 3 + 3	1 + 3(3)
4	1 + 3 + 3 + 3 + 3	1 + 4(3)
:		:
p		1 + *p*(3)

Each garden needs 1 board for the left border and 3 additional boards for each plot.
That is,
Number of boards = 1 + (Number of plots) × 3
As an equation:
$b = 1 + p(3)$
This can be rewritten as:
$b = 1 + 3p$

Addition is commutative, so $1 + 3p = 3p + 1$.
The equation gives a general pattern rule. We say the equation *generalizes* the pattern. We can use the equation to determine the value of any term.

Example 1 Writing an Equation to Represent a Written Pattern

An airplane is cruising at a height of 10 000 m. It descends to land. This table shows the height of the plane every minute after it began its descent. The height of the plane changes at a constant rate.

Time (*t* minutes)	Height (*h* metres)
0	10 000
1	9 700
2	9 400
3	9 100
4	8 800

a) Write an expression for the height in terms of the time since the plane began its descent.
b) Write an equation that relates the height of the plane to the time since it began its descent.
c) What is the height of the plane after 15 min?
d) How long after beginning its descent does the plane land?

A Solution

a) When the time increases by 1 min, the height decreases by 300 m.
Add a third column to the table and write the height in terms of time.

	Time (t minutes)	Height (h metres)	Height in Terms of Time	
	0	10 000	10 000 − 0 = 10 000	
+1	1	9 700	10 000 − 300(1) = 9700	−300
+1	2	9 400	10 000 − 300(2) = 9400	−300
+1	3	9 100	10 000 − 300(3) = 9100	−300
+1	4	8 800	10 000 − 300(4) = 8800	−300
+1	:		:	−300
+1	t		10 000 − 300(t)	−300

An expression for the height in terms of time is: $10\,000 - 300t$

b) For an equation that relates height to time, equate the expression in part a to the height, h.
An equation is: $h = 10\,000 - 300t$

c) To determine the height of the plane after 15 min, substitute $t = 15$ in the equation:

$$\begin{aligned} h &= 10\,000 - 300t \\ &= 10\,000 - 300(15) \\ &= 10\,000 - 4500 \\ &= 5500 \end{aligned}$$

After 15 min, the plane is at a height of 5500 m.

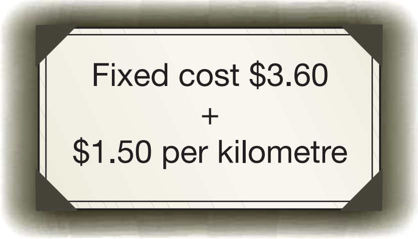
d) When the plane lands, its height is 0.
Substitute $h = 0$ in the equation $h = 10\,000 - 300t$, then solve for t.

$$\begin{aligned} h &= 10\,000 - 300t \\ 0 &= 10\,000 - 300t \\ 300t + 0 &= 10\,000 - 300t + 300t \\ 300t &= 10\,000 \\ \frac{300t}{300} &= \frac{10\,000}{300} \\ t &= 33.\overline{3} \end{aligned}$$

The plane lands about 33 min after beginning its descent.

Example 2 Writing an Equation to Represent an Oral Pattern

I called Kelly's Cabs. The cost of a ride is shown on a poster in the cab.



a) Write an expression for the fare in terms of the fixed cost and the cost per kilometre.
b) Write an equation that relates the fare to the distance travelled.
c) What is the fare for an 11-km ride?

A Solution

a) The fare is $3.60, plus $1.50 per kilometre.
That is, the fare is $3.60 + 1.50 \times$ (distance in kilometres).
Let d represent the distance in kilometres.
So, an expression for the fare is: $3.60 + 1.50 \times d$, or $3.60 + 1.50d$

b) Let F represent the fare in dollars.
Then, an equation that relates F and d is: $F = 3.60 + 1.50d$

c) To determine the cost for an 11-km trip, use the equation: $F = 3.60 + 1.50d$
Substitute: $d = 11$

$$\begin{aligned} F &= 3.60 + 1.50(11) \\ &= 3.60 + 16.50 \\ &= 20.10 \end{aligned}$$

The fare for an 11-km ride is $20.10.

Discuss the ideas

1. What different ways can you represent a relationship between two quantities?

2. What are the advantages and disadvantages of each way you described in question 1?

3. Suppose you have determined an equation that you think may describe a pattern.
   a) How could you check that your equation is correct?
   b) If you need to adjust the equation, how can you determine what needs to be changed?

Practice

Check

4. In each equation, determine the value of P when $n = 1$.

a) $P = 2n$ **b)** $P = 3n$ **c)** $P = 4n$ **d)** $P = 5n$

5. In each equation, determine the value of A when $n = 2$.

a) $A = 3n + 1$ **b)** $A = 3n + 2$

c) $A = 3n + 3$ **d)** $A = 3n + 4$

6. In a table of values for a pattern, $P = 3$ when $n = 1$; which of the following equations might represent the pattern?

a) $P = 3n$ **b)** $P = n + 3$

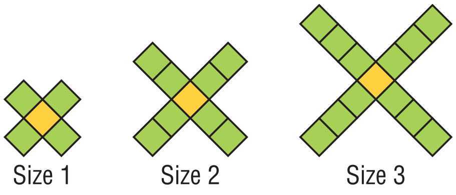
c) $P = 2n + 1$ **d)** $P = 3 - n$

7. The pattern in this table continues. Which expression below represents the number of squares in terms of the figure number?

Figure, *f*	Number of Squares, *s*
1	6
2	7
3	8
4	9
5	10

a) $5f$ **b)** $2f$ **c)** $f + 5$ **d)** $s + 5$

8. This pattern of squares continues. Which equation below relates the number of squares, n, in a picture to the size number, s?



a) $n = s + 4$ **b)** $n = 4s$

c) $n = 4s + 1$ **d)** $s = 4n$

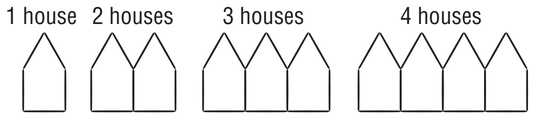
9. The pattern in this table continues. Which equation below relates the number of squares to the figure number?

Figure, *f*	Number of Squares, *s*
1	5
2	7
3	9
4	11
5	13

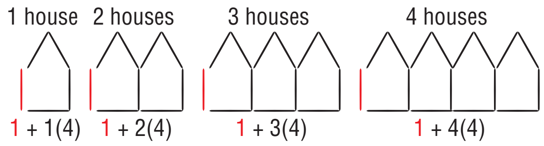
a) $s = 4f + 1$ **b)** $s = 2f + 3$

c) $s = f + 2$ **d)** $f = 2s + 3$

10. Here is a pattern made with toothpicks. The pattern continues.



Here are the toothpicks rearranged to show what stays the same and what changes in each picture.



a) Explain how the numbers in the expression below each picture describe the arrangement of toothpicks in the picture.

b) Let n represent the number of houses in a picture. Write an expression for the number of toothpicks in n houses.

c) Write an equation that relates the number of toothpicks, t, to n.

d) Verify the equation by showing that it produces the correct number of toothpicks for the first four pictures in the pattern.

Apply

11. The pattern in each table below continues.
For each table:

i) Describe the pattern that relates v to t.

ii) Write an expression for v in terms of t.

iii) Write an equation that relates v to t.

iv) Verify your equation by substituting values from the table.

a)

Term Number, t	Term Value, v
1	11
2	22
3	33
4	44

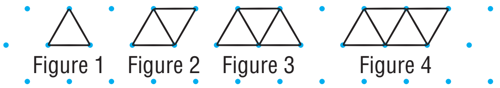
b)

Term Number, t	Term Value, v
1	5
2	8
3	11
4	14

c)

Term Number, t	Term Value, v
1	7
2	6
3	5
4	4

12. Here is a pattern of triangles made with congruent toothpicks.
The pattern continues.



a) Make a table of values for the figure number and the number of toothpicks in a figure. What patterns do you see?

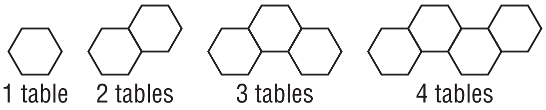
b) Write an expression for the number of toothpicks, t, in figure n.

c) Determine the number of toothpicks in figure 45.

d) Write an equation that relates t to n.

e) Which figure has 17 toothpicks?
How could you check your answer?

13. Assessment Focus Hexagonal tables are arranged as shown. The pattern continues. One person sits at each side of a table.



a) Determine the number of people who can be seated at each table arrangement. Record your results in a table.

b) Describe the patterns in the table.

c) What strategies can you use to determine the number of people who could be seated at any table arrangement in the pattern?

d) Write an equation that relates the number of people, p, who can be seated at n tables. How can you check that your equation is correct?

e) How many tables are needed to seat 41 people? How could you check your answer?

Show your work.

14. The cost to print brochures is the sum of a fixed cost of \$250, plus \$1.25 per brochure.

a) Write an equation that relates the total cost, C dollars, to the number of brochures, n.

b) What is the cost of printing 2500 brochures?

c) How many brochures can be printed for \$625?

Justify your answers.

15. A pizza with tomato sauce and cheese costs $9.00.
Each additional topping costs $0.75.

a) Create a table that shows the costs of a pizza for up to 5 toppings.

b) Write an equation that relates the cost, C dollars, to the number of toppings, n. Verify your equation by substituting values of n from the table.

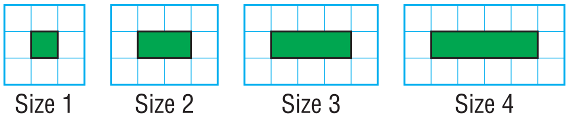
c) Suppose a pizza costs $15.00. How many toppings were ordered? What strategy did you use? Try a different strategy to check your answer.

16. Clint has a window cleaning service.
He charges a fixed cost of $12, plus $1.50 per window.

a) Write an equation that relates the total cost to the number of windows cleaned. How do you know that your equation is correct?

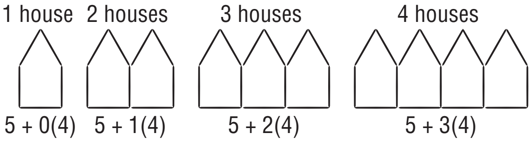
b) Clint charged $28.50 for a job.
How many windows did he clean?
How do you know?

17. A landscaper uses square patio stones as a border around a rectangular garden.
The number of patio stones needed depends on the size of the garden.
This pattern continues.



The landscaper uses 152 stones. What size of garden does she make?
How can you check your answer?

18. Here is another way to rearrange the toothpicks in question 10.



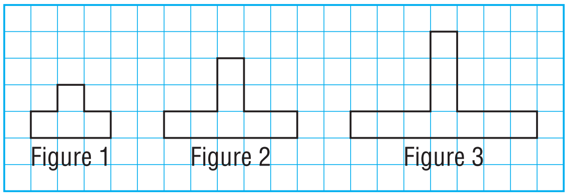
a) Explain how the expression below each picture describes the number of toothpicks in the picture.

b) Suppose n represents the number of houses. Write an equation that relates the number of toothpicks, t, to the number of houses, n.

c) Compare the equation in part b with the equation in question 10c. How can two different equations represent the same pattern? Explain.

Take It Further

19. Here is a pattern of squares. Each square has side length 1 cm. The pattern continues.



a) Make a table that shows each figure number, its perimeter, and its area.

b) Write an equation that can be used to determine the perimeter of any figure in the pattern. Verify the equation. How did you do this?

c) Write an equation that can be used to determine the area of any figure in the pattern. Verify the equation.

d) Determine the perimeter and area of figure 50.

e) Which figure has a perimeter of 100 cm?

f) Which figure has an area of 100 cm^2?

20. The pattern in this table continues.

Term Number, t	Term Value, v
1	80
2	76
3	72
4	68

a) Write an equation that relates v to t.

b) Verify your equation by substituting values from the table.

21. Marcel has a sheet of paper. He cuts the paper in half to produce two pieces. Marcel places one piece on top of the other. He then cuts these pieces in half. The pattern continues. The table below shows some of Marcel's results.

Number of Cuts	1	2	3	4	5	6	7	8	9	10
Number of Pieces	2	4	8							

a) Copy and complete the table.

b) What patterns do you see in the numbers of pieces?

c) Determine the number of pieces after 15 cuts.

d) Write an equation that relates the number of pieces, P, to the number of cuts, n.

e) How many cuts have to be made to get more than 50 000 pieces? Explain how you found out.

Reflect

Describe some different ways to represent a pattern.
Which way do you prefer when you use a pattern to solve a problem?
Explain your choice.

Tables of Values and Graphing

FOCUS

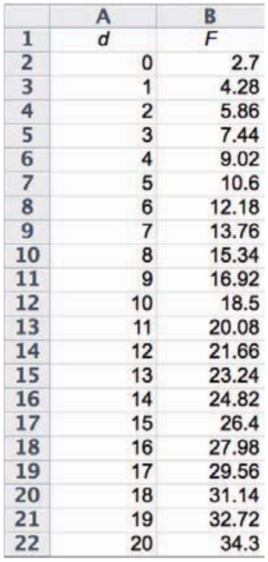
- Use a spreadsheet to create a table of values and a graph.

A spreadsheet can be used to create a table of values or to graph a relation.

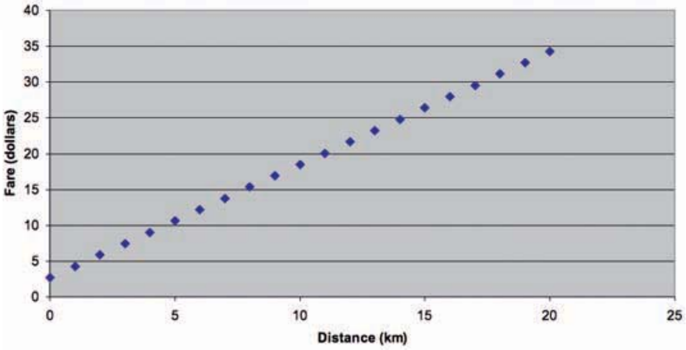
A taxi company charges a fixed cost of \$2.70, plus \$1.58 per kilometre travelled.

To create a table of values, first write an equation.
Let d represent the distance travelled in kilometres and F represent the fare in dollars.
Then the equation that relates d and F is: $F = 2.70 + 1.58d$

Generate a table of values.
Graph the data. If you need to, use the Help menu to show you how to do this with your software.



	A	B
1	d	F
2	0	2.7
3	1	4.28
4	2	5.86
5	3	7.44
6	4	9.02
7	5	10.6
8	6	12.18
9	7	13.76
10	8	15.34
11	9	16.92
12	10	18.5
13	11	20.08
14	12	21.66
15	13	23.24
16	14	24.82
17	15	26.4
18	16	27.98
19	17	29.56
20	18	31.14
21	19	32.72
22	20	34.3



Your table and graph should look similar to these.

Check

1. A second taxi company charges a fixed cost of \$4.20, plus \$1.46 per kilometre.

a) Write an equation that relates the fare to the distance travelled.

b) Use a spreadsheet to generate a table of values.

c) Use the spreadsheet to graph the equation.

2. In Lesson 4.1, you solved problems involving equations. Choose two questions from *Practice*. For each question:

a) Use a spreadsheet to generate a table of values and solve the problem.

b) Use the spreadsheet to graph the equation.

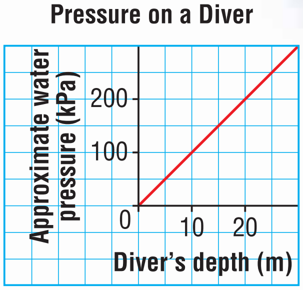
4.2 Linear Relations

FOCUS

- Analyze the graph of a linear relation.

When a scuba diver goes under water, the weight of the water exerts pressure on the diver.

Diver's Depth (m)	Approximate Water Pressure (kiloPascals)
0	0
5	50
10	100
15	150
20	200



What patterns do you see in the table and in the graph?
What do these patterns tell you about the relationship between depth and water pressure?

Investigate

A local phone company offers a cell phone plan that has a fixed cost per month and a cost related to the number of text messages sent. The fixed cost is $20 and each message sent costs 10¢.

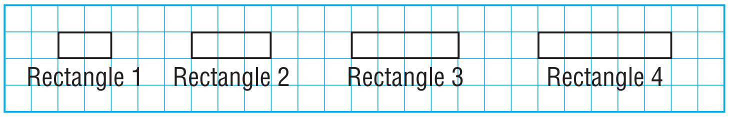
Represent the relation between the total cost and the number of text messages sent, as many different ways as you can.

Reflect & Share

Compare your representations with those of another pair of students.
Did you use the same way to represent the pattern? If your patterns are different, explain your pattern to the other students.
If you represented the relation in a different way from your classmates, explain your way to them.

Connect

The first 4 rectangles in a pattern are shown below. The pattern continues.
Each small square has side length 1 cm.



The perimeter of a rectangle is related to the rectangle number.
We can use words, a table, a graph, and an equation to represent this relationship.
Each representation tells us about the relationship between the rectangle number and its perimeter.

In Words

Rectangle 1 has perimeter 6 cm; then, as the rectangle number increases by 1, its perimeter increases by 2 cm.

In a Table

	Rectangle Number, n	Perimeter, P (cm)	
	1	$6 = 2(1) + 4$	
+1			+2
	2	$8 = 2(2) + 4$	
+1			+2
	3	$10 = 2(3) + 4$	
+1			+2
	4	$12 = 2(4) + 4$	

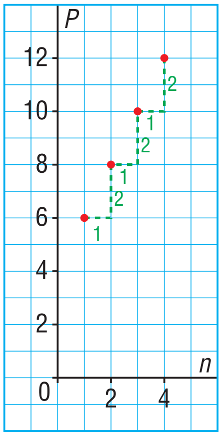
As the rectangle number increases by 1, the perimeter increases by 2 cm.

In a Graph

The graph also shows the pattern. After the first point, each point on the graph is 1 unit right and 2 units up from the preceding point. If we place a transparent ruler on the points, we see that they lie on a straight line.

We do not join the points because the data are discrete.

Graph of P against n



In an Equation

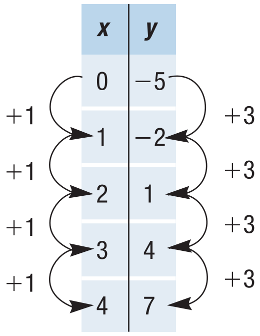
For rectangle n, the perimeter will be $2n + 4$.
The equation is: $P = 2n + 4$
The equation tells us that we can calculate the perimeter of any rectangle in the pattern by multiplying the rectangle number by 2, then adding 4.
The value of the variable P *depends* on the value of the variable n.
We say that P is the **dependent variable** and we plot it on the vertical axis.
The **independent variable** n is plotted on the horizontal axis.
When two variables are related, we have a **relation**.

Linear Relation

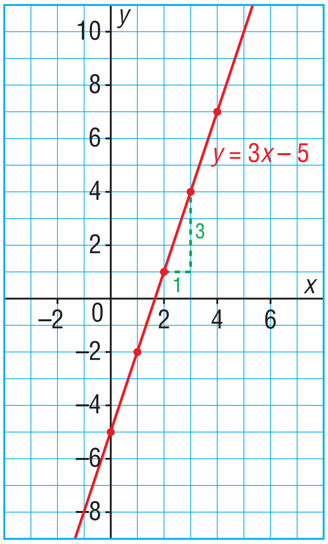
When the graph of the relation is a straight line, we have a **linear relation**.
In a linear relation, a constant change in one quantity produces a constant change in the related quantity.

In the relation above, a constant change of 1 in n produced a constant change of 2 cm in P.

Here is the equation of a linear relation: $y = 3x - 5$
x is the independent variable and it is plotted on the horizontal axis.
y is the dependent variable and it is plotted on the vertical axis.
Here are the table and graph that represent this equation.



x	y
0	−5
1	−2
2	1
3	4
4	7



Write the equation on the grid.

When x increases by 1, y increases by 3. This is shown in the table and on the graph.
Since the points lie on a straight line, the equation $y = 3x - 5$ represents a linear relation.
Since we are not told that the data are discrete, we join the points with a line.

Example 1 Graphing a Linear Relation from a Table of Values

The table of values shows the cost of renting DVDs at an online store.

a) Graph the data. Does it make sense to join the points on the graph? Explain.

b) Is the relation linear? Justify your answer.

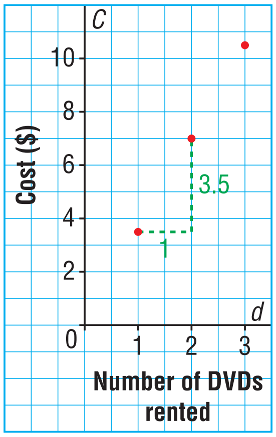
c) Use the table to describe the pattern in the rental costs. How is this pattern shown in the graph?

Number of DVDs Rented, *d*	Cost, *C* ($)
1	3.50
2	7.00
3	10.50
4	14.00
5	17.50

A Solution

a) Plot the points on a grid.

Cost to Rent DVDs



Since the cost depends on the number of DVDs rented, plot *d* horizontally and *C* vertically.

The number of DVDs rented is a whole number. We cannot rent 1.5 DVDs or any other fractional number of DVDs. So, it does not make sense to join the points.

b) The points on the graph lie on a straight line, so the relation is linear.

c) As the number of DVDs rented increases by 1, the rental cost increases by $3.50. Each point on the graph is 1 unit right and 3.5 units up from the previous point. The pattern of increases in the table produces a graph that is a straight line.

Example 2 Graphing a Linear Relation from an Equation

A relation has the equation: $y = 6 - 3x$

a) Create a table of values for the relation for values of x from -3 to 3.

b) Graph the relation. Does it make sense to join the points on the graph? Explain.

c) What patterns are in the graph? How do these patterns relate to the table of values?

d) Is the relation linear? Justify your answer.

Solutions

Method 1

a), b) To create a table of values, substitute the given values of x in the equation:

$y = 6 - 3x$

Substitute: $x = -3$

$y = 6 - 3(-3)$
$= 6 + 9$
$= 15$

Substitute: $x = -2$

$y = 6 - 3(-2)$
$= 6 + 6$
$= 12$

Substitute: $x = -1$

$y = 6 - 3(-1)$
$= 6 + 3$
$= 9$

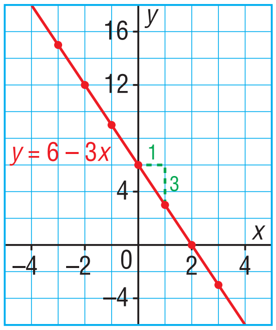
Substitute: $x = 0$

$y = 6 - 3(0)$
$= 6 - 0$
$= 6$

Use mental math to repeat the above process for $x = 1$, $x = 2$, and $x = 3$.

Write the values of x and y in a table.

x	y
−3	15
−2	12
−1	9
0	6
1	3
2	0
3	−3

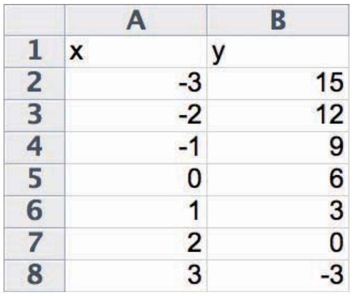


Since the data are not discrete, join the points to form a line.

Method 2

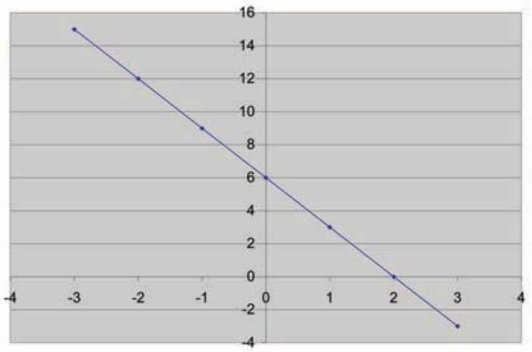
Use a spreadsheet.

a) Input the equation and make a table.



	A	B
1	x	y
2	-3	15
3	-2	12
4	-1	9
5	0	6
6	1	3
7	2	0
8	3	-3

b) Highlight the table.
Graph the data.



c) On the graph, to get from one point to the next, move 1 unit right and 3 units down.
In the table, when x increases by 1, y decreases by 3.

d) The relation is linear because its graph is a straight line.

Example 3 Solving Problems Using a Linear Relation

The student council is planning to hold a dance. The profit in dollars is 4 times the number of students who attend, minus \$200 for the cost of the music.

a) Write an equation that relates the profit to the number of students who attend.
b) Create a table of values for this relation.
c) Graph the data in the table. Does it make sense to join the points? Explain.
d) How many students have to attend to make a profit?

A Solution

a) Profit in dollars = 4 × number of students who attend − 200
Choose variables to represent the numbers that change.
Let n represent the number of students who attend.
Let P represent the profit in dollars.
An equation is: $P = 4n - 200$

b) Choose 3 values for n, then calculate the corresponding values of P.
Use the equation: $P = 4n - 200$

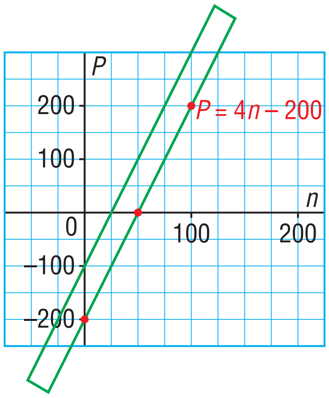
Substitute: $n = 0$
$P = 4(0) - 200$
$= 0 - 200$
$= -200$

Substitute: $n = 50$
$P = 4(50) - 200$
$= 200 - 200$
$= 0$

Substitute: $n = 100$
$P = 4(100) - 200$
$= 400 - 200$
$= 200$

n	**P**
0	−200
50	0
100	200

c) Plot the points on a grid.



A straightedge verifies that the points lie on a straight line.
Some values between the plotted points are permitted, but not others.
For example, there could be 82 students attending the dance, but not 82.5.
So, the points are not joined.

d) When P is negative, a loss is made.
When $P = 0$, $n = 50$, and the profit is 0.
When $P > 0$, $n > 50$, and there is a profit.
So, 51 or more students have to attend before a profit can be made.

Discuss the ideas

1. **a)** How do you know whether a graph represents a linear relation?
b) How do you know whether a table of values represents a linear relation?

2. **a)** How many points do you need to graph a line?
b) Why do we often use 3 points? Should we use more points? Explain.

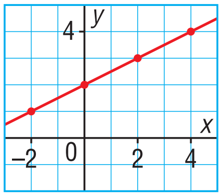
3. How do you know when to connect the points on a graph?

Practice

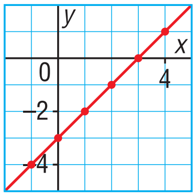
Check

4. Which graphs represent a linear relation? How do you know?

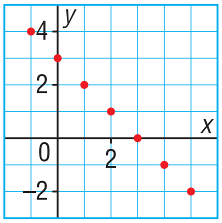
a)



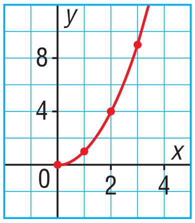
b)



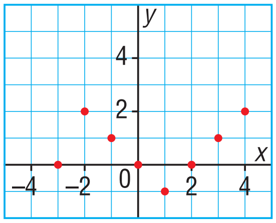
c)



d)



e)



Apply

5. For each table of values below:
i) Does it represent a linear relation?
ii) If the relation is linear, describe it.
iii) If the relation is not linear, explain how you know.

a)

x	*y*
1	4
2	13
3	22
4	31
5	40

b)

x	*y*
9	8
8	11
7	14
6	17
5	20

c)

x	*y*
0	0
1	2
2	6
3	12
4	20

d)

x	*y*
1	3
4	5
7	7
10	9
13	11

6. Graph the linear relations you identified in question 5. How does each graph verify your answers to question 5?

7. Copy and complete each table of values.

a) $y = 2x$

x	y
1	
2	
3	
4	

b) $y = x + 2$

x	y
1	
2	
3	
4	

c) $y = -2x$

x	y
2	
4	
6	
8	

d) $y = x - 2$

x	y
4	
5	
6	
7	

8. Here is a partially completed table of values for a linear relation.

x	2	3	4	5	6	7	8
y				15	18		

a) Determine the missing values of y. Explain how you found these values.

b) Describe the patterns in the table.

c) Write an equation that represents the linear relation. How do you know that your equation is correct?

d) Graph the data. How are the patterns you described in part b shown in the graph?

e) Suppose you want to determine the value of y when $x = -1$. How could you use the table and equation to do this? What is the value of y when $x = -1$?

9. Each table of values represents a linear relation. Copy and complete each table. Explain your reasoning.

a)

x	y
2	11
3	14
4	
5	
6	

b)

x	y
1	
3	8
5	9
7	
9	

c)

x	y
−4	
−2	7
0	3
2	
4	

d)

x	y
4	
6	−7
8	−4
10	
12	

10. Create a table of values for each linear relation, then graph the relation. Use values of x from -2 to 2.

a) $y = 3x$

b) $y = x + 3$

c) $y = x - 3$

d) $y = 5 - x$

e) $y = 1 - 4x$

f) $y = -2x - 3$

11. Jin is cycling at an average speed of 4 m/s. He travels a distance, d metres, in t seconds.

a) Write an equation that relates d and t.

b) Create a table of values for this relation.

c) Graph the data. Should you join the points? Explain your reasoning.

d) Is the relation between distance and time linear?

  i) How do you know from the table of values?

  ii) How you know from the graph?

e) How far does Jin travel in 3.5 h?

f) What time does it take Jin to travel 17 km?

12. In 2008, the Goods and Services Tax (GST) was 5%. To determine the tax, T dollars, charged on a given purchase price, p dollars, multiply the purchase price by 0.05.

a) Write an equation that relates T to p.

b) Copy and complete this table of values.

p	0	10	20	30	40
T					

c) What patterns do you see in the table?

d) Graph the data.
Which variable will you plot on the horizontal axis? Explain your reasoning.

e) Should you connect the points on the graph? Explain.

f) How are the patterns in the table shown in the graph?

13. An amusement park charges an admission fee of $10, plus $2 per ride.

a) Choose variables to represent the total cost in dollars and the number of rides that are taken. Write an equation that relates the total cost to the number of rides.

b) Graph the equation.

c) What is the total cost for 7 rides?

d) How many rides can be taken for a total cost of $38?

14. **Assessment Focus** Danica is having a party. She estimates that she will need 3 pieces of pizza for each guest invited, and 6 extra pieces in case someone shows up unexpectedly.

a) Explain why this situation can be represented by the equation $P = 3n + 6$. What do P and n represent in the equation?

b) Make a table of values for the relation.

c) Graph the data. Will you join the points on the graph? Explain.

d) Is the relation linear?

  i) How do you know from the table of values?

  ii) How do you know from the graph?

e) If the relation is linear, explain what this means in the context of this situation.

15. A small plane is at a height of 1800 m when it starts descending to land.
The plane's height changes at an average rate of 150 m per minute.

a) Choose variables to represent the height in metres and the time in minutes since the plane began its descent. Write an equation that relates the height to the time.

b) Graph the equation.

c) What is the height of the plane 6 min after it began its descent?

d) When is the plane 100 m above the ground?

16. Jada rollerblades from Regina to Saskatoon to raise funds for cancer research. The trip is 250 km. Jada estimates that she can rollerblade at an average speed of 8 km/h.

a) Choose variables to represent the time Jada has travelled in hours and the distance in kilometres that she has yet to travel. Write an equation that relates the distance to the time.

b) Graph the equation.

c) How far has Jada still to travel after 12 h?

d) How many hours will it take Jada to complete the trip?

17. Describe a situation that could be represented by each equation.

a) $M = 2n + 5$

b) $E = 3.50n$

c) $C = 12 + 5d$

d) $H = 100 - 5n$

Take It Further

18. This table of values represents a linear relation. Copy and complete the table. Explain your reasoning.

x	−3	−1	2	5	9	14	20
y	29		23				

Reflect

What does it mean when we say that the relation between two quantities is linear? What patterns are there in the table of values and in the graph of a linear relation? Include examples in your explanation.

Math Link

Science

When an object falls to the ground, it accelerates due to the force of gravity. The relation between the speed of the object and the time it falls is linear.

4.3 Another Form of the Equation for a Linear Relation

FOCUS

- Recognize the equations of horizontal, vertical, and oblique lines, and graph them.

Investigate

Suppose you have a piece of ribbon 20 cm long.

➤ How many different ways could you cut it into two pieces?
What are the possible lengths of the two pieces?

➤ How are the lengths of the two pieces related?
Show this relation:

- in words
- in a table
- in a graph
- as an equation

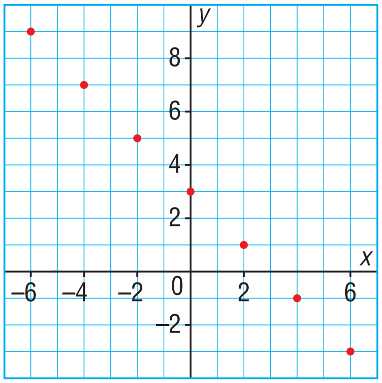
Reflect & Share

Share your different forms of the relation with another pair of students.
If any forms are different, is one of the forms incorrect?
How could you find out?
If you wrote your equation the same way, try to think of a different way to write it.

Connect

Two integers have a sum of 3.
Let x and y represent the two integers.
Here is a table of values and a graph to represent the relation.

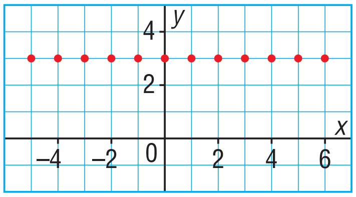
First Integer, x	Second Integer, y
−6	9
−4	7
−2	5
0	3
2	1
4	−1
6	−3



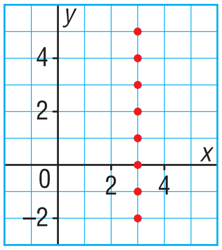
The points lie on a straight line, so the relation is linear.
We can write this linear relation as:
First integer + second integer = 3
Then, the linear relation is: $x + y = 3$
This equation has both variables on the left side of the equation.
It illustrates another way to write the equation of a linear relation.

Suppose one variable does not appear in the equation.

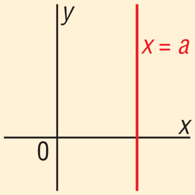
- Suppose x does not appear in $x + y = 3$.
  Then we have the equation $y = 3$.
  To graph this equation on a grid,
  plot points that have a y-coordinate of 3.
  All the points lie on a horizontal line
  that is 3 units above the x-axis.



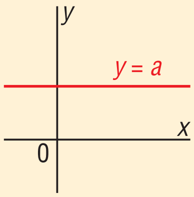
- Suppose y does not appear in $x + y = 3$.
  Then we have the equation $x = 3$.
  To graph this equation on a grid,
  plot points that have an x-coordinate of 3.
  All the points lie on a vertical line that is
  3 units to the right of the y-axis.



The graph of the equation $x = a$, where a is a constant, is a vertical line. Every point on the graph has an x-coordinate of a.



The graph of the equation $y = a$, where a is a constant, is a horizontal line. Every point on the graph has a y-coordinate of a.



Example 1 Graphing and Describing Horizontal and Vertical Lines

For each equation below:

i) Graph the equation.

ii) Describe the graph.

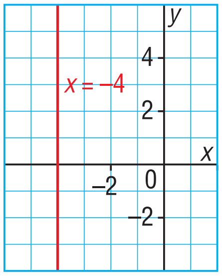
a) $x = -4$ **b)** $y + 2 = 0$ **c)** $2x = 5$

A Solution

a) $x = -4$

i) The x-coordinate of every point on this line is -4.

ii) The graph is a vertical line that intersects the x-axis at -4.

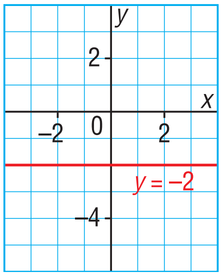


b)

$$\begin{aligned} y + 2 &= 0 && \text{Solve for } y. \\ y + 2 - 2 &= 0 - 2 && \text{Subtract 2 from each side.} \\ y &= -2 \end{aligned}$$

i) The y-coordinate of every point on this line is -2.

ii) The graph is a horizontal line that intersects the y-axis at -2.



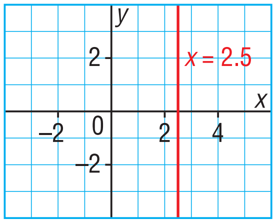
c) $2x = 5$ Solve for x.

$\frac{2x}{2} = \frac{5}{2}$ Divide both sides by 2.

$x = 2.5$

i) The x-coordinate of every point on this line is 2.5.

ii) The graph is a vertical line that intersects the x-axis at 2.5.



Example 2 Graphing an Equation in the Form $ax + by = c$

For the equation $3x - 2y = 6$:

a) Make a table of values for $x = -4$, 0, and 4.

b) Graph the equation.

A Solution

a) $3x - 2y = 6$

Substitute each value of x, then solve for y.

Substitute: $x = -4$

$$\begin{aligned}3(-4) - 2y &= 6\\ -12 - 2y &= 6\\ -2y &= 6 + 12\\ -2y &= 18\\ y &= -9\end{aligned}$$

Substitute: $x = 0$

$$\begin{aligned}3(0) - 2y &= 6\\ 0 - 2y &= 6\\ -2y &= 6\\ y &= -3\end{aligned}$$

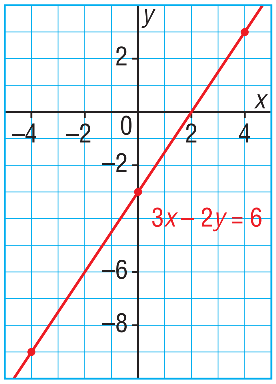
Substitute: $x = 4$

$$\begin{aligned}3(4) - 2y &= 6\\ 12 - 2y &= 6\\ -2y &= 6 - 12\\ -2y &= -6\\ y &= 3\end{aligned}$$

x	y
–4	–9
0	–3
4	3

b) Plot the points on a grid.

Join the points.



Discuss the ideas

1. The graph of an equation such as $3x - 2y = 6$ is a slanted or an *oblique* line. How are the equations for oblique lines different from the equations for horizontal and vertical lines?

2. Students often mistakenly think that $x = 3$ is a horizontal line instead of a vertical line.
Why might they make this mistake? How might the students reason to avoid making this mistake?

3. How do you recognize the equation of:
a) a vertical line? **b)** a horizontal line?

Practice

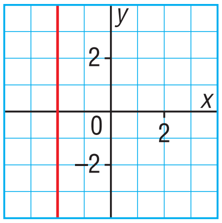
Check

4. Which equation describes each graph?

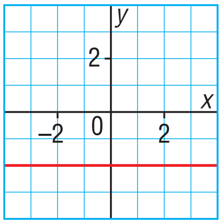
i) $x = -2$ **ii)** $x = 2$

iii) $y = -2$ **iv)** $y = 2$

a)



b)



5. Does each equation describe a vertical line, a horizontal line, or an oblique line? Describe each horizontal and vertical line.

a) $y = 7$ **b)** $x - y = 3$

c) $x = -5$ **d)** $x + 9 = 0$

e) $2y = 5$ **f)** $y = 6 - 2x$

Apply

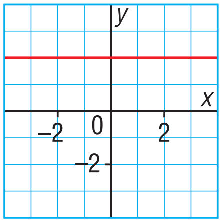
6. Describe the graph of each line. Graph each line to check your description.

a) $y = 5$ **b)** $x = -1$

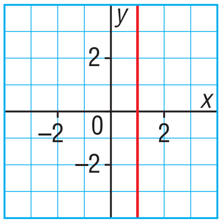
c) $x = -5$ **d)** $y = 7$

7. Write an equation to describe each line.

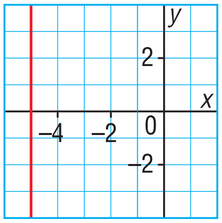
a)



b)



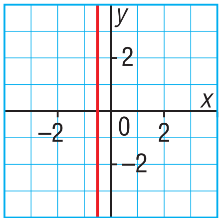
c)



8. Which equation best describes the graph below? Explain your choice.

a) $x - 2 = 0$ **b)** $2x + 1 = 0$

c) $2y - 1 = 0$ **d)** $2x - 1 = 0$



9. The sum of two numbers is 15. Let p and q represent the two numbers.

a) Complete a table for 6 different values of p.

b) Graph the data. Should you join the points? Explain.

c) Write an equation that relates p and q.

10. a) For each equation below:

- Make a table of values for $x = -2$, 0, and 2.
- Graph the equation.

i) $x + y = 6$ **ii)** $x - y = 6$

iii) $x + y = -6$ **iv)** $x - y = -6$

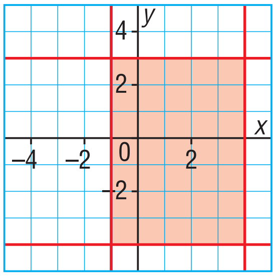
b) How are the graphs in part a alike? How are they different?

11. Graph each line. Explain your work.

a) $y + 3 = -2$ **b)** $2x = 7$

c) $3x + 1 = -5$ **d)** $2y - 2 = 10$

12. Write the equations of the lines that intersect to form the shaded rectangle.



13. Assessment Focus

a) Graph the following lines on the same grid. What shape do they form?

i) $x = -3$ **ii)** $y = 2$

iii) $x - 1 = 0$ **iv)** $y + 2 = 0$

b) Construct a congruent shape on the grid with one of its vertices at the origin.

c) Write the equations of the lines that form the shape you drew.

d) Is there more than one shape you can draw in part b? If your answer is yes, draw any more possible shapes.

Show your work.

14. The distance between Edmonton and Calgary is about 300 km. Kate leaves Calgary to drive to Edmonton. Let t kilometres represent the distance Kate has travelled. Let e kilometres represent the distance she has yet to travel to Edmonton.

a) Copy and complete this table for 6 different values of t.

Distance Travelled, t (km)	Distance to Edmonton, e (km)
0	300

b) What is the greatest value of t that could be in the table? Explain.

c) Graph the data. Should you join the points? Explain.

d) Write an equation that relates t and e.

15. For each equation below:

- Make a table for the given values of x.
- Graph the equation.

a) $2x + y = 6$; for $x = -3, 0, 3$

b) $3x - y = 2$; for $x = -2, 0, 2$

c) $x + 2y = -6$; for $x = -4, 0, 4$

d) $3x - 2y = -6$; for $x = -2, 0, 2$

16. a) On a grid, draw horizontal and vertical lines to construct a shape that satisfies the following conditions:

- The shape is a square with area 9 square units.
- One vertex is at the origin.

b) Write the equations of the lines that form the square.

c) Is it possible to draw another square that satisfies the conditions in part a? If your answer is yes, draw this square and write the equations of the lines that form it.

17. The difference of two numbers is 6. Let a represent the greater number and b the lesser number.

a) Complete a table for 6 different values of a.

b) Graph the data. Should you join the points? Explain.

c) Write an equation that relates b and a.

18. a) Graph these equations on the same grid:

$x = 2 \qquad y = 1 \qquad x + y = 8$

b) What shape is formed by the lines in part a? How do you know?

Take It Further

19. The sum of two rational numbers is $2\frac{1}{2}$.

a) Choose two variables to represent these rational numbers. Make a table to show 5 possible pairs of numbers that satisfy this relation.

b) Graph the data. Describe your graph.

c) Write an equation for the relation.

20. The difference of two rational numbers is -7.5.

a) Choose two variables to represent these rational numbers. Make a table to show 5 possible pairs of numbers that satisfy this relation.

b) Graph the data. Describe your graph.

c) Write an equation for the relation.

21. For each equation below:

- Make a table for 3 values of x.
- Graph the equation.

a) $\frac{1}{2}x + y = 4$

b) $\frac{1}{3}x - y = 2$

c) $\frac{1}{2}x + \frac{1}{3}y = 6$

d) $\frac{1}{3}x - \frac{1}{2}y = -1$

e) $\frac{1}{3}x + \frac{1}{2}y = -3$

f) $\frac{1}{4}x - \frac{1}{2}y = 1$

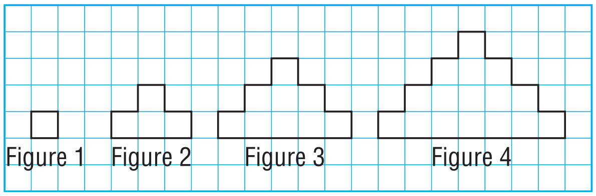
Reflect

How are the equations of horizontal and vertical lines alike?
How are they different?
How can you recognize the equation of each line?

Mid-Unit Review

4.1

1. This pattern of squares continues.



a) Make a table that shows the figure number, n, and the perimeter of a figure, P. What patterns do you see?
b) Write an expression for the perimeter of figure n.
c) What is the perimeter of figure 40?
d) Write an equation that relates P to n.
e) Which figure has a perimeter of 136 units? How do you know?

2. A phone company charges a fixed cost of \$10 per month, plus \$0.25 per minute for long distance calling.
a) Write an equation that relates the monthly cost, C dollars, to t, the time in minutes.
b) In one month, the time for the long distance calls was 55 minutes. What was the monthly cost?
c) For one month, the cost was \$22.50. How many minutes of long distance calls were made?

4.2

3. Create a table of values for each linear relation, then graph the relation. Use values of x from -3 to 3.
a) $y = -3x$
b) $y = 2x$
c) $y = 2 - 4x$
d) $y = -2x + 4$
e) $y = -3 + x$
f) $y = -x + 3$

4. Alicia buys a \$300-jacket on lay away. She made a down payment of \$30 and is paying \$15 per week. The total paid, P dollars, after n weeks can be represented by the equation $P = 15n + 30$.
a) Create a table of values to show the total paid in each of the first 5 weeks.
b) Graph the data. Should you join the points on the graph? Explain.
c) How do the patterns in the graph relate to the patterns in the table?

5. Each table of values represents a linear relation. Copy and complete each table. Explain your reasoning.

a)

x	*y*
1	10
2	14
3	
4	
5	

b)

x	*y*
1	
3	−10
5	−14
7	
9	

c)

x	*y*
−2	
−1	
0	−3
1	3
2	

d)

x	*y*
2	
4	−2
6	−5
8	
10	

4.3

6. a) Graph each equation.
i) $y = 1$
ii) $x = -4$
iii) $x + y = 8$
iv) $2x - y = 12$
b) For which equations in part a did you *not* need to make a table of values? Explain why.

7. The difference of two numbers is 1. Let g represent the greater number and n the lesser number.
a) Complete a table for 4 different values of n.
b) Graph the data. Should you join the points? Explain.
c) Write an equation that relates n and g.

GAME

What's My Point?

You will need

- grid paper
- a ruler
- a pencil

Number of Players

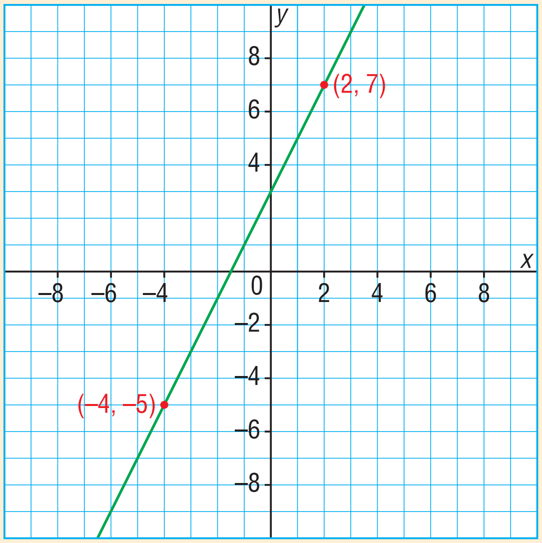
- 2

Goal of the Game

- To have the lesser score at the end of 3 rounds

How to Play

Write the equations of three different linear relations.
Graph each equation on a grid like this.
Plot all points that have integer coordinates.



1. Player A chooses two points on one line.
   She keeps these points secret.
   Player A tells Player B the equation of the line.
   Player B tells the coordinates of a point on the line.
   Player A says whether Player B's point is:
   - one of the chosen points
   - on the line and above the chosen points
   - on the line and below the chosen points, or
   - between the chosen points

   Player B continues to name points on the line until he names both chosen points.
   Each guess counts as 1 point.
2. Player A and B switch roles, with Player A guessing the points selected by Player B.
3. Play continues until all three graphs have been used.
   The player with fewer points wins.
4. Suppose your opponent gave you the equation $y = 5x - 6$.
   Which two points might you guess? Explain.
5. Create a graph that might make it difficult for your opponent to guess your two points.
   Explain why it would be difficult.

4.4

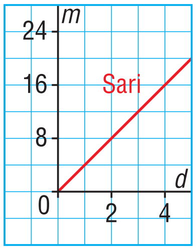
Matching Equations and Graphs

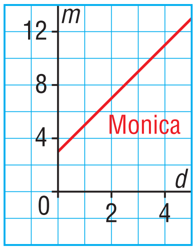
FOCUS

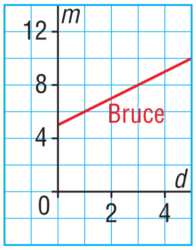
- Match equations and graphs of linear relations.

Investigate

Bruce, Monica, and Sari participate in a 5-km walk for charity.
Each student has a different plan to raise money from her or his sponsors.
These graphs show how the amount of money a sponsor owes is related to the distance walked.







- ➤ Match each graph with its equation: $m = 2d + 3$ $\quad m = 4d \quad$ $m = d + 5$
  Explain your strategy.
- ➤ Describe each person's sponsorship plan.

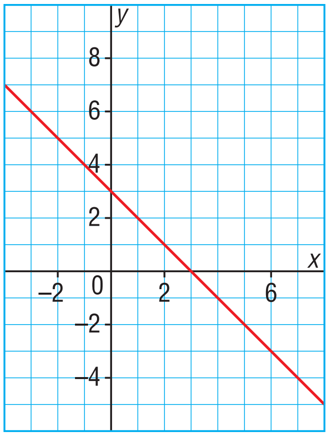
Reflect & Share

Compare your strategies and descriptions with those of another pair of students.
Did you use the same strategies to match each graph and its equation?
If not, explain your strategies to the other students.

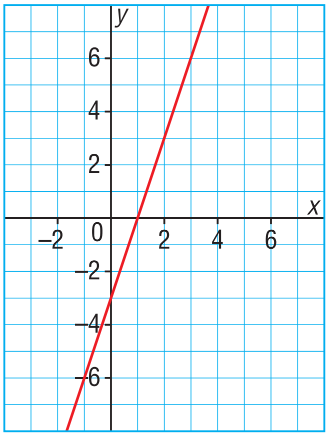
Connect

The 3 graphs below have these equations, but the graphs are not in order:
$y = 3x + 3$ $\quad x + y = 3$ $\quad y = 3x - 3$

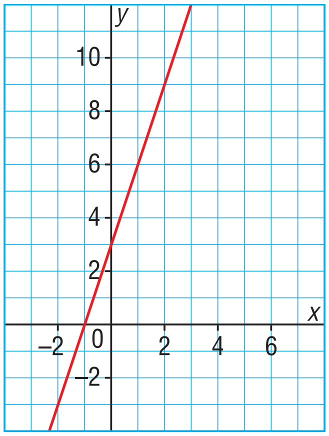
Graph A



Graph B



Graph C



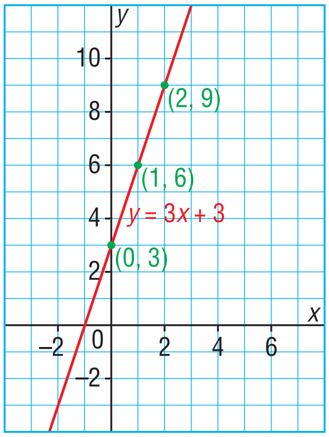
To match each equation with its graph,
use the equation to determine the coordinates of 3 points.
Then find which graph passes through those 3 points.

➤ For $y = 3x + 3$

Substitute: $x = 0$
$y = 3(0) + 3$
$y = 3$
One point is: (0, 3)

Substitute: $x = 1$
$y = 3(1) + 3$
$y = 6$
One point is: (1, 6)

Substitute: $x = 2$
$y = 3(2) + 3$
$y = 9$
One point is: (2, 9)



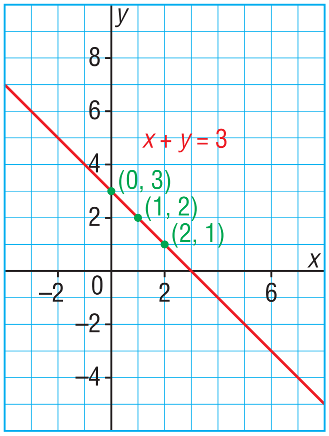
The graph that passes through these 3 points is Graph C.

➤ For $x + y = 3$

Substitute: $x = 0$
$0 + y = 3$
$y = 3$
One point is: (0, 3)

Substitute: $x = 1$
$1 + y = 3$
$y = 2$
One point is: (1, 2)

Substitute: $x = 2$
$2 + y = 3$
$y = 1$
One point is: (2, 1)



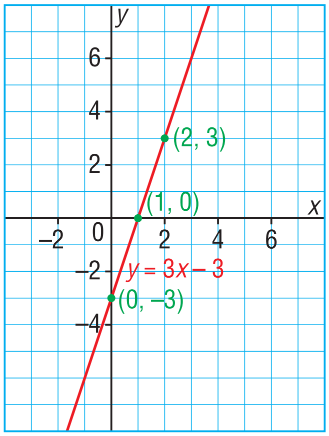
The graph that passes through these 3 points is Graph A.

So, the equation $y = 3x - 3$ must match Graph B. Substitute to check.

Substitute: $x = 0$
$y = 3(0) - 3$
$y = -3$
One point is: (0, −3)

Substitute: $x = 1$
$y = 3(1) - 3$
$y = 0$
One point is: (1, 0)

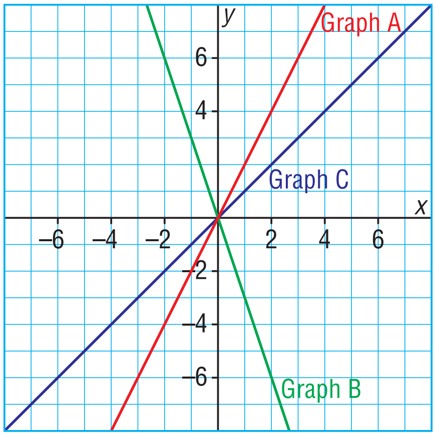
Substitute: $x = 2$
$y = 3(2) - 3$
$y = 3$
One point is: (2, 3)



The graph that passes through these 3 points is Graph B.

Example 1 Matching Equations with Graphs that Pass through the Origin

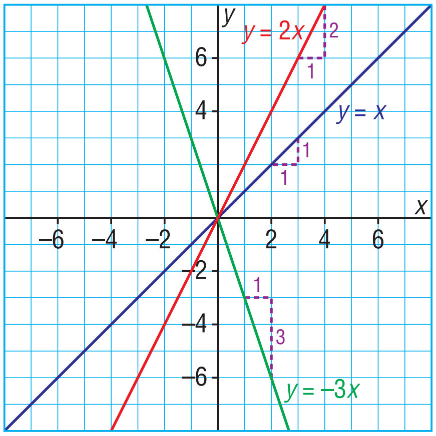
Match each graph on the grid with its equation below.



$y = x$
$y = 2x$
$y = -3x$

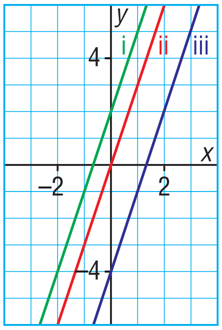
A Solution

Rewrite $y = x$ as $y = 1x$. The coefficient of x represents the pattern of the points on the graph.
In the equation $y = 1x$, the 1 indicates that when x increases by 1 unit, y also increases 1 unit.
This matches Graph C.
In the equation $y = 2x$, the 2 indicates that when x increases by 1 unit, y increases by 2 units.
This matches Graph A.
In the equation $y = -3x$, the -3 tells us that when x increases by 1 unit, y decreases by 3 units.
This matches Graph B.



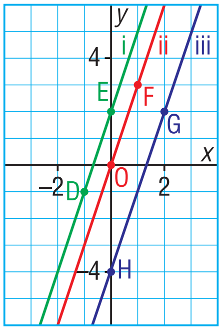
Example 2 Identifying a Graph Given Its Equation

Which graph on this grid has the equation $y = 3x - 4$?
Justify the answer.



A Solution

Pick 2 points on each graph and check to see if their coordinates satisfy the equation.



Two points on Graph i have coordinates
D(−1, −1) and E(0, 2).
Substitute $x = -1$ and $y = -1$ in $y = 3x - 4$.
Left side: $y = -1$ Right side: $3x - 4 = 3(-1) - 4$
$= -7$
The left side does not equal the right side.
So, these coordinates do not satisfy the equation and
Graph i does not have equation $y = 3x - 4$.

Two points on Graph ii have coordinates O(0, 0) and F(1, 3).
Substitute $x = 0$ and $y = 0$ in $y = 3x - 4$.
Left side: $y = 0$ Right side: $3x - 4 = 3(0) - 4$
$= -4$
The left side does not equal the right side.
So, these coordinates do not satisfy the equation and Graph ii does not have
equation $y = 3x - 4$.

Two points on Graph iii have coordinates G(2, 2) and H(0, −4).
Substitute $x = 2$ and $y = 2$ in $y = 3x - 4$.
Left side: $y = 2$ Right side: $3x - 4 = 3(2) - 4$
$= 2$
The left side does equal the right side, so the coordinates of G satisfy the equation.
Substitute $x = 0$ and $y = -4$ in $y = 3x - 4$.
Left side: $y = -4$ Right side: $3x - 4 = 3(0) - 4$
$= -4$
The left side does equal the right side, so the coordinates of H satisfy the equation.
Since both pairs of coordinates satisfy the equation, Graph iii has equation
$y = 3x - 4$.

Discuss the ideas

1. When we match an equation to a graph by determining coordinates of points on the graph, why is it helpful to check 3 points, even though 2 points are enough to identify a line?

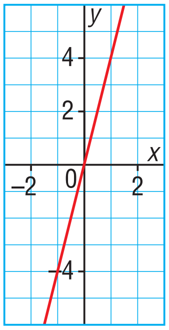
2. When we choose points on a graph to substitute their coordinates in an equation, what is an advantage of choosing the points where the graph intersects the axes?

Practice

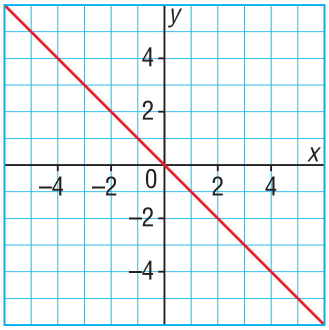
Check

3. Match each equation with a graph below.
   a) $y = 2x$ **b)** $y = 4x$ **c)** $y = -x$

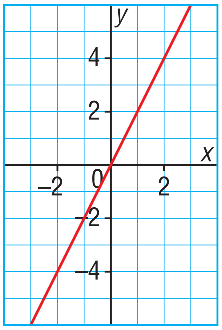
i)



ii)

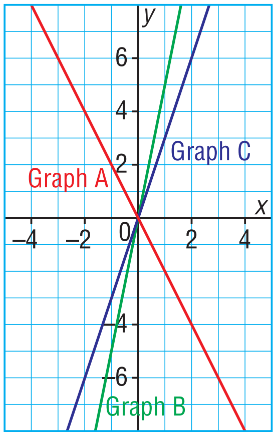


iii)



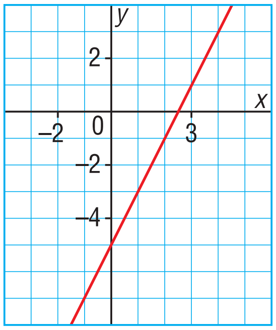
Apply

4. Match each equation with a graph on the grid below.
   a) $y = 3x$ **b)** $y = 5x$ **c)** $y = -2x$

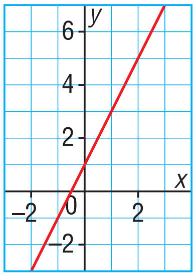


5. Match each equation with a graph below. Which strategy did you use?
   a) $y = 2x + 1$ **b)** $y = 2x + 3$ **c)** $y = 2x - 5$

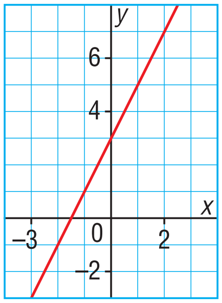
i)



ii)



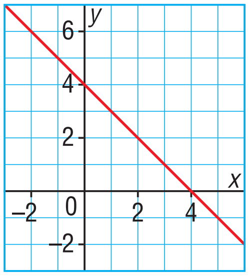
iii)



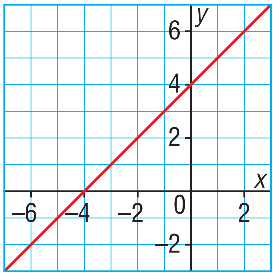
6. Match each equation with a graph below. Justify your answers.

a) $x + y = 4$ **b)** $x - y = 4$ **c)** $x - y = -4$

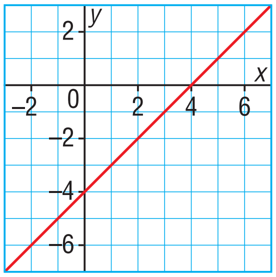
i)



ii)

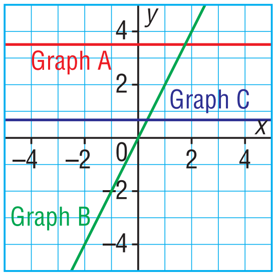


iii)

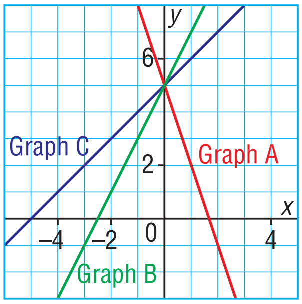


7. Match each equation with its graph below. Explain your strategy.

a) $y = 2x$ **b)** $2y = 7$ **c)** $3y = 2$



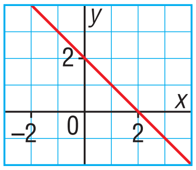
8. Which graph on this grid has equation $y = 2x + 5$? Justify your answer.



9. Which equation describes each graph? Justify your answers.

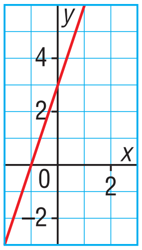
a) **i)** $y = 2x + 1$ **ii)** $y = 2x + 3$

iii) $y = x - 2$ **iv)** $y = -x + 2$



b) **i)** $x + 3y = 1$ **ii)** $3x - y = -3$

iii) $3x + y = 1$ **iv)** $3x - y = 3$



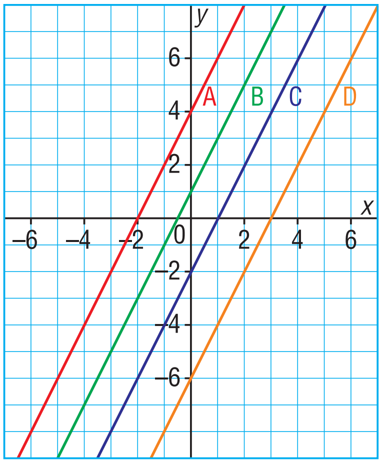
10. a) Write the equations of 3 different lines.

b) Graph the lines on the same grid. Write the equations below the grid.

c) Trade grids with a classmate. Match your classmates' graphs and equations.

11. Assessment Focus

a) How are these 4 graphs alike?



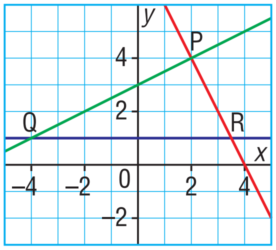
b) How are the graphs different?

c) Match each graph to its equation.

- **i)** $y = 2x - 2$
- **ii)** $y = 2x + 4$
- **iii)** $2x - y = 6$
- **iv)** $2x - y = -1$

d) Did you use the same strategy each time?
If your answer is yes, what strategy did you use and why?
If your answer is no, explain why you used different strategies.
Show your work.

12. The lines on the grid below intersect to form ΔPQR. The equations of the lines are: $y = 1$, $2x + y = 8$, and $2y - x = 6$

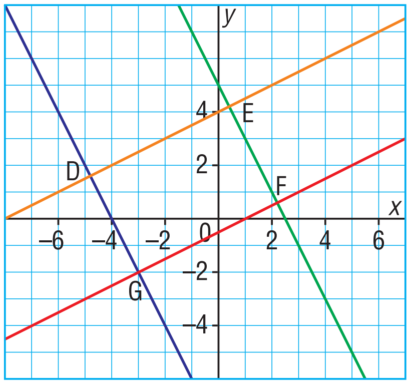


What is the equation of the line on which each side of the triangle lies?

a) PQ **b)** QR **c)** RP

Take It Further

13. The lines on the grid below intersect to form rectangle DEFG.
The equations of the lines are: $y = \frac{1}{2}x - \frac{1}{2}$; $y = -2x + 5$; $y = -2x - 8$; and $x - 2y = -8$



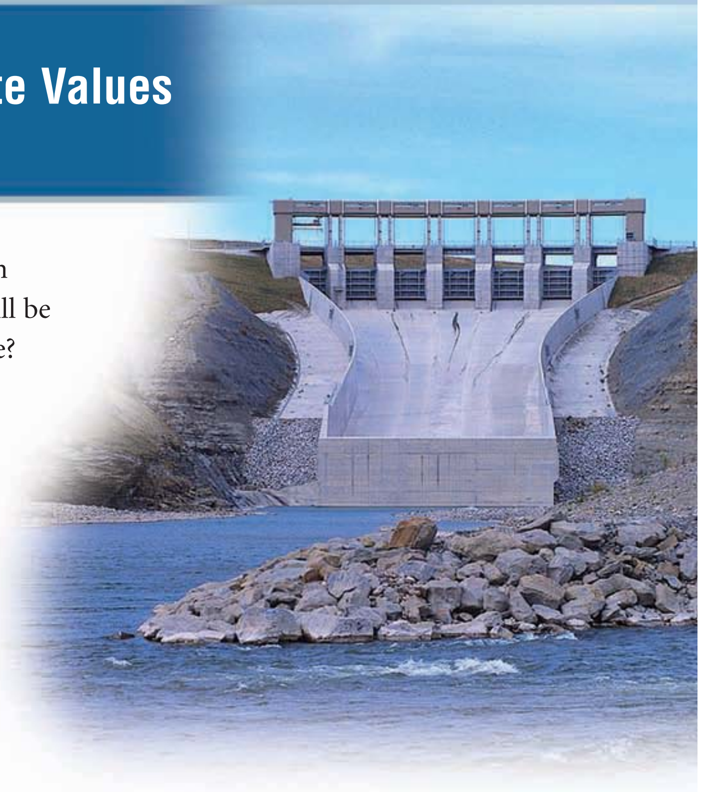
What is the equation of the line on which each side of the rectangle lies?

a) DE **b)** DG **c)** EF **d)** FG

Reflect

What strategies have you learned to match an equation with its graph? When might you use each strategy? Include examples in your explanation.

4.5 Using Graphs to Estimate Values



How do you think city planners can predict the volume of water that will be needed by its residents in the future?

FOCUS

- Use interpolation and extrapolation to estimate values on a graph.

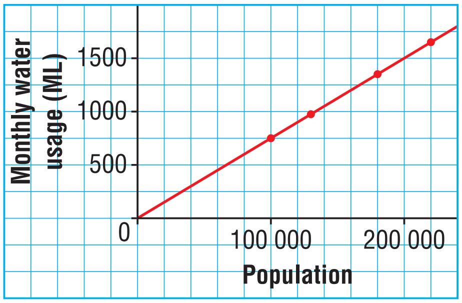
Investigate

A city has grown over the past few years. This table and graph show how the volume of water used each month is related to the population.

Population	Monthly Water Usage (ML)
100 000	750
130 000	975
180 000	1350
220 000	1650

1 ML is 1 000 000 L.

Water Usage in One City



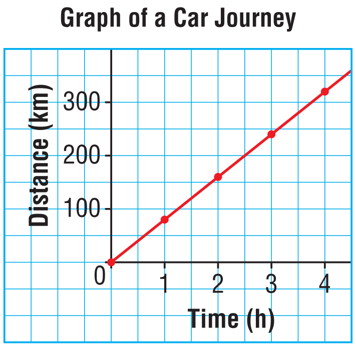
Use these data to:

- Estimate the monthly water usage for a population of 150 000 people.
- Estimate the population when the monthly water usage is 1400 ML.
- Predict the water usage for 250 000 people.

Reflect & Share

Share your answers and strategies for solving the problems with another pair of students.
Did you use the table to estimate? Did you use the graph?
Are your estimates the same? Should they be? Explain.
Why do we call these numbers "estimates"?

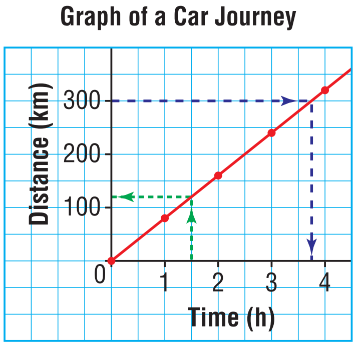
Connect



This graph shows how the distance travelled by a car on the highway changes over a 4-h period.

To draw the graph, we plotted the distance travelled every hour, then drew a line through the points.

We can use **interpolation** to estimate values that lie *between* 2 data points on the graph.



To estimate the distance travelled in 1.5 h:

- Begin at 1.5 on the *Time* axis.
- Draw a vertical line to the graph.
- Then draw a horizontal line from the graph to the *Distance* axis.
  This line intersects the axis at about 120 km.

So, the distance travelled in 1.5 h is about 120 km.

To estimate the time it takes to travel 300 km:

- Begin at 300 on the *Distance* axis.
- Draw a horizontal line to the graph.
- Then draw a vertical line from the graph to the *Time* axis.
  This line intersects the axis at about 3.75 h, which is 3 h 45 min.

So, it takes about 3 h 45 min to travel 300 km.

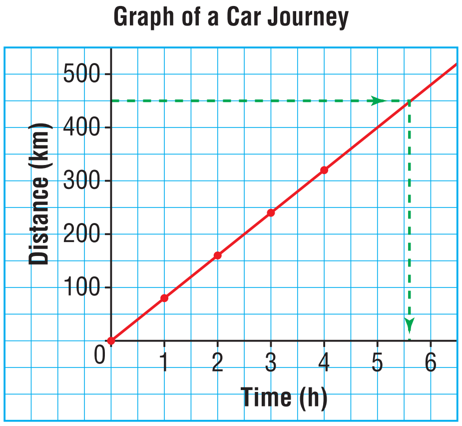
Suppose the car maintains the same average speed. We can extend the graph to predict how far the car will travel in a given time or to predict the time it takes to travel a given distance.

This is called **extrapolation**. When we use a graph to predict in this way, we assume that the relation is linear and will continue to be linear.
We use a ruler to extend the graph.

To estimate the time it takes to travel 450 km:

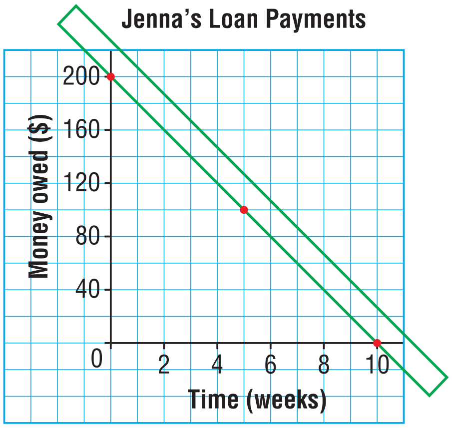
- Extend the grid so the *Distance* axis shows at least 450 km.
  Use a ruler to extend the graph.
- Repeat the process to estimate the time to travel 450 km.

It takes a little more than 5.5 h,
or about 5 h 40 min to travel 450 km.



Example 1 Using Interpolation to Solve Problems

Jenna borrows money from her parents for a school trip. She repays the loan by making regular weekly payments. The graph shows how the money is repaid over time. The data are discrete because payments are made every week.

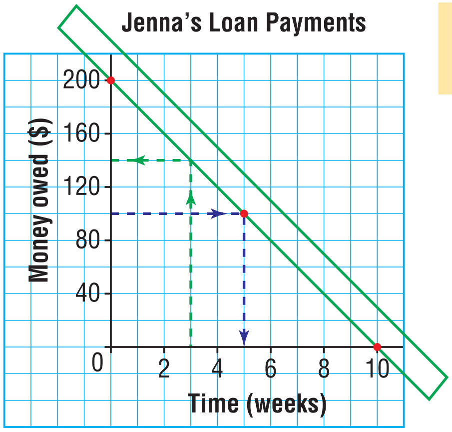


a) How much money did Jenna originally borrow?

b) How much money does she still owe after 3 weeks?

c) How many weeks will it take Jenna to repay one-half of the money she borrowed?

A Solution



Use a straightedge to help.

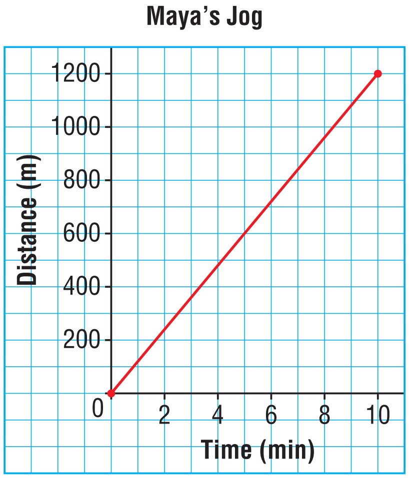
a) The money borrowed is the amount when the repayment time is 0.
This is the point where the graph intersects the *Money owed* axis.
Jenna originally borrowed $200.

b) Begin at 3 on the *Time* axis.
Draw a vertical line to the graph, then a horizontal line to the *Money owed* axis.
The amount owed is about halfway between 120 and 160.
So, Jenna owes about $140 after 3 weeks.

c) Jenna borrowed $200.
After she repays one-half of this amount, she still owes $100.
Begin at 100 on the *Money owed* axis.
Draw a horizontal line to the graph, then a vertical line to the *Time* axis.
It will take Jenna about 5 weeks to repay one-half of the money.

Example 2 Using Extrapolation to Solve Problems

Maya jogs on a running track. This graph shows how far she jogs in 10 min.
Assume Maya continues to jog at the same average speed.



Use the graph.

a) Predict how long it will take Maya to jog 2000 m.

b) Predict how far Maya will jog in 14 min.

c) What assumption did you make?

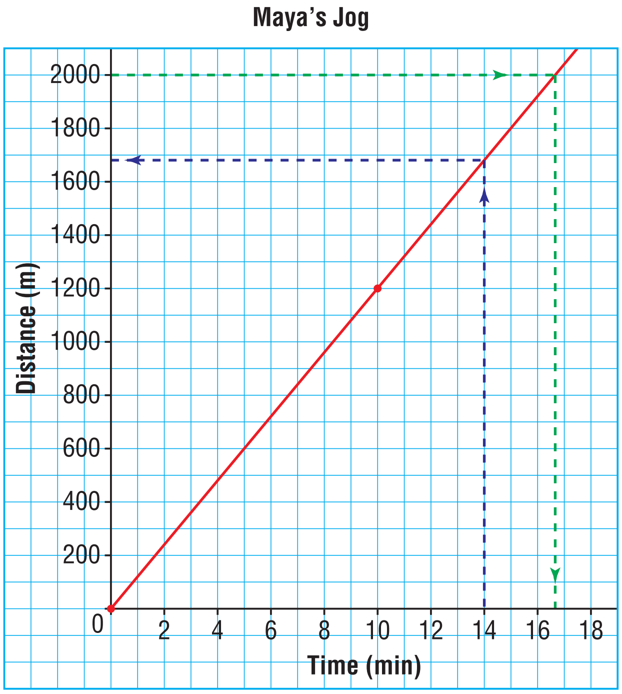
A Solution

Extend the graph to include 2000 m vertically and 14 min horizontally.

a) Begin at 2000 on the *Distance* axis.
Move across to the graph then down to the *Time* axis.
It will take Maya between 16 and 17 min to jog 2000 m.

b) Begin at 14 on the *Time* axis.
Move up to the graph then across to the *Distance* axis.
The distance is about 1700 m.
In 14 min, Maya will jog about 1700 m.

c) I assume that Maya will continue to jog at the same average speed as before.

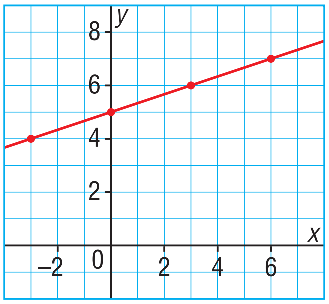


Example 3 Interpolating and Extrapolating to Determine Values of Variables from a Graph

Use this graph of a linear relation.

a) Determine the value of x when $y = 3$.

b) Determine the value of y when $x = 5$.

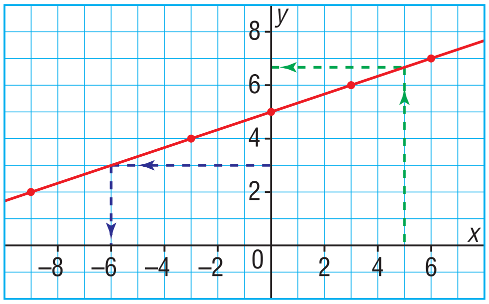


A Solution

Extend the graph to the left to be able to extrapolate for $y = 3$.
Label the extended x-axis.

a) Begin at 3 on the y-axis.
Move across to the graph, then down to the x-axis.
When $y = 3$, $x = -6$

b) Begin at 5 on the x-axis.
Move up to the graph, then across to the y-axis.
The value of y is between 6 and 7, but closer to 7.
When $x = 5$, $y \doteq 6\frac{2}{3}$



Discuss the ideas

1. a) What is interpolation? When do we use it?

b) What is extrapolation? When do we use it?

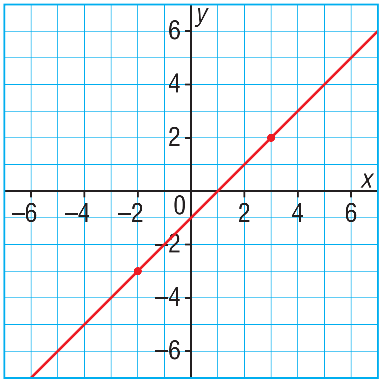
2. When we extrapolate, why is it important to know that the data represent a linear relation?

3. What problems might there be if you extrapolate far beyond the last data point?

Practice

Check

4. This graph represents a linear relation.



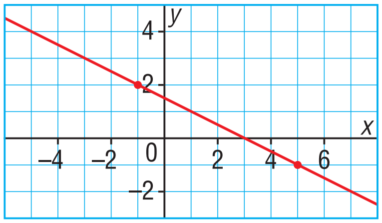
a) Determine each value of x for:

i) $y = 5$ **ii)** $y = -1$ **iii)** $y = -2$

b) Determine each value of y for:

i) $x = -4$ **ii)** $x = 2$ **iii)** $x = 5$

5. This graph represents a linear relation.



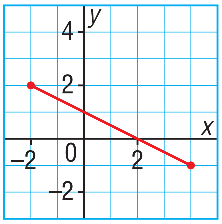
a) Determine each value of x for:

i) $y = 3$ **ii)** $y = 1$ **iii)** $y = -2$

b) Determine each value of y for:

i) $x = -3$ **ii)** $x = 3$ **iii)** $x = 6$

6. This graph represents a linear relation.



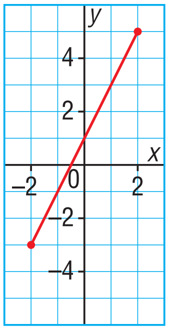
a) Determine each value of x for:

i) $y = 6$ **ii)** $y = -4$ **iii)** $y = -8$

b) Determine each value of y for:

i) $x = -6$ **ii)** $x = 6$ **iii)** $x = 9$

7. This graph represents a linear relation.



a) Determine each value of x for:

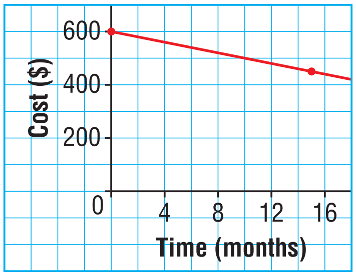
i) $y = 6$ **ii)** $y = -4$ **iii)** $y = -7$

b) Determine each value of y for:

i) $x = -5$ **ii)** $x = 3$ **iii)** $x = 5$

Apply

8. This graph shows how the price of a new game console changes with time.


Cost of a Game Console

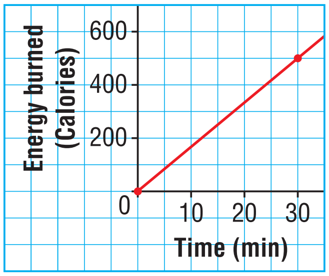
Use the graph.

a) Estimate the cost of the game console 5 months after it is released.

b) How many months is it until the console costs $500?

c) Estimate the price of the console one year after it was released.

9. This graph shows the energy in Calories that Kendall burns when he works out on an elliptical machine.


Energy Burned on an Elliptical Machine

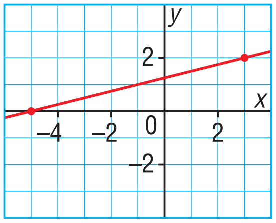
Use the graph.

a) Estimate how many Calories Kendall burns in 20 min.

b) Estimate for how long Kendall must exercise to burn 400 Calories.

c) Estimate how many Calories Kendall burns in 6 min.

10. This graph represents a linear relation.

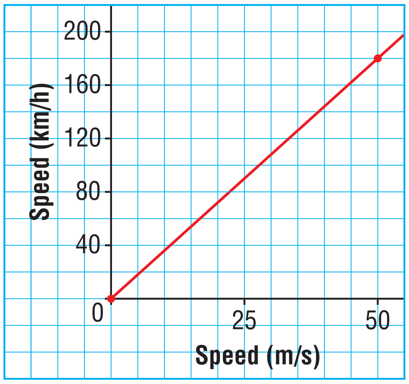


Estimate the value of y when:

a) $x = -3$ **b)** $x = 0$ **c)** $x = 1$

Explain how you estimated.

11. **Assessment Focus** This graph shows how a speed in metres per second relates to a speed in kilometres per hour.


Graph for Converting Speeds

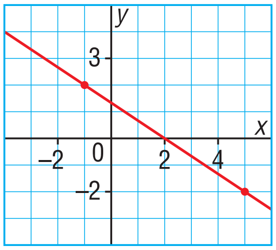
a) Estimate the speed, in metres per second, of:
   i) a car that is travelling at 70 km/h
   ii) a train that is travelling at 110 km/h

b) Estimate the speed, in kilometres per hour, of:
   i) a racing car that is travelling at 60 m/s
   ii) a bicycle that is travelling at 8 m/s

c) For which of parts a and b did you use:
   i) interpolation?
   ii) extrapolation?
   Explain how you know.

d) Explain why your answers are estimates and not exact.

12. This graph represents a linear relation.



Estimate the value of x when:

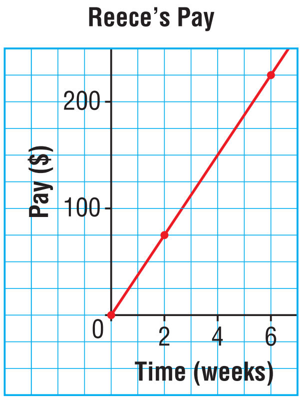
i) $y = 3$

ii) $y = 1$

iii) $y = -1$

Explain how you estimated.

13. Reece works for 5 h each week at a clothing store. This graph shows how her pay relates to the number of weeks she works.

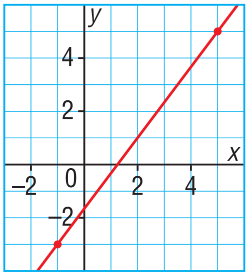


a) Estimate Reece's earnings after 8 weeks.

b) Estimate how long it will take Reece to earn \$400. What assumption did you make?

c) What conditions could change that would make this graph no longer valid?

14. This graph represents a linear relation.



a) Estimate the value of y when:

i) $x = -3$ ii) $x = -5$ iii) $x = 10$

b) Estimate the value of x when:

i) $y = -5$ ii) $y = 8$ iii) $y = 10$

Explain how you estimated.

Take It Further

15. A local convenience store sells 3 different sizes of drinks. The price of each drink is listed below. The store owner plans to introduce 2 new sizes of drinks. She wants the prices and sizes to be related to the drinks she sells already.

Size (mL)	Price (¢)
500	79
750	89
1000	99

a) Graph the data.

b) What should the store owner charge for a 1400-mL drink?

c) What should be the size of a drink that costs 65¢?

Justify your answers.

Reflect

What is the difference between interpolation and extrapolation?
When might you use each process? Use examples in your explanation.

Interpolating and Extrapolating

FOCUS

- Use a graphing calculator to create a graph, then interpolate and extrapolate values.

We can use a graphing calculator to graph the data in a table of values. We can then interpolate and extrapolate from the graph to estimate or predict values that are not in the table.

The table at the right shows the costs of gas for 5 customers at a gas station.
What is the cost of 20 L of gas?
How much gas can be bought for $20.00?
What is the cost of 30 L of gas?

Volume of Gas (L)	Cost ($)
6	5.10
10	8.50
16	13.60
18	15.30
24	20.40

To graph the relation:

- ➢ Enter the data in a graphing calculator.
- ➢ Set up the calculator to plot the points.
- ➢ Display the graph.

To interpolate or extrapolate:

- ➢ To determine the cost of 20 L of gas, use the table feature or trace along the graph to interpolate.
  The cost of 20 L of gas is $17.00.
- ➢ To determine how much gas can be bought for $20.00, find where the horizontal line $y = 20$ meets the graph. Input the equation $y = 20$, then use the trace or intersection feature to determine the coordinates of the point where the lines intersect.
  About 23.5 L of gas can be bought for $20.00.
- ➢ To determine the cost of 30 L of gas, extend the table or trace along the graph to extrapolate. You may need to adjust the window before you trace along the graph.
  The cost of 30 L of gas is $25.50.

Check

Follow the steps above to graph the data.

1. Use the graph to estimate each value.
  a) the cost of:
    i) 10 L of gas **ii)** 50 L of gas
  b) the volume of gas that can be purchased for:
    i) $65.00 **ii)** $12.00
  Did you interpolate or extrapolate to determine these values? Explain.

Study Guide

Generalize a Pattern

Term Number, n	Term Value, v	Pattern
1	3	2(1) + 1
2	5	2(2) + 1
3	7	2(3) + 1
:	:	:
n		2(n) + 1

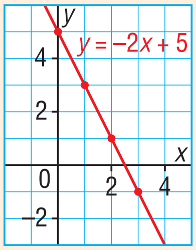
Each term value is 2 more than the preceding term value.
Start with the expression $2n$ and adjust it as necessary to produce the numbers in the table.
The expression is: $2n + 1$
The equation is: $v = 2n + 1$

Linear Relations

- The graph of a linear relation is a straight line.
  To graph a linear relation, first create a table of values.
  For example, to graph the linear relation: $y = -2x + 5$

x	y
0	5
1	3
2	1

Choose 3 values of x, then use the equation to calculate corresponding values of y.

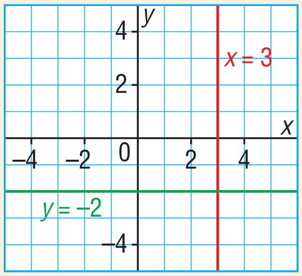


Each point on the graph is 1 unit right and 2 units down from the preceding point.

Another form of the equation of the graph above is $2x + y = 5$.

Horizontal and Vertical Lines

- The graph of the equation $x = a$, where a is a constant, is a vertical line.
  The graph of the equation $y = a$, where a is a constant, is a horizontal line.



Interpolation and Extrapolation

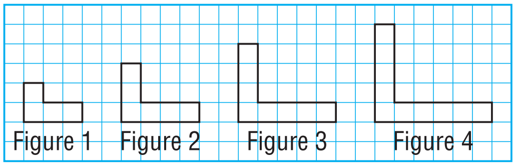
- Interpolation is determining data points *between* given points on the graph of a linear relation.

  Extrapolation is determining data points *beyond* given points on the graph of a linear relation.
  When we extrapolate, we assume that the linear relation continues.

Review

4.1

1. This pattern continues.



a) Determine the perimeter of each figure.
b) Draw the next 3 figures on grid paper.
c) Make a table to show the number of each figure and its perimeter.
d) Write an expression for the perimeter in terms of the figure number, *n*.
e) Write an equation that relates the perimeter *P* to *n*.
f) Determine the perimeter of figure 30.
g) Determine the figure number that has perimeter 90 units.

2. The pattern in this table continues.

Term Number, *n*	Term Value, *v*
1	−5
2	−2
3	1
4	4

a) Describe the patterns in the table.
b) Use *n* to write an expression for the term value.
c) Write an equation that relates *v* and *n*.
d) Verify the equation by substituting a pair of values from the table.
e) Determine the value of the 21st term.
f) Which term number has a value of 106? How do you know?

3. The first number in a pattern has the value 75. As the term number increases by 1, its value decreases by 4.
a) Create a table for this pattern.
b) Write an expression for the value of the term in terms of the term number *n*.

4.2

4. Norman has $140 in his savings account. Each month he deposits $20 into this account. Let *t* represent the time in months and *A* the account balance in dollars.
a) Create a table to show several values of *t* and *A*.
b) Graph the data. Will you join the points? Explain.
c) Is this relation linear? Justify your answer.
d) Describe the pattern in the table. How are these patterns shown in the graph?
e) Write an equation that relates *A* and *t*.

5. Copy and complete each table of values. Describe the patterns in the table.
a) $y = 4x$ **b)** $y = 10 - 2x$ **c)** $y = 3x + 4$

x	*y*
1	
2	
3	

x	*y*
0	
1	
2	

x	*y*
−3	
−2	
−1	

6. Graph the data from each table in question 5. For each graph, explain how the patterns in the graph match the patterns in the table.

4.3

7. A piece of string is 25-cm long. The string is cut into 2 pieces.
a) Make a table that shows 6 possible lengths for the two pieces of string.
b) Graph the data.
  i) Is the relation linear? How do you know?
  ii) Should you join the dots? Explain.
c) Choose 2 variables to represent the lengths of the longer and shorter pieces.
  i) Write an equation that relates the variables.
  ii) How could you check your equation?

8. Graph each equation. Do you need to make a table of values each time? Explain.

a) $x = -2$ **b)** $y = 3$

c) $x = 5$ **d)** $y = -1$

9. For each equation below:

- Make a table for the given values of x.
- Graph the equation.

a) $3x + y = 9$; for $x = -3, 0, 3$

b) $2x - y = 4$; for $x = -2, 0, 2$

c) $2x + y = -6$; for $x = -4, 0, 4$

d) $x - 2y = -6$; for $x = -2, 0, 2$

10. Does each equation represent a vertical line, a horizontal line, or an oblique line? How can you tell without graphing?

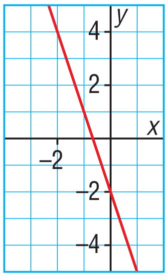
a) $x = 6$ **b)** $x - y = 3$

c) $y + 8 = 0$ **d)** $2x + 9 = 0$

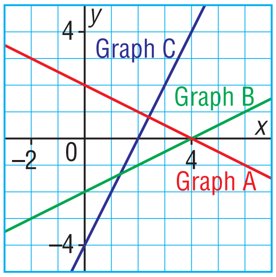
4.4 **11.** Which equation describes the graph below? Justify your answer.

a) $y = -2x + 3$ **b)** $y = 2x - 3$

c) $y = 3x - 2$ **d)** $y = -3x - 2$



12. Which graph represents the equation $x - 2y = 4$? How do you know?



13. Match each equation with its graph below. Explain your strategy.

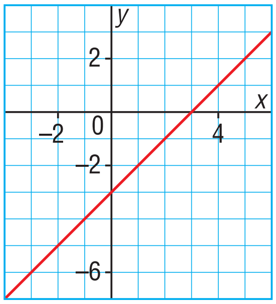
a) $x + 2y = 6$

b) $y = x - 3$

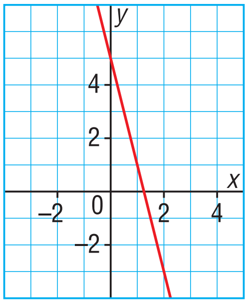
c) $y = 2x - 3$

d) $y = -4x + 5$

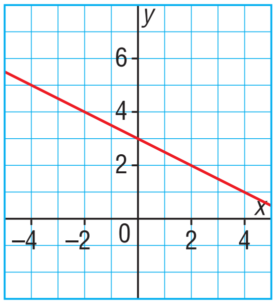
i)



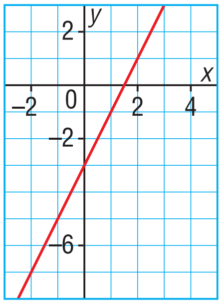
ii)



iii)

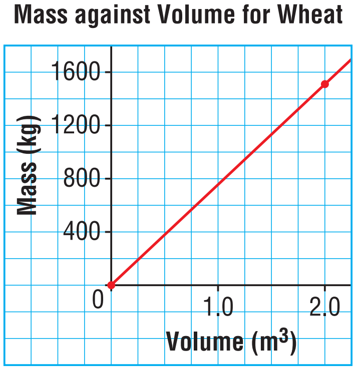


iv)



4.5

14. This graph shows how the mass of wheat changes with its volume.

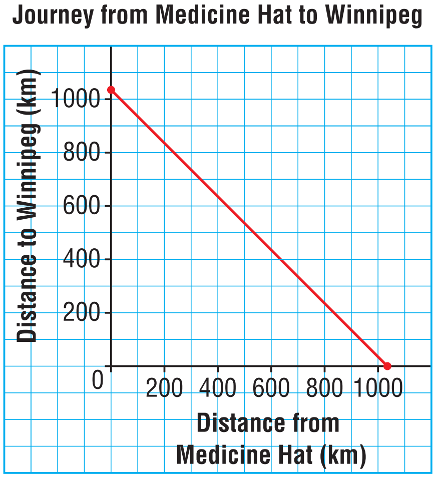


Use the graph.

a) Estimate the volume of 2000 kg of wheat.

b) Estimate the mass of 2.5 m^3 of wheat.

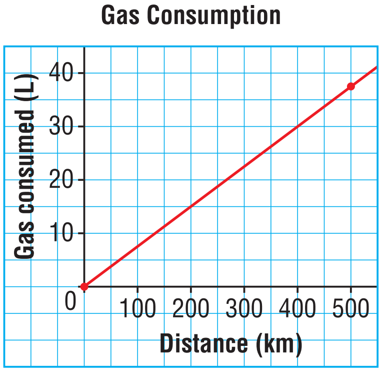
15. Harold and Jenny are driving from Medicine Hat to Winnipeg. The graph shows the distance travelled and the distance yet to go.



a) About how far is it from Medicine Hat to Winnipeg? How can you tell from the graph?

b) When Jenny and Harold have travelled 450 km, about how far do they still have to go?

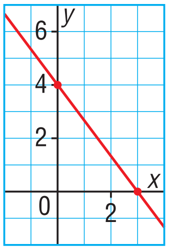
16. The Dubois family lives in Regina. The family is planning a family holiday to the West Coast. This graph shows the gas consumption of the family's car.



a) The distance from Regina to Vancouver is 1720 km. Estimate the volume of gasoline needed to travel from Regina to Vancouver. Explain how you did this.

b) To travel from Regina to Prince Albert, the car used about 30 L of gasoline. About how far is it between these two towns?

17. This graph represents a linear relation.



a) Estimate the value of y when:

  i) $x = -4$

  ii) $x = 2$

  iii) $x = 5$

b) Estimate the value of x when:

  i) $y = 7$

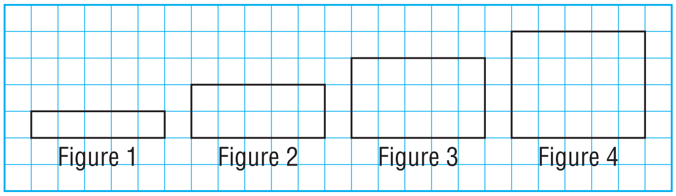
  ii) $y = 2$

  iii) $y = -3$

Explain how you estimated.

Practice Test

1. Here is a pattern made from square tiles.



a) Make a table that shows how the number of square tiles, *s*, in a figure relates to the figure number, *f*.

b) Write an expression for the number of square tiles in terms of *f*.

c) Write an equation that relates *s* and *f*.
Verify the equation by substituting the values from the table.

d) How are the expression and equation alike? How are they different?

e) Which figure has 225 tiles? Explain how you know.

2. **a)** Make a table of values for this equation: $y = -2x + 7$

b) Graph the relation.

c) Explain how the patterns in the graph match those in the table.

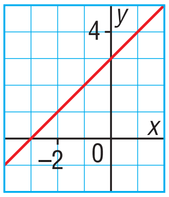
3. Does each equation describe a vertical, a horizontal, or an oblique line? How do you know?

a) $x = 6$ **b)** $2y - 7 = 3$ **c)** $2x + 9 = 0$

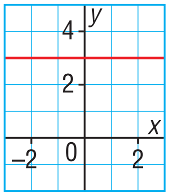
4. Match each equation with its graph below. Explain your strategy.

a) $y = x + 3$ **b)** $y = 3$ **c)** $x + y = 3$ **d)** $x = -3$

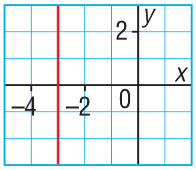
i)



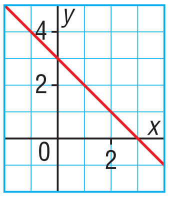
ii)



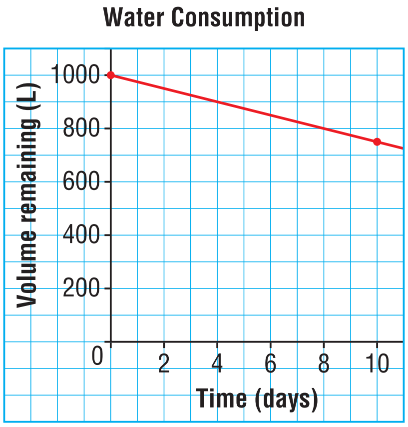
iii)



iv)



5. A family uses a cistern for drinking water at its cabin. The graph shows how the volume of drinking water in the cistern changes during a 10-day period. Suppose the pattern in the water usage continues.



a) How many days did it take to use 200 L of water?

b) Estimate the volume of water in the cistern after 22 days.

c) Estimate how much water is used in the first 14 days.

d) What assumptions did you make?

Unit Problem Predicting Music Trends

The format in which music is produced and sold has changed over the past 30 years.

Part 1

The table shows the sales of cassette tapes in North America.

Year	Cassette Sales (billions)
1993	$2.9
1995	$2.3
1998	$1.4

a) Graph the data. Do the data represent a linear relation? How do you know?

b) Describe how the sales of cassettes changed over time.

c) Let t represent the number of years after 1993 and S the sales in billions of dollars. Write an equation that relates S and t.

d) Use the equation to determine the sales in 1997. Does the answer agree with the value in the graph? Explain.

e) Use the graph to predict the year in which the sales of cassettes were $0.

f) Cassettes were sold until 2004. Explain why this is different from the year predicted in the graph.

Part 2

As the sale of cassettes was decreasing, the sales of CDs were increasing. Assume the growth in CDs sales, from 1996 to 2000, was linear.

Year	CD Sales (millions)
1996	$9 935
2000	$13 215

a) Graph the data. Use the graph to estimate the CD sales for 1997, 1998, and 1999. Is this interpolation or extrapolation? Explain.

b) Estimate the total CD sales for this 5-year period.

c) Estimate the CD sales in 2001. Is this interpolation or extrapolation? Explain.

d) Use the graph to estimate the CD sales for 2005.

e) Which answer in parts c and d is more likely to be the closer estimate? Justify your answer.

Your work should show:

- accurate and labelled graphs
- how you wrote and used the equation
- clear explanations of your thinking

Reflect on Your Learning

What is a linear relation? How may a linear relation be described? What can you determine when you know a relation is linear? Include examples.

Project

Number Systems

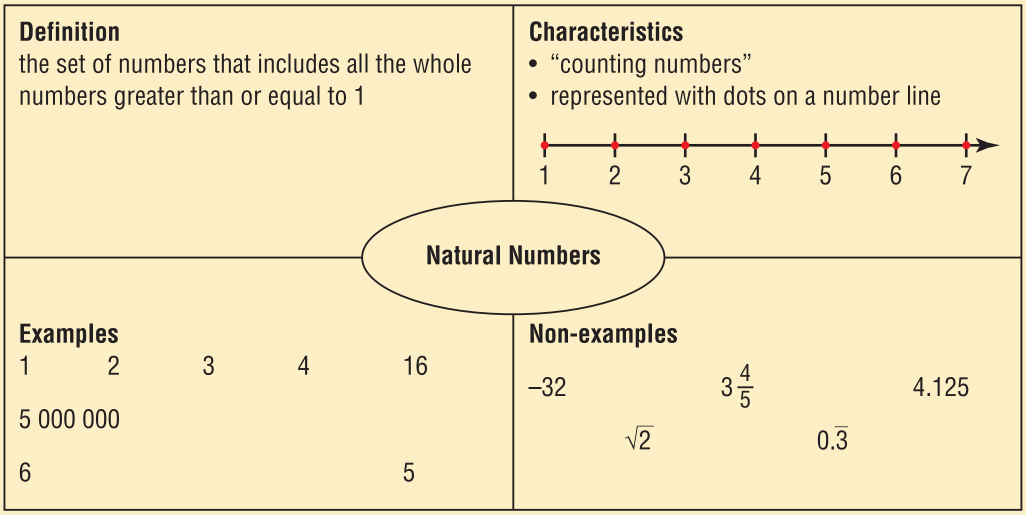
Materials

- copies of a Frayer model

Work with a partner.

Part 1

This is a Frayer model for **natural numbers**.



Natural Numbers

Definition
the set of numbers that includes all the whole numbers greater than or equal to 1

Characteristics

- "counting numbers"
- represented with dots on a number line

Examples
1 2 3 4 16
5 000 000
6 5

Non-examples
–32 $3\frac{4}{5}$ 4.125
$\sqrt{2}$ $0.\overline{3}$

➤ Complete a Frayer model for each of these number systems:

- whole numbers
- integers
- rational numbers

Each of these number systems is part of a larger system of numbers called the **real numbers**.

Part 2

➤ Is it possible for a number to belong to more than one number system? Explain. Use examples to support your answer.

➤ Is it possible for an entire number system to belong to another number system? Explain.

➤ Draw a diagram to show how the number systems in Part 1 overlap.

Part 3

Choose a number system from Part 1; this is the name of your club.
Choose a number from your number system.

You are in charge of memberships for your club. It is your job to either accept or reject a number that wishes to join your club. Write a letter of acceptance or rejection for your partner's number. If the number belongs in your number system, it must be allowed membership.

Your letter should be written as a business letter.
It must address the following points:

- examples of other numbers in your club and what their characteristics are
- how your partner's number either fits or does not fit the characteristics of your club
- if you are accepting the number, why your club wants that number
- if you are rejecting the number, what other clubs the number could contact and why

Take It Further

- Numbers that are not rational numbers are called *irrational numbers*. Create a Frayer model for *irrational numbers*.
- Amend your diagram from Part 2 to include these numbers.

UNIT 5

Polynomials

How could you solve this problem?

Denali and Mahala weed the borders on the north and south sides of their rectangular yard. Denali starts first and has weeded 1 m on the south side when Mahala says he should be weeding the north side. So, Denali moves to the north side. Mahala finishes weeding the south side. Then she moves to the north side where she weeds 2 m. Both students have then finished. Which student weeded more of the borders? How much more?

What You'll Learn

- Recognize, write, describe, and classify polynomials.
- Use algebra tiles, pictures, and algebraic expressions to represent polynomials.
- Strategies to add and subtract polynomials.
- Strategies to multiply and divide a polynomial by a monomial.

Why It's Important

Just as numbers are the building blocks of arithmetic, polynomials are the building blocks of algebra.

In later grades, you will use polynomials to model real-world situations in business, science, medicine, and engineering. The skills, understanding, and language that you develop in this unit will lay the foundation for this work.

Key Words

- polynomial
- term
- coefficient
- degree
- constant term
- monomial
- binomial
- trinomial
- like terms

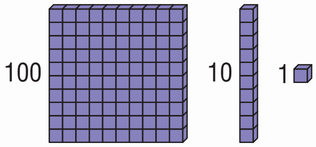
5.1

Modelling Polynomials

FOCUS

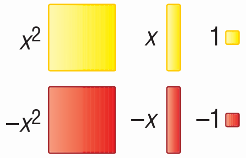
- Model, write, and classify polynomials.

In arithmetic, we use Base Ten Blocks to model whole numbers. How would you model the number 234?



In algebra, we use algebra tiles to model integers and variables.

Yellow represents positive tiles. Red represents negative tiles.



How are Base Ten Blocks and algebra tiles alike?

Investigate

Use algebra tiles.

➤ Model each expression. Sketch the tiles.
How do you know which tiles to use?
How do you know how many of each tile to use?

- $x^2 + x - 3$
- $-2x^2 - 3$
- $2x^2 + 3x$
- $-2x^2 - 3x + 1$
- $-3x + 3$

➤ Write your own expression.
Have your partner model it with tiles.
Model your partner's expression with tiles.

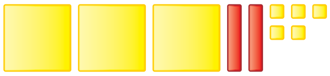
Reflect & Share

For the first activity, compare your sketches with those of another pair of students.
Did you use the same tiles each time? If not, is one of you wrong?
Could both of you be correct? Explain.
Did the order in which you laid out the tiles matter? Explain.

Connect

We can use algebra tiles to model an expression such as $3x^2 - 2x + 5$.

To model $3x^2 - 2x + 5$, we use three x^2-tiles, two $-x$-tiles, and five 1-tiles.



A **polynomial** is one term or the sum of terms whose variables have whole-number exponents.

The expression $3x^2 - 2x + 5 = 3x^2 + (-2)x + 5$ is an example of a polynomial in the variable x.
This polynomial has 3 terms: $3x^2$, $(-2)x$, and 5
Terms are numbers, variables, or the product of numbers and variables.
The **coefficients** of the variable are 3 and -2.

The term with the greatest exponent determines the **degree** of the polynomial.
This polynomial has degree 2.

$$3x^2 - 2x + 5$$

The term 5 is a **constant term**.
Its value does not change when the value of x changes.
A constant term has degree 0.

The term $-2x$ has degree 1 because $-2x = -2x^1$.

We can use any variable to write a polynomial and to describe the tiles that model it.
For example, the tiles that model the polynomial $-5n^2 + 7n - 1$ also model the polynomial $-5p^2 + 7p - 1$.

We can also classify a polynomial by the number of terms it has.
Polynomials with 1, 2, or 3 terms have special names.
A **monomial** has 1 term; for example: $4a$, 6, $-2p^2$
A **binomial** has 2 terms; for example: $2c - 5$, $2m^2 + 3m$
A **trinomial** has 3 terms; for example: $2h^2 - 6h + 4$

A polynomial is usually written in descending order; that is,
the exponents of the variable decrease from left to right;
for example, the polynomial $2k - 4k^2 + 7$ is written as $-4k^2 + 2k + 7$.

An algebraic expression that contains a term with a variable in the denominator, such as $\frac{3}{n}$, or the square root of a variable, such as $\sqrt{n}$, is *not* a polynomial.

Example 1 Recognizing the Same Polynomials in Different Variables

Which of these polynomials can be represented by the same algebra tiles?

a) $3x^2 - 5x + 6$ **b)** $-5 + 6r + 3r^2$ **c)** $-5m + 6 + 3m^2$

Justify the answer.

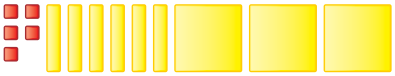
A Solution

a) $3x^2 - 5x + 6$

Use three x^2-tiles, five $-x$-tiles, and six 1-tiles.

b) $-5 + 6r + 3r^2$

Use five -1-tiles, six r-tiles, and three r^2-tiles.



c) $-5m + 6 + 3m^2$

Use five $-m$-tiles, six 1-tiles, and three m^2-tiles.

In parts a and c, the same algebra tiles are used.

So, the polynomials $3x^2 - 5x + 6$ and $-5m + 6 + 3m^2$ can be represented by the same tiles.

Example 2 Modelling Polynomials with Algebra Tiles

Use algebra tiles to model each polynomial.

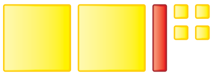
Is the polynomial a monomial, binomial, or trinomial? Explain.

a) $-2x^2$ **b)** $2b^2 - b + 4$ **c)** $5a - 3$

A Solution

a) To represent $-2x^2$, use two $-x^2$-tiles. Since there is only one type of tile, $-2x^2$ is a monomial.

b) To represent $2b^2 - b + 4$, use two b^2-tiles, one $-b$-tile, and four 1-tiles. Since there are 3 types of tiles, $2b^2 - b + 4$ is a trinomial.



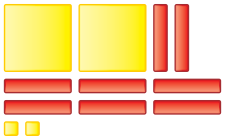
c) To represent $5a - 3$, use five a-tiles and three -1-tiles. Since there are 2 types of tiles, $5a - 3$ is a binomial.

Two polynomials are *equivalent* when they can be represented by identical algebra tiles.

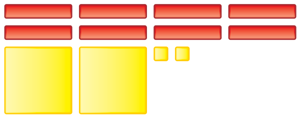
Example 3 Recognizing Equivalent Polynomials

a) Which polynomial does each group of algebra tiles represent?

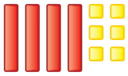
Model A



Model B



Model C



b) Which of the polynomials in part a are equivalent? How do you know?

A Solution

a) Use a table.

Model	Description of Tiles	Polynomial
A	two x^2-tiles, eight $-x$-tiles, and two 1-tiles	$2x^2 - 8x + 2$
B	eight $-x$-tiles, two x^2-tiles, and two 1-tiles	$-8x + 2x^2 + 2$
C	four $-x$-tiles and six 1-tiles	$-4x + 6$

b) Both models A and B contain the same tiles. The polynomials represented by these tiles have the same degree, and the same terms: $2x^2$, $-8x$, and 2

Both polynomials can be written as: $2x^2 - 8x + 2$

So, $2x^2 - 8x + 2$ and $-8x + 2x^2 + 2$ are equivalent polynomials.

Model C has no x^2-tiles, so its degree is different from that of models A and B.

Discuss the ideas

1. In the polynomial $3 + 2p$, which term is the constant term? How are constant terms modelled with algebra tiles?

2. Suppose you are given an algebra tile model of a polynomial. How can you identify the terms, the coefficients, and the degree of the polynomial? How can you identify the constant term?

3. What do we mean by "equivalent polynomials"? How can you determine whether two polynomials are equivalent?

Practice

Check

4. Which of the following expressions are polynomials? Explain how you know.

a) $2 + 3n$ **b)** $3\sqrt{x}$
c) $-5m + 1 + 2m^2$ **d)** 7
e) $\frac{1}{x^2} + \frac{1}{x} + 1$ **f)** $\frac{1}{2}s$

5. Is each expression a monomial, binomial, or trinomial? Explain how you know.

a) $3t + 4t^2 - 2$ **b)** $5 - 3g$
c) $9k$ **d)** 11

6. Name the coefficient, variable, and degree of each monomial.

a) $-7x$ **b)** $14a^2$
c) m **d)** 12

7. Identify the degree of each polynomial. Justify your answers.

a) $7j^2 + 4$ **b)** $9x$
c) $2 - 5p + p^2$ **d)** -10

Apply

8. Identify the polynomials that can be represented by the same set of algebra tiles.

a) $x^2 + 3x - 4$
b) $-3 + 4n - n^2$
c) $4m - 3 + m^2$
d) $-4 + r^2 + 3r$
e) $-3m^2 + 4m - 3$
f) $-h^2 - 3 + 4h$

9. Name the coefficients, variable, and degree of each polynomial. Identify the constant term if there is one.

a) $5x^2 - 6x + 2$ **b)** $7b - 8$
c) $12c^2 + 2$ **d)** $12m$
e) 18 **f)** $3 + 5x^2 - 8x$

10. One student says, "$4a$ is a monomial." Another student says, "$4a$ is a polynomial." Who is correct? Explain.

11. Use algebra tiles to model each polynomial. Sketch the tiles.

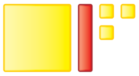
a) $4x - 3$
b) $-3n - 1$
c) $2m^2 + m + 2$
d) $-7y$
e) $-d^2 - 4$
f) 3

12. Match each polynomial with its corresponding algebra tile model.

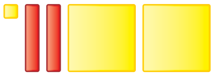
a) $r^2 - r + 3$
b) $-t^2 - 3$
c) $-2v$
d) $2w + 2$
e) $2s^2 - 2s + 1$

Model A

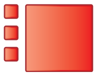
Model B



Model C



Model D



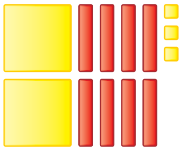
Model E

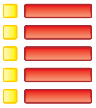
13. Which polynomial does each collection of algebra tiles represent?
Is the polynomial a monomial, binomial, or trinomial? Explain.

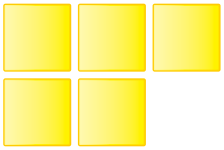
a)

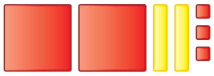

b)

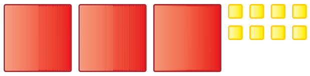

c)


d)


e)


f)


g)


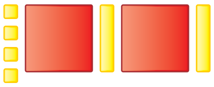
h)


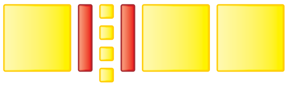
14. Write a polynomial with the given degree and number of terms. Use algebra tiles to model the polynomial. Sketch the tiles.

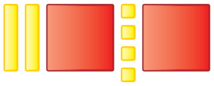
a) degree 1, with 2 terms
b) degree 0, with 1 term
c) degree 2, with 1 term
d) degree 2, with 3 terms and constant term 5

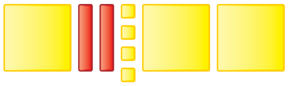
15. Identify which polynomials are equivalent. Explain how you know.

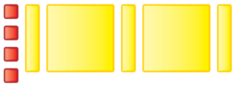
a)

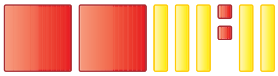

b)


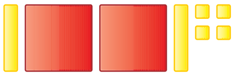
c)


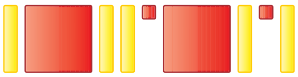
d)


e)


f)


g)


h)


i)


16. Identify which polynomials are equivalent. Justify your answers.

a) $5 - v + 7v^2$
b) $7v + 5 - v^2$
c) $5v + v^2 - 7$
d) $-7 + 5v + v^2$
e) $5 - v^2 + 7v$
f) $7v^2 + v + 5$

17. Write an expression that is *not* a polynomial. Explain why it is not a polynomial.

18. Assessment Focus

a) Use algebra tiles to model each polynomial. Sketch the tiles. Identify the variable, degree, number of terms, and coefficients.

  i) $-2x - 3x^2 + 4$

  ii) $m^2 + m$

b) Write a polynomial that matches this description:

a polynomial in variable *c*, degree 2, binomial, constant term -5

c) Write another polynomial that is equivalent to the polynomial you wrote in part b. Explain how you know that the polynomials are equivalent.

19. a) Write as many polynomials as you can that are equivalent to $-8d^2 - 3d - 4$. How do you know you have written all possible polynomials?

b) Which polynomial in part a is in descending order? Why is it useful to write a polynomial in this form?

Take It Further

20. The *stopping distance* of a car is the distance the car travels between the time the driver applies the brakes and the time the car stops. The polynomial $0.4s + 0.02s^2$ can be used to calculate the stopping distance in metres of a car travelling at *s* kilometres per hour on dry pavement.

a) Determine the stopping distance for each speed:

  i) 25 km/h ii) 50 km/h iii) 100 km/h

b) Does doubling the speed double the stopping distance? Explain.

Reflect

What is a polynomial?
How can you represent a polynomial with algebra tiles and with symbols?
Include examples in your explanation.

Math Link

Your World

A polynomial can be used to model projectile motion. When a golf ball is hit with a golf club, the distance the ball travels in metres, in terms of the time *t* seconds that it is in the air, may be modelled by the polynomial $-4.9t^2 + 22.8t$.

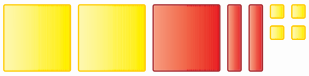
5.2 Like Terms and Unlike Terms

FOCUS

- Simplify polynomials by combining like terms.

When you work with integers,
a 1-tile and a −1-tile form a zero pair.  0

What do you think happens when you combine algebra tiles with opposite signs?
Which expression do these tiles represent?



Investigate

You will need algebra tiles and a paper bag.

➤ Put both colours of algebra tiles in a bag.
Take a handful of tiles and sketch them.
Construct a table to record your work.

Algebra Tile Model	Symbolic Record

Use symbols to write the polynomial modelled by the tiles. Remove zero pairs.
Sketch the tiles that remain.
Use symbols to write the polynomial represented by the smaller set of tiles.

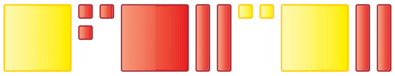
➤ Return the algebra tiles to the bag.
Repeat the activity 4 more times.

Reflect & Share

Share your results with another pair of students.
How could you verify each other's results?
When can you remove zero pairs from a set of tiles?
How does removing zero pairs help you simplify the polynomial that represents the set of tiles?

Connect

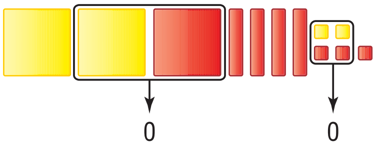
Here is a collection of red and yellow algebra tiles:



We organize the tiles by grouping like tiles:

These tiles represent the polynomial: $2x^2 - x^2 - 4x + 2 - 3$

We simplify the tile model by removing zero pairs.



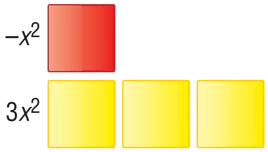
The remaining tiles represent the polynomial: $x^2 - 4x - 1$
We say that the polynomial $2x^2 - x^2 - 4x + 2 - 3$ *simplifies* to $x^2 - 4x - 1$.

A polynomial is in *simplified form* when:

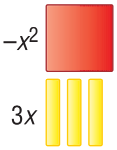
- its algebra tile model uses the fewest tiles possible
- its symbolic form contains only one term of each degree and no terms with a zero coefficient

Terms that can be represented by algebra tiles with the same size and shape are called **like terms**.

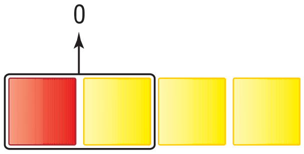
$-x^2$ and $3x^2$ are like terms.
Each term is modelled with x^2-tiles.
Each term has the same variable, x,
raised to the same exponent, 2.



$-x^2$ and $3x$ are *unlike terms*.
Each term is modelled with a different
algebra tile.
Each term has the variable x,
but the exponents are different.



To simplify a polynomial, we group like terms and remove zero pairs.
$-x^2 + 3x^2$ simplifies to $2x^2$.

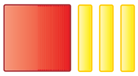


We can also simplify a polynomial by adding the coefficients of like terms. This is called *combining like terms*.

$$\begin{aligned} -x^2 + 3x^2 &= -1x^2 + 3x^2 && \text{Add the integer coefficients: } -1 + 3 = 2 \\ &= 2x^2 \end{aligned}$$

The polynomials $-x^2 + 3x^2$ and $2x^2$ are *equivalent*.
So, a polynomial in simplified form is also the equivalent polynomial in which all the like terms have been combined.

$-x^2 + 3x$ cannot be simplified.
We may not add coefficients when we have unlike terms.



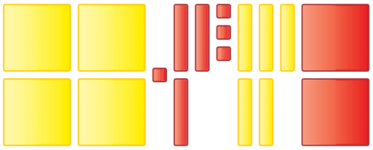
Example 1 Using Algebra Tiles to Simplify a Polynomial

Use algebra tiles to simplify the polynomial $4n^2 - 1 - 3n - 3 + 5n - 2n^2$. Record the process symbolically.

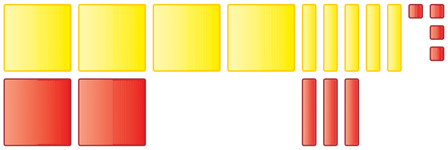
A Solution

Tile Model

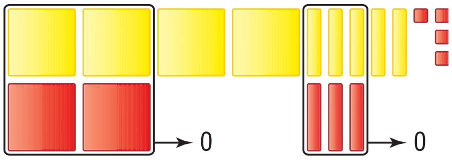
Display $4n^2 - 1 - 3n - 3 + 5n - 2n^2$.



Group like tiles.



Remove zero pairs.



The remaining tiles represent $2n^2 + 2n - 4$.

Symbolic Record

$4n^2 - 1 - 3n - 3 + 5n - 2n^2$

Group like terms:

$$\underbrace{4n^2 - 2n^2} + \underbrace{5n - 3n} \underbrace{- 1 - 3}$$

Combine like terms:

$2n^2 + 2n - 4$

Example 2 Simplifying a Polynomial Symbolically

Simplify: $14x^2 - 11 + 30x + 3 + 15x - 25x^2$

A Solution

We need many tiles to model this polynomial.
So, we simplify it symbolically.

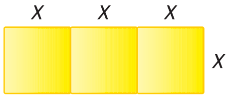
$14x^2 - 11 + 30x + 3 + 15x - 25x^2$ Group like terms.
$= 14x^2 - 25x^2 + 30x + 15x - 11 + 3$ Add the coefficients of like terms.
$= -11x^2 + 45x - 8$

In *Example* 2, the polynomials $14x^2 - 11 + 30x + 3 + 15x - 25x^2$ and $-11x^2 + 45x - 8$ are equivalent.
Polynomials can be used to represent measures such as the side lengths of shapes.

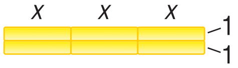
Example 3 Investigating Situations that Represent Polynomials

a) Write a polynomial to represent the perimeter of each rectangle.

i)



ii)

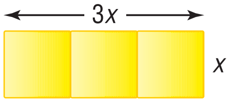


b) Each polynomial represents the perimeter of a rectangle.
Use algebra tiles to make the rectangle.

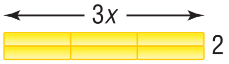
i) $4a + 2$ **ii)** $10b$

A Solution

a) i) The dimensions of the rectangle are $3x$ and x. So, the perimeter of the rectangle is:
$3x + x + 3x + x = 8x$



ii) The dimensions of the rectangle are $3x$ and 2. So, the perimeter of the rectangle is:
$3x + 2 + 3x + 2 = 6x + 4$

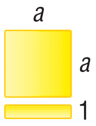


b) i) The perimeter is $4a + 2$.
Work backward.
Write the polynomial as the sum of equal pairs of terms.
$4a + 2 = 2a + 2a + 1 + 1$
The dimensions of the rectangle could be $2a$ and 1.

Another solution is:

$4a + 2 = a + (a + 1) + a + (a + 1)$

The dimensions of the rectangle could be a and $a + 1$.

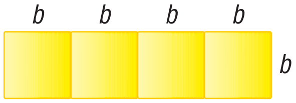


ii) The perimeter is $10b$.

Write the polynomial as the sum of equal pairs of terms.

$10b = 4b + 4b + b + b$

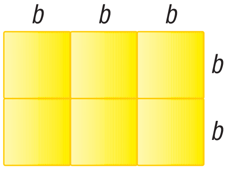
The dimensions of the rectangle could be $4b$ and b.



Another solution is:

$10b = 3b + 3b + 2b + 2b$

The dimensions of the rectangle could be $3b$ and $2b$.



A polynomial may contain more than one variable. Here is a polynomial in x and y:

$-2x^2 + 3xy + y^2 - 4x - 8y$

Example 4 Simplifying a Polynomial in Two Variables

Simplify: $4xy - y^2 - 3x^2 + 2xy - x - 3y^2$

A Solution

$4xy - y^2 - 3x^2 + 2xy - x - 3y^2$ Group like terms.

$= 4xy + 2xy - y^2 - 3y^2 - 3x^2 - x$ Combine like terms.

$= 6xy - 4y^2 - 3x^2 - x$

Discuss the ideas

1. Why can we combine like terms? Why can we not combine unlike terms?

2. How can you identify and combine like terms in an algebra tile model?

3. How can you identify and combine like terms symbolically?

Practice

Check

4. a) Use algebra tiles to model $3d$ and $-5d$. Sketch the tiles.

b) Are $3d$ and $-5d$ like terms? How can you tell from the tiles? How can you tell from the monomials?

5. a) Use algebra tiles to model $4p$ and $2p^2$. Sketch the tiles.

b) Are $4p$ and $2p^2$ like terms? How can you tell from the tiles? How can you tell from the monomials?

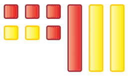
Apply

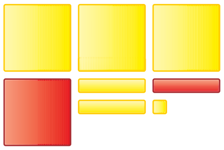
6. From the list, which terms are like $8x$?
$-3x$, $5x^2$, 4, $3x$, 9, $-11x^2$, $7x$, -3
Explain how you know they are like terms.

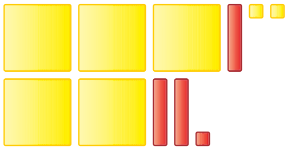
7. From the list, which terms are like $-2n^2$?
$3n$, $-n^2$, -2, $4n$, $2n^2$, -2, 3, $5n^2$
Explain how you know they are like terms.

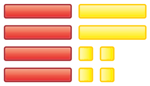
8. For each part, combine tiles that represent like terms.
Write the simplified polynomial.

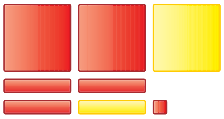
a)


b)


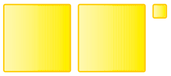
c)


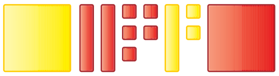
d)


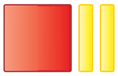
e)


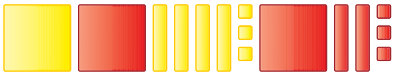
f)


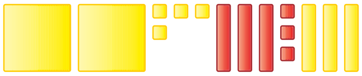
9. Identify the equivalent polynomials in the diagrams below. Justify your answers.

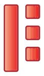
a)


b)


c)


d)


e)


f)


10. A student made these mistakes on a test.

- The student simplified $2x + 3x$ as $5x^2$.
- The student simplified $4 + 3x$ as $7x$.

Use algebra tiles to explain what the student did wrong.
What are the correct answers?

11. Use algebra tiles to model each polynomial, then combine like terms. Sketch the tiles.

a) $2c + 3 + 3c + 1$

b) $2x^2 + 3x - 5x$

c) $3f^2 + 3 - 6f^2 - 2$

d) $3b^2 - 2b + 5b + 4b^2 + 1$

e) $5t - 4 - 2t^2 + 3 + 6t^2$

f) $4a - a^2 + 3a - 4 + 2a^2$

12. Simplify each polynomial.

a) $2m + 4 - 3m - 8$

b) $4 - 5x + 6x - 2$

c) $3g - 6 - 2g + 9$

d) $-5 + 1 + h - 4h$

e) $-6n - 5n - 4 - 7$

f) $3s - 4s - 5 - 6$

13. Simplify each polynomial.

a) $6 - 3x + x^2 + 9 - x$

b) $5m - 2m^2 - m^2 + 5m$

c) $5x - x^2 + 3x + x^2 - 7$

d) $3p^2 - 2p + 4 + p^2 + 3$

e) $a^2 - 2a - 4 + 2a - a^2 + 4$

f) $-6x^2 + 17x - 4 - 3x^2 + 8 - 12x$

14. Simplify each polynomial.

a) $3x^2 + 5y - 2x^2 - 1 - y$

b) $pq - 1 - p^2 + 5p - 5pq - 2p$

c) $5x^2 + 3xy - 2y - x^2 - 7x + 4xy$

d) $3r^2 - rs + 5s + r^2 - 2rs - 4s$

e) $4gh + 7 - 2g^2 - 3gh - 11 + 6g$

f) $-5s + st - 4s^2 - 12st + 10s - 2s^2$

15. Identify the equivalent polynomials. Justify your answers.

a) $1 + 5x$

b) $6 - 2x + x^2 - 1 - x + x^2$

c) $4x^2 - 7x + 1 - 7x^2 + 2x + 3$

d) $4 - 5x - 3x^2$

e) $2x^2 - 3x + 5$

f) $3x + 2x^2 + 1 - 2x^2 + 2x$

16. Write 3 different polynomials that simplify to $-2a^2 + 4a - 8$.

17. Write a polynomial with degree 2 and 5 terms, which has only 2 terms when it is simplified.

18. **Assessment Focus**

a) A student is not sure whether $x + x$ simplifies to $2x$ or x^2.
Explain how the student can use algebra tiles to determine the correct answer.
What is the correct answer?

b) Simplify each polynomial. How do you know that your answers are correct?

  i) $-2 + 4r - 2r + 3$

  ii) $2t^2 - 3t + 4t^2 - 6t$

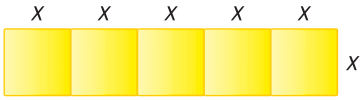
  iii) $3c^2 + 4c + 2 + c^2 + 2c + 1$

  iv) $15x^2 - 12xy + 5y + 10xy - 8y - 9x^2$

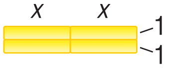
c) Create a polynomial that cannot be simplified. Explain why it cannot be simplified.

19. Write a polynomial to represent the perimeter of each rectangle.

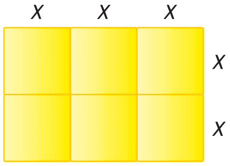
a)



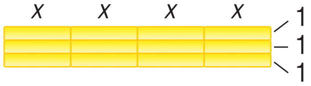
b)



c)



d)



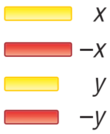
20. Each polynomial below represents the perimeter of a rectangle. Use algebra tiles to make the rectangle. Sketch the tiles. How many different rectangles can you make each time?

a) $6c + 4$ **b)** $4d$ **c)** $8 + 2m$

d) $12r$ **e)** $6s$ **f)** $4a + 10$

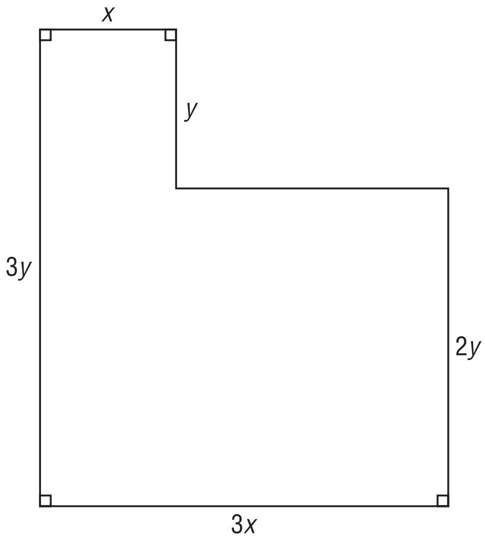
Take It Further

21. Many algebra tile kits contain x-tiles and y-tiles.



What do you think an xy-tile looks like? Sketch your idea and justify your picture.

22. Write a polynomial for the perimeter of this shape. Simplify the polynomial.



Reflect

Explain how like terms can be used to simplify a polynomial. Use diagrams and examples in your explanation.

Math Link

Your World

On a forward somersault dive, a diver's height above the water, in metres, in terms of the time t seconds after the diver leaves the board may be modelled by the polynomial $-4.9t^2 + 6t + 3$.

5.3 Adding Polynomials

FOCUS

- Use different strategies to add polynomials.

Investigate

2

You will need algebra tiles and a paper bag.
Conduct the activity 3 times.

Put both colours of algebra tiles in a paper bag.
Each person removes a handful of algebra tiles and writes the simplified polynomial that the tiles model.
Add the two polynomials.
Record your work as an addition sentence.

Reflect & Share

Compare your strategies for adding two polynomials with those of another pair of students.
If you used different strategies, explain your strategies.
If you used the same strategies, find a pair of students who used a different strategy.
Which terms can be combined when you add polynomials?
Why can these terms be combined?

Connect

To add polynomials, we combine the algebra tiles that represent each polynomial and record the process symbolically. This develops a strategy to add polynomials without algebra tiles.

When we write the sum of two polynomials, we write each polynomial in brackets.

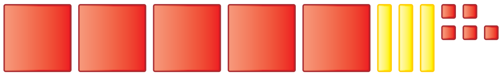
To determine the sum of $3x^2 + 2x + 4$ and $-5x^2 + 3x - 5$, we write:

$(3x^2 + 2x + 4) + (-5x^2 + 3x - 5)$

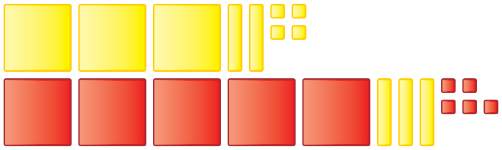
Tile Model

Display: $3x^2 + 2x + 4$

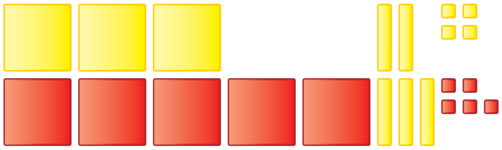
Display: $-5x^2 + 3x - 5$



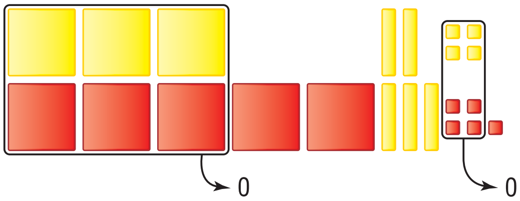
Combine the displays.



Group like tiles.



Remove zero pairs.



The remaining tiles represent $-2x^2 + 5x - 1$.

Symbolic Record

The sum is:

$(3x^2 + 2x + 4) + (-5x^2 + 3x - 5)$

This is written as:

$3x^2 + 2x + 4 - 5x^2 + 3x - 5$

Group like terms:

$3x^2 - 5x^2 + 2x + 3x + 4 - 5$

Combine like terms:

$-2x^2 + 5x - 1$

Example 1 Adding Polynomials Symbolically

Add: $(7s + 14) + (-6s^2 + s - 6)$

Solutions

Add the polynomials by adding the coefficients of like terms. In the second polynomial, the term s has coefficient 1, so write s as $1s$.

Method 1

Add horizontally.

$(7s + 14) + (-6s^2 + 1s - 6)$	Remove the brackets.
$= 7s + 14 - 6s^2 + 1s - 6$	Group like terms.
$= -6s^2 + 7s + 1s + 14 - 6$	Combine like terms by adding their coefficients.
$= -6s^2 + 8s + 8$	

Method 2

Add vertically. Align like terms, then add their coefficients.

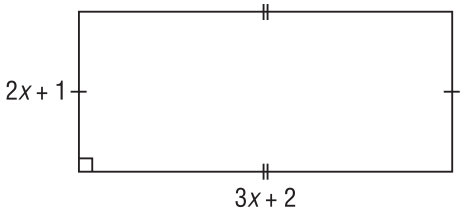
$$\begin{array}{rrrr} & & 7s & + 14 \\ + & -6s^2 & + 1s & - \quad 6 \\ \hline & -6s^2 & + 8s & + \quad 8 \end{array}$$

So, $(7s + 14) + (-6s^2 + 1s - 6) = -6s^2 + 8s + 8$

Example 2 Determining a Polynomial for the Perimeter of a Rectangle

a) Write a polynomial for the perimeter of this rectangle.
Simplify the polynomial.

b) Substitute to check the answer.



A Solution

a) The perimeter is the sum of the measures of the four sides.

$$\begin{array}{rr} & 2x + 1 \\ + & 2x + 1 \\ + & 3x + 2 \\ + & 3x + 2 \\ \hline & 10x + 6 \end{array}$$

The perimeter is $10x + 6$.

b) Choose a value for x, such as $x = 1$.

Write the addition sentence:

$2x + 1 + 2x + 1 + 3x + 2 + 3x + 2 = 10x + 6$

Substitute $x = 1$.

Left side:

$$\begin{aligned} & 2x + 1 + 2x + 1 + 3x + 2 + 3x + 2 \\ &= 2(1) + 1 + 2(1) + 1 + 3(1) + 2 + 3(1) + 2 \\ &= 2 + 1 + 2 + 1 + 3 + 2 + 3 + 2 \\ &= 16 \end{aligned}$$

Right side:

$$\begin{aligned} 10x + 6 &= 10(1) + 6 \\ &= 10 + 6 \\ &= 16 \end{aligned}$$

Since the left side equals the right side, the polynomial for the perimeter is correct.

Example 3 Adding Polynomials in Two Variables

Add: $(2a^2 + a - 3b - 7ab + 3b^2) + (-4b^2 + 3ab + 6b - 5a + 5a^2)$

A Solution

$(2a^2 + a - 3b - 7ab + 3b^2) + (-4b^2 + 3ab + 6b - 5a + 5a^2)$	Remove brackets.
$= 2a^2 + a - 3b - 7ab + 3b^2 - 4b^2 + 3ab + 6b - 5a + 5a^2$	Group like terms.
$= 2a^2 + 5a^2 + a - 5a - 3b + 6b - 7ab + 3ab + 3b^2 - 4b^2$	Combine like terms.
$= 7a^2 - 4a + 3b - 4ab - b^2$	

Discuss the ideas

1. How can you use what you know about adding integers to add polynomials?

2. How is adding polynomials like simplifying a polynomial?

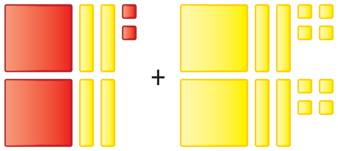
Practice

Check

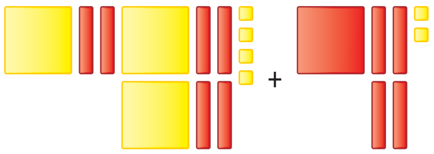
3. Write the polynomial sum modelled by each set of tiles.

a)

b)



c)



4. Explain how to use algebra tiles to determine $(3x^2 + 2) + (x^2 - 1)$. What is the sum?

5. Use algebra tiles to model each sum of binomials. Record your answer symbolically.

a) $(5g + 3) + (2g + 4)$
b) $(3 - 2j) + (-4 + 2j)$
c) $(p + 1) + (5p - 6)$
d) $(7 + 4m) + (-5m + 4)$

6. Add these polynomials. Visualize algebra tiles if it helps.

a)
$$\begin{array}{r} 2x + 4 \\ + \underline{3x - 5} \end{array}$$

b)
$$\begin{array}{r} 3x^2 + 5x \\ + \underline{-2x^2 - 8x} \end{array}$$

c)
$$\begin{array}{r} 3x^2 + 5x + 7 \\ + \underline{-8x^2 - 3x + 5} \end{array}$$

7. Do you prefer to add vertically or horizontally? Give reasons for your choice.

Apply

8. Use a personal strategy to add.

a) $(6x + 3) + (3x + 4)$
b) $(5b - 4) + (2b + 9)$
c) $(6 - 3y) + (-3 - 2y)$
d) $(-n + 7) + (3n - 2)$
e) $(-4s - 5) + (6 - 3s)$
f) $(1 - 7h) + (-7h - 1)$
g) $(8m + 4) + (-9 + 3m)$
h) $(-8m - 4) + (9 - 3m)$

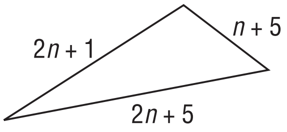
9. Add. Which strategy did you use each time?

a) $(4m^2 + 4m - 5) + (2m^2 - 2m + 1)$
b) $(3k^2 - 3k + 2) + (-3k^2 - 3k + 2)$
c) $(-7p - 3) + (p^2 + 5)$
d) $(9 - 3t) + (9t + 3t^2 - 6t)$
e) $(3x^2 - 2x + 3) + (2x^2 + 4)$
f) $(3x^2 - 7x + 5) + (6x - 6x^2 + 8)$
g) $(6 - 7x + x^2) + (6x - 6x^2 + 10)$
h) $(1 - 3r + r^2) + (4r + 5 - 3r^2)$

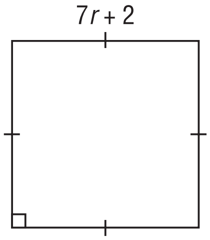
10. a) For each shape below, write the perimeter:

- as a sum of polynomials
- in simplest form

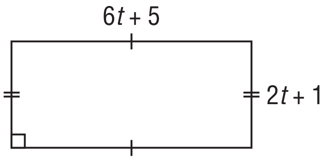
i)



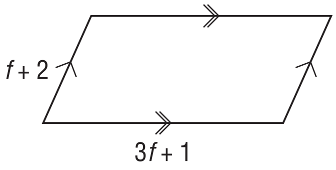
ii)



iii)



iv)

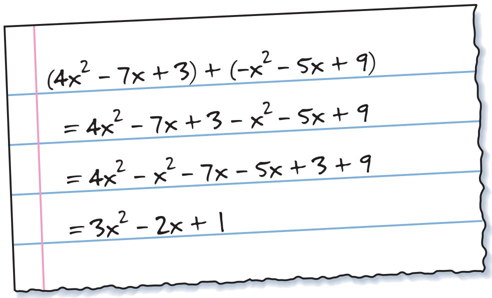


b) Use substitution to check each answer in part a.

11. Sketch 2 different shapes whose perimeter could be represented by each polynomial.

a) $8 + 6r$
b) $3s + 9$
c) $4 + 12t$
d) $20u$
e) $7 + 5v$
f) $4y + 6$
g) $9 + 9c$
h) $15m$

12. A student added $(4x^2 - 7x + 3)$ and $(-x^2 - 5x + 9)$ as follows.



$(4x^2 - 7x + 3) + (-x^2 - 5x + 9)$

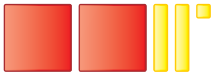
$= 4x^2 - 7x + 3 - x^2 - 5x + 9$

$= 4x^2 - x^2 - 7x - 5x + 3 + 9$

$= 3x^2 - 2x + 1$

Is the student's work correct?
If not, explain where the student made any errors and write the correct answer.

13. Assessment Focus
These tiles represent the sum of two polynomials.



a) What might the two polynomials be?
Explain how you found out.

b) How many different pairs of polynomials can you find? List all the pairs you found.

14. The sum of two polynomials is $12m^2 + 2m + 4$.
One polynomial is $4m^2 - 6m + 8$.
What is the other polynomial?
Explain how you found your answer.

15. Create a polynomial that is added to $3x^2 + 7x + 2$ to get each sum.

a) $5x^2 + 10x + 1$
b) $2x^2 + 5x + 8$
c) $4x^2 + 3x$
d) $-x^2 + x - 1$
e) $2x + 3$
f) 4

16. a) What polynomial must be added to $5x^2 + 3x - 1$ to obtain a sum of 0?
Justify your answer.

b) How are the coefficients of the two polynomials related?
Will this relationship be true for all polynomials with a sum of 0? Explain.

17. Add.

a) $(3x^2 - 2y^2 + xy) + (-2xy - 2y^2 - 3x^2)$
b) $(-5q^2 + 3p - 2q + p^2) + (4p + q + pq)$
c) $(3mn + m^2 - 3n^2 + 5m) + (7n^2 - 8n + 10)$
d) $(3 - 8f + 5g - f^2) + (2g^2 - 3f + 4g - 5)$

Take It Further

18. a) The polynomials $4x - 3y$ and $2x + y$ represent the lengths of two sides of a triangle. The perimeter of the triangle is $9x + 2$. Determine the length of the third side.

b) Use substitution to check your solution in part a.

19. The polynomial $5y + 3x + 7$ represents the perimeter of an isosceles triangle.
Write three polynomials that could represent the side lengths of the triangle.
Find as many answers as you can.

Reflect

What strategies can you use for adding polynomials?
Which strategy do you prefer?
How can you check that your answers are correct?
Include examples in your explanation.

5.4 Subtracting Polynomials

FOCUS

- Use different strategies to subtract polynomials.

What strategies do you know to subtract two integers, such as $-2 - 3$? How could these strategies help you subtract two polynomials?

Investigate

2

Use algebra tiles.

➤ Write two like monomials.
Subtract the monomials.
Write the subtraction sentence.
Subtract the monomials in the reverse order.
Write the new subtraction sentence.
Sketch the tiles you used.

➤ Repeat the process above for two binomials, then for two trinomials.

➤ Subtract. Use a strategy of your choice.
$(5x) - (3x)$
$(2x^2 + 3x) - (4x^2 - 6x)$
$(3x^2 - 6x + 4) - (x^2 + 3x - 2)$
Use a different strategy to verify your answer.

Reflect & Share

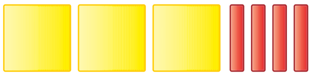
Compare your answers and strategies with those of a pair of students who used a different strategy.
Explain your strategies to each other.
Work together to write an addition sentence that corresponds to each subtraction sentence.

Connect

Here are two strategies to subtract polynomials.

- ➤ Using algebra tiles

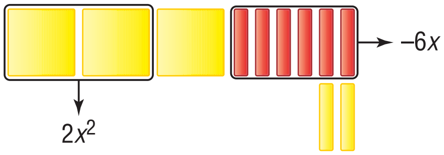
To subtract: $(3x^2 - 4x) - (2x^2 - 6x)$
Use algebra tiles to model $3x^2 - 4x$.



To subtract $2x^2 - 6x$, we need to:

- Take away two x^2-tiles from three x^2-tiles.
- Take away six $-x$-tiles from four $-x$-tiles.
  To do this, we need 2 more $-x$-tiles.
  So, we add 2 zero pairs of x-tiles.

Now we can take away the tiles for $2x^2 - 6x$.



The remaining tiles represent $x^2 + 2x$.
So, $(3x^2 - 4x) - (2x^2 - 6x) = x^2 + 2x$

- ➤ Using the properties of integers

We know that −6 is the opposite of 6.
Subtracting −6 from an integer is the same as adding 6 to that integer.
The same process is true for like terms.

To subtract: $(3x^2 - 4x) - (2x^2 - 6x)$

$$\begin{aligned}(3x^2 - 4x) - (2x^2 - 6x) &= 3x^2 - 4x - (2x^2) - (-6x) && \text{Subtract each term.}\\ &= 3x^2 - 4x - 2x^2 - (-6x) && \text{Add the opposite term.}\\ &= 3x^2 - 4x - 2x^2 + 6x && \text{Collect like terms.}\\ &= 3x^2 - 2x^2 - 4x + 6x && \text{Combine like terms.}\\ &= x^2 + 2x\end{aligned}$$

Example 1 Subtracting Two Trinomials

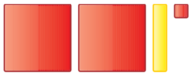
Subtract: $(-2a^2 + a - 1) - (a^2 - 3a + 2)$

Solutions

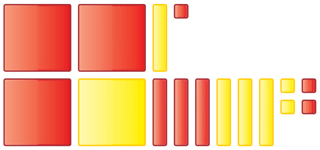
$(-2a^2 + a - 1) - (a^2 - 3a + 2)$

Method 1

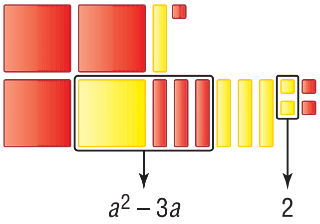
Use algebra tiles.
Display: $-2a^2 + a - 1$



To subtract a^2, add a zero pair of a^2-tiles.
To subtract $-3a$, add 3 zero pairs of a-tiles.
To subtract 2, add 2 zero pairs of 1-tiles.



Now remove tiles for $a^2 - 3a + 2$.



The remaining tiles represent $-3a^2 + 4a - 3$.

Method 2

Use the properties of integers.

$$\begin{aligned}
&(-2a^2 + a - 1) - (a^2 - 3a + 2)\\
&= -2a^2 + a - 1 - (a^2) - (-3a) - (+2)\\
&= -2a^2 + a - 1 - a^2 + 3a - 2\\
&= -2a^2 - a^2 + a + 3a - 1 - 2\\
&= -3a^2 + 4a - 3
\end{aligned}$$

To check the difference when two numbers are subtracted,
we add the difference to the number that was subtracted;
for example, to check that $23 - 5 = 18$ is correct, we add: $5 + 18 = 23$

We can use the same process to check the difference of two polynomials.

Example 2 Subtracting Trinomials in Two Variables

Subtract: $(5x^2 - 3xy + 2y^2) - (8x^2 - 7xy - 4y^2)$
Check the answer.

A Solution

$$\begin{aligned}(5x^2 - 3xy + 2y^2) - (8x^2 - 7xy - 4y^2) &= 5x^2 - 3xy + 2y^2 - (8x^2) - (-7xy) - (-4y^2)\\ &= 5x^2 - 3xy + 2y^2 - 8x^2 + 7xy + 4y^2\\ &= 5x^2 - 8x^2 - 3xy + 7xy + 2y^2 + 4y^2\\ &= -3x^2 + 4xy + 6y^2\end{aligned}$$

To check, add the difference to the second polynomial:

$$\begin{aligned}(-3x^2 + 4xy + 6y^2) + (8x^2 - 7xy - 4y^2) &= -3x^2 + 4xy + 6y^2 + 8x^2 - 7xy - 4y^2\\ &= -3x^2 + 8x^2 + 4xy - 7xy + 6y^2 - 4y^2\\ &= 5x^2 - 3xy + 2y^2\end{aligned}$$

The sum is equal to the first polynomial.
So, the difference is correct.

Discuss the ideas

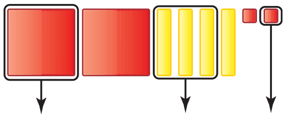
1. How is subtracting polynomials like subtracting integers?
2. How is subtracting polynomials like adding polynomials? How is it different?
3. When might using algebra tiles not be the best method to subtract polynomials?

Practice

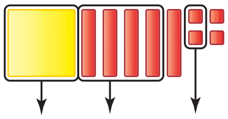
Check

4. Write the subtraction sentence that these algebra tiles represent.

a)



b)



5. Use algebra tiles to subtract.
Sketch the tiles you used.
   - a) $(5r) - (3r)$
   - b) $(5r) - (-3r)$
   - c) $(-5r) - (3r)$
   - d) $(-5r) - (-3r)$
   - e) $(3r) - (5r)$
   - f) $(-3r) - (5r)$
   - g) $(3r) - (-5r)$
   - h) $(-3r) - (-5r)$

Apply

6. Use algebra tiles to model each difference of binomials. Record your answer symbolically.
   - a) $(5x + 3) - (3x + 2)$
   - b) $(5x + 3) - (3x - 2)$
   - c) $(5x + 3) - (-3x + 2)$
   - d) $(5x + 3) - (-3x - 2)$

7. Use algebra tiles to model each difference of trinomials. Record your answer symbolically.

a) $(3s^2 + 2s + 4) - (2s^2 + s + 1)$
b) $(3s^2 - 2s + 4) - (2s^2 - s + 1)$
c) $(3s^2 - 2s - 4) - (-2s^2 + s - 1)$
d) $(-3s^2 + 2s - 4) - (2s^2 - s - 1)$

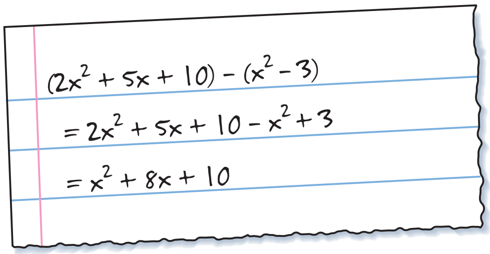
8. Use a personal strategy to subtract. Check your answers by adding.

a) $(3x + 7) - (-2x - 2)$
b) $(b^2 + 4b) - (-3b^2 + 7b)$
c) $(-3x + 5) - (4x + 3)$
d) $(4 - 5p) - (-7p + 3)$
e) $(6x^2 + 7x + 9) - (4x^2 + 3x + 1)$
f) $(12m^2 - 4m + 7) - (8m^2 + 3m - 3)$
g) $(-4x^2 - 3x - 11) - (x^2 - 4x - 15)$
h) $(1 - 3r + r^2) - (4r + 5 - 3r^2)$

9. The polynomial $4n + 2500$ represents the cost, in dollars, to produce n copies of a magazine in colour. The polynomial $2n + 2100$ represents the cost, in dollars, to produce n copies of the magazine in black-and-white.

a) Write a polynomial for the difference in the costs of the two types of magazines.
b) Suppose the company wants to print 3000 magazines. How much more does it cost to produce the magazine in colour instead of black-and-white?

10. A student subtracted $(2x^2 + 5x + 10) - (x^2 - 3)$ like this:

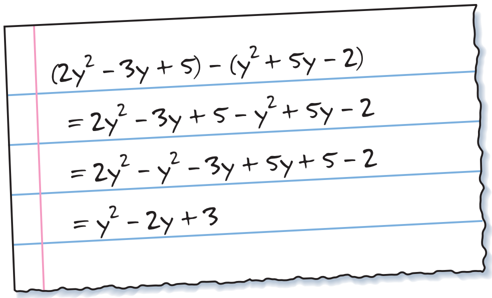


$(2x^2 + 5x + 10) - (x^2 - 3)$
$= 2x^2 + 5x + 10 - x^2 + 3$
$= x^2 + 8x + 10$

a) Use substitution to show that the answer is incorrect.
b) Identify the errors and correct them.

11. **Assessment Focus** Create a polynomial subtraction question. Answer your question. Check your answer. Show your work.

12. A student subtracted like this:

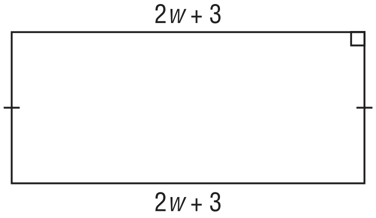


$(2y^2 - 3y + 5) - (y^2 + 5y - 2)$
$= 2y^2 - 3y + 5 - y^2 + 5y - 2$
$= 2y^2 - y^2 - 3y + 5y + 5 - 2$
$= y^2 - 2y + 3$

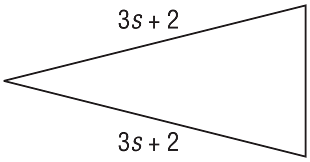
a) Explain why the solution is incorrect.
b) What is the correct answer? Show your work.
c) How could you check that your answer is correct?
d) What could the student do to avoid making the same mistakes in the future?

13. The perimeter of each polygon is given. Determine each unknown length.

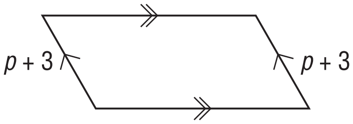
a) $6w + 14$



b) $7s + 7$



c) $10p + 8$



14. a) Write two polynomials, then subtract them.

b) Subtract the polynomials in part a in the reverse order.

c) How do the answers in parts a and b compare? Why are the answers related this way?

15. Subtract.

a) $(r^2 - 3rs + 5s^2) - (-2r^2 - 3rs - 5s^2)$

b) $(-3m^2 + 4mn - n^2) - (5m^2 + 7mn + 2n^2)$

c) $(5cd + 8c^2 - 7d^2) - (3d^2 + 6cd - 4c^2)$

d) $(9e + 9f - 3e^2 + 4f^2) -$
$(-f^2 - 2e^2 + 3f - 6e)$

e) $(4jk - 7j - 2k + k^2) - (2j^2 + 3j - jk)$

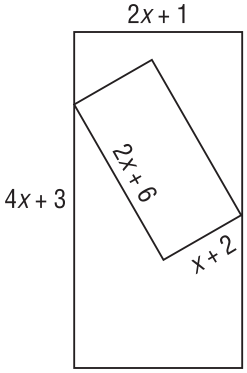
16. The difference of two polynomials is $3x^2 + 4x - 7$.
One polynomial is $-8x^2 + 5x - 4$.

a) What is the other polynomial?

b) Why are there two possible answers to part a?

Take It Further

17. The diagram shows one rectangle inside another rectangle. What is the difference in the perimeters of the rectangles?



18. One polynomial is subtracted from another. The difference is $-4x^2 + 2x - 5$.
Write two polynomials that have this difference. How many different pairs of polynomials can you find? Explain.

Reflect

What strategy or strategies do you use to subtract polynomials?
Why do you prefer this strategy or strategies?

Math Link

Your World

On a suspension bridge, the roadway is hung from huge cables passing through the tops of high towers. Here is a photograph of the Lions Gate Bridge in Vancouver. The position of any point on the cable can be described by its horizontal and vertical distance from the centre of the bridge. The vertical distance in metres is modelled by the polynomial $0.0006x^2$, where x is the horizontal distance in metres.

Mid-Unit Review

5.1

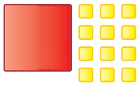
1. In each polynomial, identify:
the variable, number of terms, coefficients, constant term, and degree.

a) $3m - 5$
b) $4r$
c) $x^2 + 4x + 1$

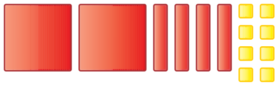
2. Create a polynomial that meets these conditions:
trinomial in variable m, degree 2, constant term is -5

3. Which polynomial is represented by each set of algebra tiles? Is the polynomial a monomial, binomial, or trinomial? How do you know?

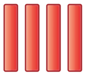
a)



b)



c)



4. Use algebra tiles to represent each polynomial. Sketch the tiles you used.
a) $4n - 2$
b) $-t^2 + 4t$
c) $2d^2 + 3d + 2$

5.2

5. For each pair of monomials, which are like terms? Explain how you know.
a) $2x, -5x$
b) $3, 4g$
c) $10, 2$
d) $2q^2, -7q^2$
e) $8x^2, 3x$
f) $-5x, -5x^2$

6. Simplify $3x^2 - 7 + 3 - 5x^2 - 3x + 5$.
Explain how you did this.

7. Renata simplified a polynomial and got $4x^2 + 2x - 7$. Her friend simplified the same polynomial and got $-7 + 4x^2 + 2x$. Renata thinks her friend's answer is wrong. Do you agree? Explain.

8. Cooper thinks that $5x - 2$ simplifies to $3x$. Is he correct? Explain.
Use algebra tiles to support your explanation.

9. Identify the equivalent polynomials.
Justify your answers.
a) $1 + 3x - x^2$
b) $1 + 3x^2 - x^2 + 2x - 2x^2 + x - 2$
c) $x^2 - 3x - 1$
d) $6 + 6x - 6x^2 - 4x - 5 + 2x^2 + x^2 - 4$
e) $3x - 1$
f) $-3x^2 + 2x - 3$
g) $6x^2 - 6x - 6 + x - 5x^2 - 1 + 2x + 4$
h) $3x - x^2 + 1$

5.3
5.4

10. Use algebra tiles to add or subtract.
Sketch the tiles you used.
a) $(4f^2 - 4f) + (-2f^2)$
b) $(3r^2 + 2r + 5) + (-7r^2 + r - 3)$
c) $(-2v + 5) - (-9v + 3)$
d) $(-2g^2 - 12) - (-6g^2 + 4g - 1)$

11. Add or subtract. Use a strategy of your choice.
a) $(3w^2 + 17w) + (12w^2 - 3w)$
b) $(5m^2 - 3) + (m^2 + 3)$
c) $(-3h - 12) - (-9h - 6)$
d) $(6a^2 + 2a - 2) + (-7a^2 + 4a + 11)$
e) $(3y^2 + 9y + 7) - (2y^2 - 4y + 13)$
f) $(-14 + 3p^2 + 2p) - (-5p + 10 - 7p^2)$

12. **a)** Which polynomial must be added to $5x^2 + 3x - 2$ to get $7x^2 + 5x + 1$?
b) Which polynomial must be subtracted from $5x^2 + 3x - 2$ to get $7x^2 + 5x + 1$?
Justify your answers.

Start Where You Are

How Can I Summarize What I Have Learned?

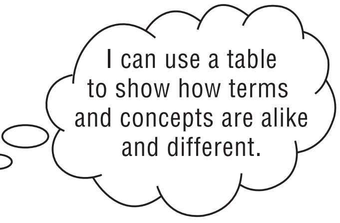
Suppose I want to summarize what I know about polynomials.

➤ What tools could I use to do this?

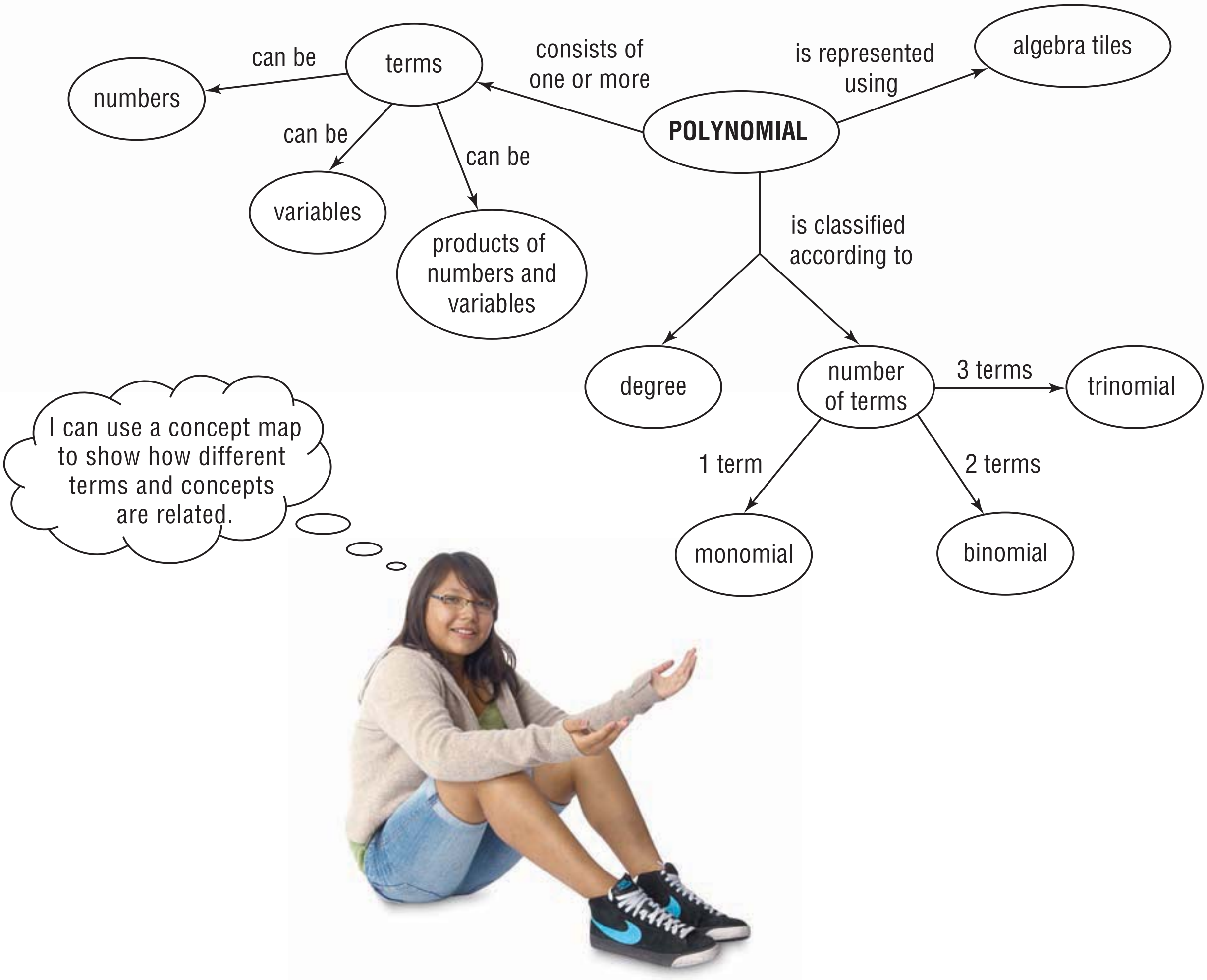
- a Frayer model
- a table
- a concept map

Definition Like terms have the same variable raised to the same exponent.	**Facts/Characteristics** Like terms are represented by algebra tiles with the same size and shape. I can combine like terms by adding their coefficients.
Examples $-3x$ and $4x$ $5b^2$ and $2b^2$	**Non-examples** $-3c$ and 4 $5n^2$ and $2n$

Like terms



Polynomial	Number of Terms	Name by Number of Terms	Degree
12	1	monomial	0
$8a$	1	monomial	1
$-4b^2 + 9$	2	binomial	2
$2c - 7$	2	binomial	1
$3d^2 - 4d + 6$	3	trinomial	2



Check

Use the tools *you* find most helpful to summarize the important ideas and concepts you have learned about polynomials.

1. Choose another term or concept. Make a Frayer model to show what you know about that term or concept.

2. What other types of polynomials could you include in the table on page 238?

3. a) What could you add to the concept map above?
   b) Think of another way to draw a concept map about polynomials.

Add to your Frayer model, table, or concept map as you work through this unit.

GAME

Investigating Polynomials that Generate Prime Numbers

A prime number is any whole number, greater than 1, that is divisible by only itself and 1.

In 1772, Leonhard Euler, a Swiss mathematician, determined that the polynomial $n^2 - n + 41$ generates prime numbers for different values of n.
Use a calculator to check that this is true:

- Choose a value of n between 1 and 10. Substitute this number for n in the polynomial. Is the number you get a prime number? How do you know?
- Repeat the process for other values of n between 1 and 10.
- Choose a value of n between 10 and 40. Substitute this number for n in the polynomial. Is the number you get a prime number? How do you know?
- Repeat the process for other values of n between 10 and 40.
- Substitute $n = 41$. Is the number you get a prime number? How can you tell?
- List the values of n and the resulting primes in a table.

In 1879, E. B. Escott, an American mathematician, determined the polynomial $n^2 - 79n + 1601$ for generating prime numbers.
Test this polynomial:

- Substitute different values of n, and check that the numbers you get are prime. List the values of n and the resulting primes in a table. What patterns do you see?
- Substitute $n = 80$. Did you get a prime number? Explain.
- Determine other values of n for which Escott's polynomial does *not* generate prime numbers.

Currently, there is no known polynomial that generates only prime numbers. And, there is no known polynomial that generates all the prime numbers.

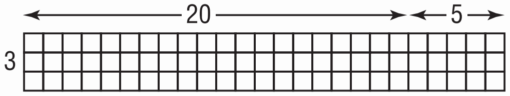
- Determine a value of n for which each of these polynomials does *not* generate a prime number:
    - $n^2 - n + 41$, other than $n = 41$
    - $n^2 - n + 17$
    - $n^2 + n - 1$

5.5 Multiplying and Dividing a Polynomial by a Constant

FOCUS

- Use different strategies to multiply and divide a polynomial by a constant.

How does this diagram model the product 3 × 25?



What property is illustrated by this diagram?
How could you use the diagram above to model division?

Investigate

2

Use any strategy or materials you wish.

➤ Determine each product. Write a multiplication sentence.

- $2(3x)$
- $3(2x + 1)$
- $2(2x^2 + x + 4)$
- $-2(3x)$
- $-3(2x + 1)$
- $-2(2x^2 + x + 4)$

➤ Determine each quotient. Write a division statement.

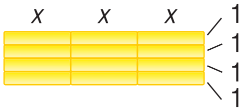
- $9x \div 3$
- $(8x + 12) \div 4$
- $(5x^2 + 10x + 20) \div 5$
- $9x \div (-3)$
- $(8x + 12) \div (-4)$
- $(5x^2 + 10x + 20) \div (-5)$

Reflect & Share

Compare your answers and strategies with those of another pair of students.
If your answers are different, find out why.
Look at your multiplication and division sentences.
What relationships do you see among the original terms and the answers?
How could you use these relationships to multiply and divide without using algebra tiles?

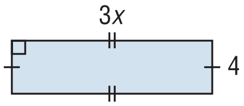
Connect

➤ The expression $4(3x)$ is a product statement.
It represents the product of the constant, 4, and the monomial, $3x$.
We can model the product as 4 rows of three x-tiles.



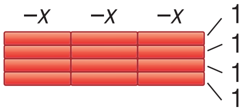
So, $4(3x) = 3x + 3x + 3x + 3x$ This is repeated addition.
$= 12x$

We can also model $4(3x)$ as the area of a rectangle with dimensions 4 and $3x$.



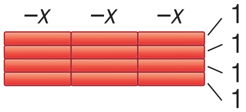
So, $4(3x) = 4(3)(x)$
$= 12x$

➤ $4(-3x)$ is the product of 4 and the monomial $-3x$.
We can model the product as 4 rows of three $-x$-tiles.



So, $4(-3x) = -3x - 3x - 3x - 3x$
$= -12x$

➤ $-4(3x)$ is the opposite of $4(3x)$.
We can model this by flipping the tiles we used to model $4(3x)$.



So, $-4(3x) = -(12x)$
$= -12x$

We can use the same strategy with algebra tiles to multiply a binomial or a trinomial by a constant. To determine the product symbolically, we use the *distributive property*.

Example 1 Multiplying a Binomial and a Trinomial by a Constant

Determine each product.

a) $3(-2m + 4)$

b) $-2(-n^2 + 2n - 1)$

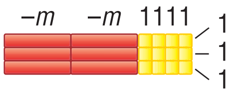
Solutions

Method 1

Use algebra tiles.

a) $3(-2m + 4)$

Display 3 rows of two $-m$-tiles and four 1-tiles.

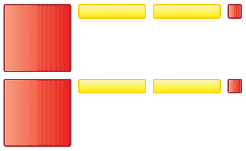


There are six $-m$-tiles and twelve 1-tiles.

So, $3(-2m + 4) = -6m + 12$

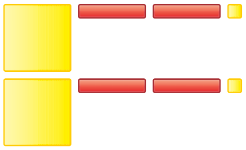
b) $-2(-n^2 + 2n - 1)$

Display 2 rows of one $-n^2$-tile, two n-tiles, and one –1-tile.



This shows $2(-n^2 + 2n - 1)$.

Flip all the tiles.



There are two n^2-tiles, four $-n$-tiles, and two 1-tiles.

So, $-2(-n^2 + 2n - 1) = 2n^2 - 4n + 2$

Method 2

Use the distributive property.

Multiply each term in the brackets by the term outside the brackets.

a) $3(-2m + 4) = 3(-2m) + 3(4)$

$= -6m + 12$

b) $-2(-n^2 + 2n - 1)$

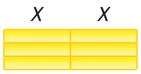
$= (-2)(-1n^2) + (-2)(2n) + (-2)(-1)$

$= 2n^2 + (-4n) + 2$

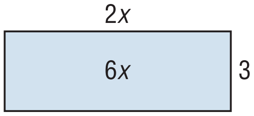
$= 2n^2 - 4n + 2$

Multiplication and division are inverse operations. To divide a polynomial by a constant, we reverse the process of multiplication.

➤ The expression $6x \div 3$ is a division statement.
It represents the quotient of the monomial, $6x$, and the constant 3.
To model $6x \div 3$,
we arrange six x-tiles in 3 rows.
Each row contains two x-tiles.
So, $6x \div 3 = 2x$



We can also model $6x \div 3$ as
one dimension of a rectangle with
an area of $6x$ and the other dimension 3.
Then, $6x \div 3 = \frac{6x}{3}$
$= 2x$

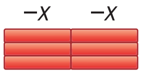


We can use what we know about division as a fraction and integer division to determine the quotient.

$$\begin{aligned}\frac{6x}{3} &= \frac{6}{3} \times x\\ &= 2 \times x\\ &= 2x\end{aligned}$$

➤ $(-6x) \div 3$ is the quotient of the monomial, $-6x$, and the constant 3.

Using a model:
We arrange six $-x$-tiles in 3 rows.



Each row contains two $-x$-tiles.
So, $(-6x) \div 3 = -2x$

Using fractions and integers:
$(-6x) \div 3 = \frac{-6x}{3}$
Simplify the fraction.

$$\begin{aligned}(-6x) \div 3 &= \frac{-6}{3} \times x\\ &= -2 \times x\\ &= -2x\end{aligned}$$

➤ $6x \div (-3)$ is the quotient of the monomial, $6x$, and the constant -3.
Using fractions and integers:
$6x \div (-3) = \frac{6x}{-3}$
Simplify the fraction.

$$\begin{aligned}6x \div (-3) &= \frac{6}{-3} \times x\\ &= -2 \times x\\ &= -2x\end{aligned}$$

Example 2 Dividing a Binomial and a Trinomial by a Constant

Determine each quotient.

a) $\frac{4s^2 - 8}{4}$ **b)** $\frac{-3m^2 + 15mn - 21n^2}{-3}$

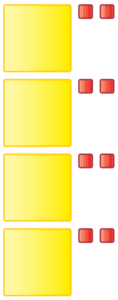
Solutions

Method 1

a) $\frac{4s^2 - 8}{4}$

Use algebra tiles.

Arrange four s^2-tiles and eight -1-tiles in 4 equal rows.



In each row, there is one s^2-tile and two -1-tiles.

So, $\frac{4s^2 - 8}{4} = s^2 - 2$

b) $\frac{-3m^2 + 15mn - 21n^2}{-3}$

Think multiplication.

What do we multiply -3 by to get $-3m^2 + 15mn - 21n^2$?

$(-3) \times ? = -3m^2 + 15mn - 21n^2$

Since $(-3) \times 1 = -3$,

then $(-3) \times (1m^2) = -3m^2$

Since $(-3) \times (-5) = 15$,

then $(-3) \times (-5mn) = +15mn$

Since $(-3) \times 7 = -21$,

then $(-3) \times (+7n^2) = -21n^2$

So, $\frac{-3m^2 + 15mn - 21n^2}{-3} = m^2 - 5mn + 7n^2$

Method 2

a) $\frac{4s^2 - 8}{4}$

Write the quotient expression as the sum of 2 fractions.

$$\frac{4s^2 - 8}{4} = \frac{4s^2}{4} + \frac{-8}{4}$$

Simplify each fraction.

$$= \frac{4}{4} \times s^2 + (-2)$$
$$= 1 \times s^2 - 2$$
$$= s^2 - 2$$

b) $\frac{-3m^2 + 15mn - 21n^2}{-3}$

Write the quotient expression as the sum of 3 fractions.

$$\frac{-3m^2 + 15mn - 21n^2}{-3}$$
$$= \frac{-3m^2}{-3} + \frac{15mn}{-3} + \frac{-21n^2}{-3}$$

Simplify each fraction.

$$= m^2 + (-5mn) + (7n^2)$$
$$= m^2 - 5mn + 7n^2$$

Discuss the ideas

1. How could you use multiplication to verify the quotient in a division question?

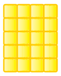
2. Why can we not use algebra tiles to divide when the divisor is negative?

Practice

Check

3. Write the multiplication sentence modelled by each set of algebra tiles.

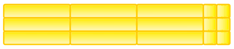
a)



b)

c)

d)



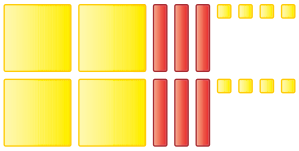
4. For each set of algebra tiles in question 3, write a division sentence.

5. a) Which of these products is modelled by the algebra tiles below?

i) $2(-2n^2 + 3n + 4)$

ii) $2(2n^2 - 3n + 4)$

iii) $-2(2n^2 - 3n + 4)$



b) In part a, two of the products were not modelled by the algebra tiles. Model each product. Sketch the tiles you used.

6. Which of these quotients is modelled by the algebra tiles below?

a) $\frac{8t - 12}{-4}$

b) $\frac{-8t - 12}{4}$

c) $\frac{8t - 12}{4}$

Apply

7. a) Multiply.

i) $3(5r)$ **ii)** $-3(5r)$

iii) $(5r)(3)$ **iv)** $-5(3r)$

v) $-5(-3r)$ **vi)** $(-3r)(5)$

b) In part a, explain why some answers are the same.

c) For which products in part a could you have used algebra tiles? For each product, sketch the tiles you could use.

8. a) Divide.

i) $\frac{12k}{4}$ **ii)** $(-12k) \div 4$

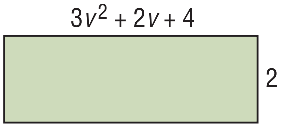
iii) $\frac{12k}{-4}$ **iv)** $(-12k) \div (-4)$

b) In part a, explain why some answers are the same.

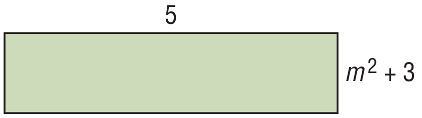
c) For which quotients in part a could you have used algebra tiles? For each quotient, sketch the tiles you could use.

9. Write the multiplication sentence modelled by each rectangle.

a)



b)



10. For each rectangle in question 9, write a division sentence.

11. Use algebra tiles to determine each product. Sketch the tiles you used. Record the product symbolically.

a) $7(3s + 1)$

b) $-2(-7h + 4)$

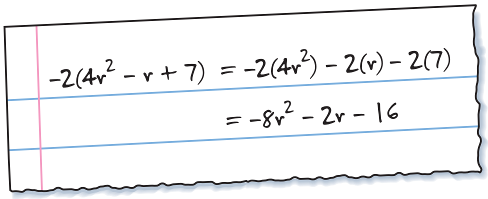
c) $2(-3p^2 - 2p + 1)$

d) $-6(2v^2 - v + 5)$

e) $(-w^2 + 3w - 5)(3)$

f) $(x^2 + x)(-5)$

12. Here is a student's solution for this question:



$-2(4v^2 - v + 7) = -2(4v^2) - 2(v) - 2(7)$

$= -8v^2 - 2v - 16$

Identify the errors in the solution, then write the correct solution.

13. Use algebra tiles to determine each quotient. Sketch the tiles you used. Record the product symbolically.

a) $\frac{12p - 18}{6}$

b) $\frac{-6q^2 - 10}{2}$

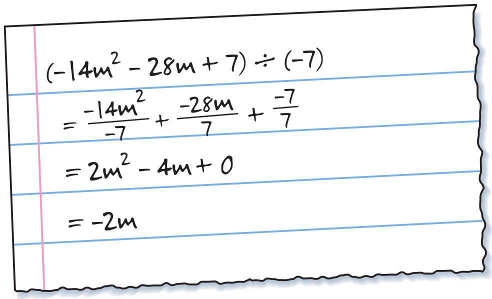
c) $\frac{5h^2 - 20h}{5}$

d) $\frac{4r^2 - 16r + 6}{2}$

e) $\frac{-8a^2 + 4a - 12}{4}$

f) $\frac{6x^2 + 3x + 9}{3}$

14. Here is a student's solution for this question:
Divide: $(-14m^2 - 28m + 7) \div (-7)$



$(-14m^2 - 28m + 7) \div (-7)$

$= \frac{-14m^2}{-7} + \frac{-28m}{7} + \frac{-7}{7}$

$= 2m^2 - 4m + 0$

$= -2m$

Identify the errors in the solution, then write the correct solution.

15. Use any strategy to determine each product.

a) $-3(-4u^2 + 16u + 8)$

b) $12(2m^2 - 3m)$

c) $(5t^2 + 2t)(-4)$

d) $(-6s^2 - 5s - 7)(-5)$

e) $4(-7y^2 + 3y - 9)$

f) $10(8n^2 - n - 6)$

16. Use any strategy to determine each quotient.

a) $\frac{24d^2 - 12}{12}$

b) $\frac{8x + 4}{4}$

c) $\frac{-10 + 4m^2}{-2}$

d) $(25 - 5n) \div (-5)$

e) $(-14k^2 + 28k - 49) \div 7$

f) $\frac{30 - 36d^2 + 18d}{-6}$

g) $\frac{-26c^2 + 39c - 13}{-13}$

17. Which pairs of expressions are equivalent? Explain how you know.

a) $5j^2 + 4$ and $5(j + 4)$

b) $10x^2$ and $3x(x + 7)$

c) $15x - 10$ and $5(-2 + 3x)$

d) $-3(-4x - 1)$ and $12x^2 - 3x$

e) $-5(3x^2 - 7x + 2)$ and $-15x^2 + 12x - 10$

f) $2x(-3x - 7)$ and $-6x^2 - 14x$

18. Assessment Focus

a) Determine each product or quotient.

i) $(3p)(4)$ ii) $\frac{-21x}{3}$

iii) $(3m^2 - 7)(-4)$

iv) $\frac{-2f^2 + 14f - 8}{2}$

v) $(6y^2 - 36y) \div (-6)$

vi) $(-8n + 2 - 3n^2)(3)$

b) List the products and quotients in part a that can be modelled with algebra tiles. Justify your selection.

c) Sketch the tiles for one product and one quotient in part a.

19. a) Determine each product.

i) $2(2x + 1)$ ii) $2(1 - 2x)$

$3(2x + 1)$ $3(1 - 2x)$

$4(2x + 1)$ $4(1 - 2x)$

$5(2x + 1)$ $5(1 - 2x)$

b) Describe the patterns in part a.

c) Predict the next 3 products in each list in part a. How do you know the products are correct?

d) Suppose you extended the lists in part a upward. Predict the preceding 3 products in each list.

20. a) The perimeter of an equilateral triangle is represented by the polynomial $15a^2 + 21a + 6$.
Determine the polynomial that represents the length of one side.

b) Determine the length of one side when $a = 4$ cm.

21. Square A has side length $4s + 1$. Square B has a side length that is 3 times as great as the side length of square A.

a) What is the perimeter of each square? Justify your answers.

b) Write a polynomial, in simplest form, to represent the difference in the perimeters of squares A and B.

22. Determine each product.

a) $2(2x^2 - 3xy + 7y^2)$

b) $-4(pq + 3p^2 + 3q^2)$

c) $(-2gh + 6h^2 - 3g^2 - 9g)(3)$

d) $5(-r^2 + 8rs - 3s^2 - 5s + 4r)$

e) $-2(4t^2 - 3v^2 + 19tv - 6v - t)$

23. Determine each quotient.

a) $(3n^2 - 12mn + 6m^2) \div 3$

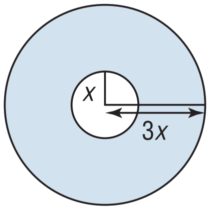
b) $\frac{-6rs - 16r - 4s}{-2}$

c) $\frac{10gh - 30g^2 - 15h}{5}$

d) $(12t^2 - 24ut - 48t) \div (-6)$

Take It Further

24. The area of a circle is given by the monomial πr^2.
Write, then simplify a polynomial for the shaded area in this diagram:



Reflect

How are multiplying and dividing a polynomial by a constant related?
Use examples to explain.

5.6 Multiplying and Dividing a Polynomial by a Monomial

FOCUS

- Use different strategies to multiply and divide a polynomial by a monomial.

You can use the strategies you know for multiplying and dividing a polynomial by a constant to multiply and divide a polynomial by a monomial.

Investigate

2

You may need algebra tiles.

➤ Determine each product.
Use a strategy of your choice.
Write a multiplication sentence.

- $2a(5a)$
- $4b(3b - 2)$
- $-3c(-5c + 1)$

➤ Determine each quotient.
Use a strategy of your choice.
Write a division sentence.

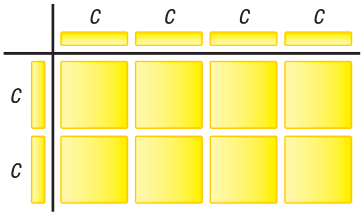
- $\dfrac{3g^2 + 9g}{3g}$
- $\dfrac{-18f^2 + 12f}{6f}$
- $\dfrac{24d^2 + 8d}{-4d}$

Reflect & Share

Compare your answers and strategies with those of another pair of students.
If you have different answers, find out why.
If you used different strategies, explain your strategies and choice of strategies.
How can you use multiplication to check your quotients?

Connect

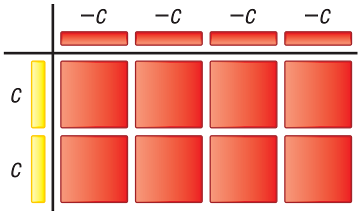
- The expression $(2c)(4c)$ is the product of two monomials.
  We interpret the product with algebra tiles arranged to form a rectangle with dimensions $2c$ and $4c$.



To help build the rectangle, we place guiding tiles to represent each dimension. Then we fill in the rectangle with tiles.

  We need eight c^2-tiles to build the rectangle.
  So, $(2c)(4c) = 8c^2$

- The expression $(2c)(-4c)$ is the product of a positive and a negative monomial.
  We form a rectangle with guiding tiles: two c-tiles along one dimension and four $-c$-tiles along the other dimension.
  We know that the product of a positive number and a negative number is negative.
  So, when we fill in the rectangle, we use $-c^2$-tiles.



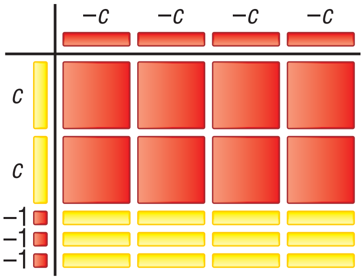
  We need eight $-c^2$-tiles to build this rectangle.
  So, $(2c)(-4c) = -8c^2$

We use similar strategies to multiply a binomial by a monomial.

- The expression $-4c(2c - 3)$ is the product of a monomial and a binomial.
  We form a rectangle with guiding tiles:
  - four $-c$-tiles along one dimension; and
  - two c-tiles and three -1-tiles along the other dimension

  The product of two numbers with opposite signs is negative.
  So, when we place a tile in a row and column headed by guiding tiles with opposite signs, the tile is negative.

The product of two numbers with the same sign is positive.
So, when we place a tile in a row and column headed by guiding tiles with the same sign, the tile is positive.



There are eight $-c^2$-tiles and twelve c-tiles.
So, $-4c(2c - 3) = -8c^2 + 12c$

Example 1 Multiplying a Binomial by a Monomial

Determine each product.

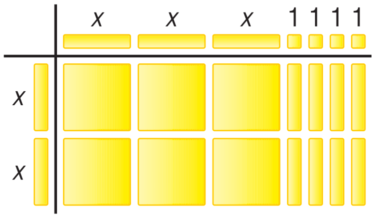
a) $2x(3x + 4)$ **b)** $-2x(-3x + 4)$

Solutions

Method 1

a) $2x(3x + 4)$

Use algebra tiles to make a rectangle with dimensions $2x$ and $3x + 4$.

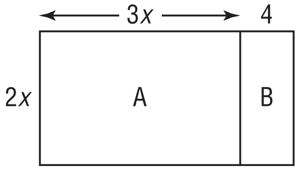


Six x^2-tiles and eight x-tiles fill the rectangle.
So, $2x(3x + 4) = 6x^2 + 8x$

Method 2

a) $2x(3x + 4)$

Use an area model.
Sketch a rectangle with dimensions $2x$ and $3x + 4$.
Divide the rectangle into 2 smaller rectangles.



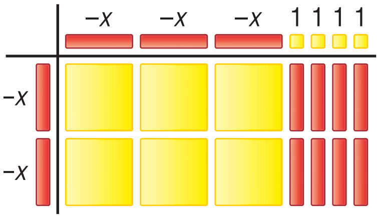
Rectangle A has area: $2x(3x) = 6x^2$
Rectangle B has area: $2x(4) = 8x$
The total area is: $6x^2 + 8x$
So, $2x(3x + 4) = 6x^2 + 8x$

b) $-2x(-3x + 4)$

Use algebra tiles.

Form a rectangle with guiding tiles:

- two $-x$-tiles along one dimension; and
- three $-x$-tiles and four 1-tiles along the other dimension



Six x^2-tiles and eight $-x$-tiles fill the rectangle.

So, $-2x(-3x + 4) = 6x^2 - 8x$

b) $-2x(-3x + 4)$

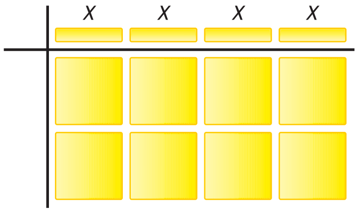
Use the distributive property.

Multiply each term in the brackets by the term outside the brackets.

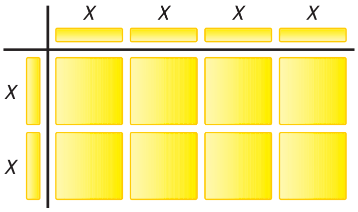
$$\begin{aligned}-2x(-3x + 4) &= -2x(-3x) + (-2x)(4)\\ &= 6x^2 - 8x\end{aligned}$$

To divide a polynomial by a monomial, we reverse the process of multiplying these polynomials.

➤ To determine the quotient of $\frac{8x^2}{4x}$, arrange eight x^2-tiles in a rectangle with one dimension $4x$.



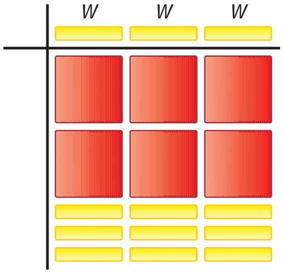
Along the left side of the rectangle, the guiding tiles are x-tiles.



There are 2 guiding x-tiles.

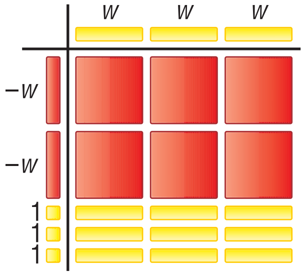
So, $\frac{8x^2}{4x} = 2x$

- To determine the quotient of $\frac{-6w^2 + 9w}{3w}$, arrange six $-w^2$-tiles and nine w-tiles in a rectangle with one dimension $3w$.



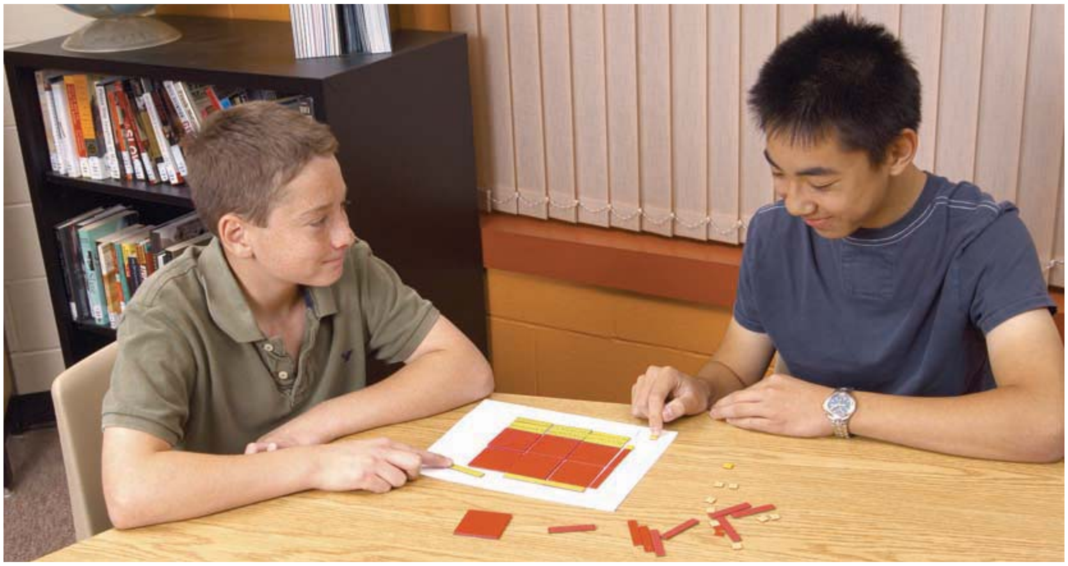
Along the left side of the rectangle:

- the guiding w-tiles are negative because they must have the sign opposite to that of the guiding tiles along the top of the rectangle
- the guiding 1-tiles are positive because they must have the same sign as the guiding tiles along the top of the rectangle



There are 2 guiding $-w$-tiles and 3 guiding 1-tiles.

So, $\frac{-6w^2 + 9w}{3w} = -2w + 3$



Example 2 Dividing a Monomial and a Binomial by a Monomial

Determine each quotient.

a) $\frac{-10m^2}{2m}$ **b)** $\frac{30k^2 - 18k}{-6k}$

Solutions

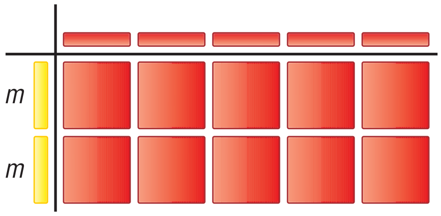
Method 1

a) $\frac{-10m^2}{2m}$

Use algebra tiles.

Arrange ten $-m^2$-tiles in a rectangle with one dimension $2m$.

The guiding tiles along the other dimension represent $-5m$.



So, $\frac{-10m^2}{2m} = -5m$

b) $\frac{30k^2 - 18k}{-6k}$

Think multiplication.

$-6k \times ? = 30k^2 - 18k$

Since $-6k \times (-5k) = 30k^2$ and

$-6k \times (+3) = -18k$

Then $-6k \times (-5k + 3) = 30k^2 - 18k$

So, $\frac{30k^2 - 18k}{-6k} = -5k + 3$

Method 2

a) $\frac{-10m^2}{2m}$

Think multiplication.

$2m \times ? = -10m^2$

Since $2 \times (-5) = -10$ and $m \times m = m^2$

Then $2m \times (-5m) = -10m^2$

So, $\frac{-10m^2}{2m} = -5m$

b) $\frac{30k^2 - 18k}{-6k}$

Write the quotient expression as the sum of two fractions.

$$\frac{30k^2 - 18k}{-6k} = \frac{30k^2}{-6k} + \frac{-18k}{-6k}$$

Simplify each fraction.

$$\begin{aligned}\frac{30k^2 - 18k}{-6k} &= \frac{30}{-6} \times \frac{k^2}{k} + \frac{-18}{-6} \times \frac{k}{k} \\ &= (-5) \times k + 3 \times 1 \\ &= -5k + 3\end{aligned}$$

Discuss the ideas

1. Why can we not use repeated addition to model the product $(2c)(4c)$?

2. Why can we not use an area model to multiply when there are negative terms in the product statement?

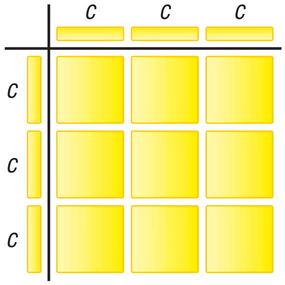
3. How could we check that a quotient is correct?

Practice

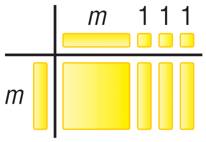
Check

4. Write the multiplication sentence modelled by each set of algebra tiles.

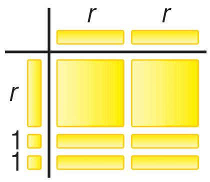
a)



b)



c)



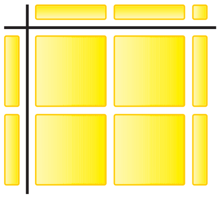
5. For each set of algebra tiles in question 4, write a division sentence.

6. Which of these multiplication sentences is modelled by the algebra tiles below?

a) $2n(n + 2)$

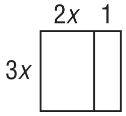
b) $2(2n^2 + 1)$

c) $2n(2n + 1)$

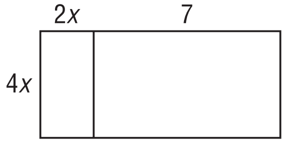


7. Write the multiplication sentence modelled by each rectangle.

a)



b)



8. For each rectangle in question 7, write a division sentence.

Apply

9. a) Multiply.

i) $(3m)(4m)$ ii) $(-3m)(4m)$

iii) $(3m)(-4m)$ iv) $(-3m)(-4m)$

v) $(4m)(3m)$ vi) $(4m)(-3m)$

b) In part a, explain why there are only two answers.

c) For which products in part a could you have used algebra tiles? For each product, sketch the tiles you could use.

10. a) Divide.

i) $\frac{12x}{2x}$ ii) $\frac{12x}{-2x}$

iii) $\frac{-12x}{2x}$ iv) $\frac{-12x}{-2x}$

v) $\frac{12x^2}{2x}$ vi) $\frac{12x^2}{2x^2}$

vii) $\frac{-12x^2}{2x^2}$ viii) $\frac{12x^2}{-2x^2}$

b) In part a, explain why some quotients are equal.

c) For which quotients in part a could you have used algebra tiles? For each quotient, sketch the tiles you could use.

11. Multiply or divide as indicated.

a) $(2r)(-6r)$

b) $(-16n^2) \div (-8n)$

c) $(-5g)(7g)$

d) $\frac{40k}{-10k}$

e) $(9h)(3h)$

f) $\frac{48p^2}{12p}$

g) $18u^2 \div (-3u^2)$

h) $\frac{-24d^2}{-8d^2}$

12. Use any strategy to determine each product.

a) $2x(x + 6)$

b) $3t(5t + 2)$

c) $-2w(3w - 5)$

d) $-x(2 + 8x)$

e) $3g(-5 - g)$

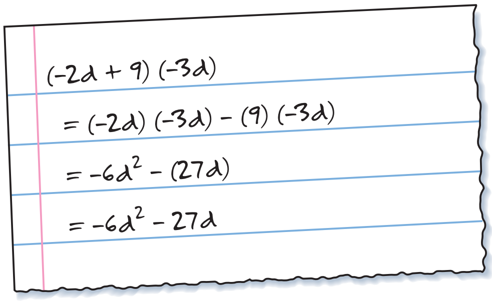
f) $(4 + 3y)(2y)$

g) $(-7s - 1)(-y)$

h) $(-3 + 6r)(2r)$

13. A student thinks that the product $2x(x + 1)$ is $2x^2 + 1$. Choose a model. Use the model to explain how to get the correct answer.

14. Here is a student's solution for this question:
Multiply: $(-2d + 9)(-3d)$



$(-2d + 9)\ (-3d)$

$= (-2d)\ (-3d) - (9)\ (-3d)$

$= -6d^2 - (27d)$

$= -6d^2 - 27d$

Identify the errors in the solution, then write the correct solution.

15. a) Describe two different strategies to simplify $\frac{3r^2 - 12r}{3r}$.

b) Which strategy do you find easier? Explain.

16. Use any strategy to determine each quotient.

a) $\frac{10x^2 + 4x}{2x}$

b) $(6x^2 + 4x) \div x$

c) $\frac{6y + 3y^2}{3y}$

d) $\frac{40x^2 - 16x}{8x}$

e) $\frac{15g - 10g^2}{5g}$

f) $\frac{-12k - 24k^2}{3k}$

g) $(24h^2 + 36h) \div (-4h)$

h) $(-8m^2 + 18m) \div (-2m)$

17. Assessment Focus

a) Determine each product or quotient. Use a different strategy each time.

i) $\frac{15n^2 + 5n}{5n}$

ii) $-3r(4 - 7r)$

iii) $(-16s^2 + 4s) \div (-2s)$

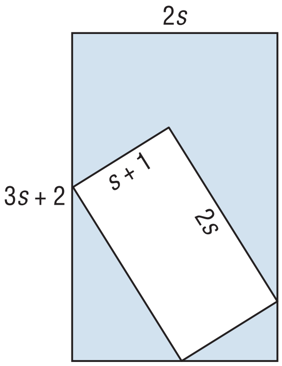
iv) $(t - 9)(4t)$

b) Choose one product and one quotient in part a. Use a different strategy to solve each problem. In each case, which strategy do you prefer? Explain why.

18. a) Use algebra tiles to model the quotient $\frac{12x^2 + 12x}{2x}$. Determine the quotient.

b) The polynomial $12x^2 + 12x$ can be represented by the areas of rectangles with different dimensions. Sketch and label the dimensions for as many different rectangles as you can. For each rectangle, write a division statement.

19. a) Write a polynomial to represent the area of each rectangle in the diagram below.



b) Determine a polynomial for the shaded area. Justify your strategy.

c) Determine the area in part b when $s = 2.5$ cm.

20. Determine each product.

a) $3m(2n + 4)$

b) $(-5 + 3f)(-2g)$

c) $7m(-6p + 7m)$

d) $(-8h - 3k)(4k)$

e) $(-2t + 3r)(4t)$

f) $(-g)(8h - 5g)$

21. Determine each quotient.

a) $(12x^2 + 6xy) \div 3x$

b) $\dfrac{12gh + 6g}{2g}$

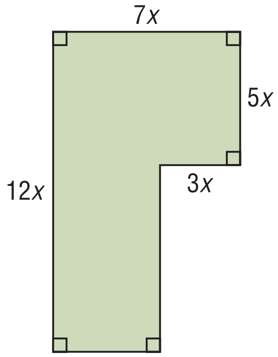
c) $(-27p^2 + 36pq) \div 9p$

d) $\dfrac{40rs - 35r}{-5r}$

e) $\dfrac{14n^2 + 42np}{-7n}$

Take It Further

22. Determine a polynomial for the area of this shape. Justify your answer.



23. a) The polynomial $54s^2$ represents the surface area of a cube. Determine a polynomial that represents the area of one face.

b) Use your answer to part a. Determine the length of an edge of the cube.

24. The product $2\pi r(r + h)$ represents the surface area of a cylinder.

a) Determine the product.

b) To check your work, determine the surface area of a cylinder with radius 5 cm and height 3 cm two ways:

- using the product
- using your answer to part a

25. Simplify:
$[(2x^2 - 8x + 3xy + 5) +$
$(24x^2 - 16x - 12xy)] \div 4x$

Reflect

Explain how the strategies for dividing a polynomial by a monomial are related to the strategies for multiplying a polynomial by a monomial. Include examples in your explanation.

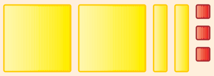
Study Guide

Polynomials

- A polynomial is one term or the sum of terms whose variables have whole-number exponents; for example, $2m^2 + 3m - 5$
- The numerical value of a term is its coefficient.
- A term that consists of only a number is a constant term.
- The degree of a polynomial in the variable m is the highest power of m in the polynomial.
- A polynomial with: 1 term is a monomial; 2 terms is a binomial; and 3 terms is a trinomial.

Algebra Tiles

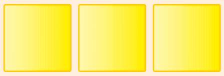
We can represent a polynomial with algebra tiles. $2p^2 + 2p - 3$



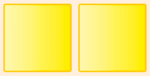
Like Terms

Like terms are represented by the same type of algebra tile. In symbolic form, like terms have the same variables raised to the same exponent. Like terms can be added or subtracted. $3x^2$ and $2x^2$ are like terms, but $-x$ and 3 are not.

$3x^2$:



$2x^2$:



$3x^2 + 2x^2$ simplifies to $5x^2$.

$-x$:

3:

$-x + 3$ cannot be simplified.

Operations with Polynomials

We can use algebra tiles to model operations with polynomials, then record the answers symbolically.

- To add polynomials, combine like terms:

$$\begin{aligned}(3r^2 + 5r) + (2r^2 - r) &= 3r^2 + 5r + 2r^2 - r \\ &= 5r^2 + 4r\end{aligned}$$

- To subtract polynomials, use a strategy for subtracting integers:

$$\begin{aligned}(3r^2 + 5r) - (2r^2 - r) &= 3r^2 + 5r - (2r^2) - (-r) \\ &= 3r^2 - 2r^2 + 5r + r \\ &= r^2 + 6r\end{aligned}$$

- To multiply a polynomial by a monomial, multiply each term of the polynomial by the monomial:

$$\begin{aligned}2t(5t - 3) &= 2t(5t) + 2t(-3) \\ &= 10t^2 - 6t\end{aligned}$$

- To divide a polynomial by a monomial, divide each term of the polynomial by the monomial:

$$\begin{aligned}\frac{21x^2 - 14x}{7x} &= \frac{21x^2}{7x} - \frac{14x}{7x} \\ &= 3x - 2\end{aligned}$$

Review

5.1

1. Use algebra tiles to model each polynomial. Sketch the tiles you used.

a) $2u^2 + 5u$ **b)** $4n^2 - 2n - 3$

2. Identify the variables, coefficients, and constant terms in each polynomial.

a) $4w - 3$ **b)** $5v^2 + 3$ **c)** $5y - 6 - y^2$

3. Classify each polynomial below:

- **i)** according to the number of terms
- **ii)** according to its degree

a) $3f + 5$ **b)** $-2g^2$ **c)** $5h - 6 - h^2$

4. Use algebra tiles to model the polynomial that fits each description. Sketch the tiles you used.

a) a second-degree trinomial in the variable y, the coefficients of the variable when the polynomial is written in descending order are -1 and -3, and with constant term 4

b) a first-degree binomial in the variable x, with constant term 4, and the coefficient of the other term is -3

5. Identify the equivalent polynomials. Explain how you know they are equivalent.

a) $-3x^2 + 3x - 11$ **b)** $3x^2 + 4x$

c) $-2 - x$ **d)** $7 + 5x$

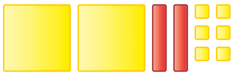
e) $5x + 7$ **f)** $x - 2$

g) $4x + 3x^2$ **h)** $3x - 11 - 3x^2$

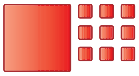
6. Which polynomial is modelled by each set of algebra tiles? State the degree of the polynomial.

a)

b)



c)



7. Jennie does not understand how the terms $2k$ and k^2 are different. Use algebra tiles to model these terms and explain the difference.

8. For each polynomial, write an equivalent polynomial.

a) $-1 - 2h$ **b)** $3j + 2j^2 - 4$ **c)** $-5p + p^2$

5.2

9. Identify like terms.

a) $5x^2, 3y^2, -2x^2, 5x, 2y$

b) $-8x, 5x, 8, -2, -x, 11$

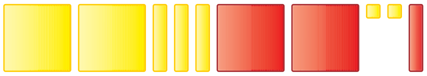
10. Match each algebra tile model below with its corresponding polynomial.

a) $n^2 - n + 3$ **b)** $-w^2 - 3$

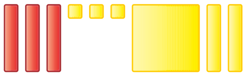
c) $-2t$ **d)** $2q + 2$

e) $2r^2 - 2r + 1$

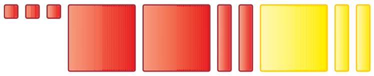
A



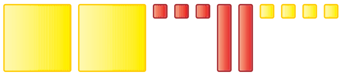
B



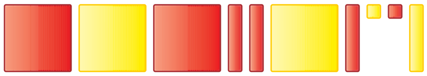
C



D



E



11. Write an expression with 5 terms that has only 3 terms when simplified.

12. Simplify by combining like terms.

a) $3x + 4 - 2x - 8 + 3x - 3$

b) $4y^2 - 2y + 3y - 11y^2$

c) $2a^2 + 7a - 3 - 2a^2 - 4a + 6$

d) $2a^2 + 3a + 3a^2 - a^2 - a - 4a^2$

5.3
5.4

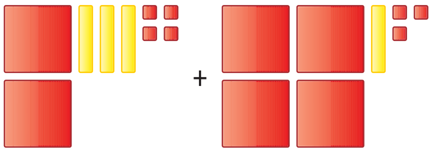
13. Students who have trouble with algebra often make these mistakes.

- They think: $x + x = x^2$
- They think: $(x)(x) = 2x$

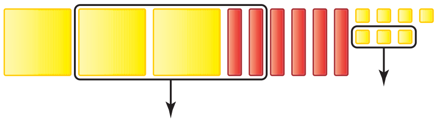
Use algebra tiles to explain the correct answers.

14. Write the polynomial sum or difference modelled by each set of tiles. Determine the sum or difference.

a)



b)



15. Add or subtract as indicated.

a) $(p^2 + 3p + 5) + (3p^2 + p + 1)$

b) $(3q^2 + 3q + 7) - (2q^2 + q + 2)$

c) $(6 - 3r + 7r^2) - (9 + 4r + 3r^2)$

d) $(5s + 3 - s^2) + (5 + 3s - 2s^2)$

e) $(-4t^2 - 3t + 9) - (-2t^2 - 5t - 1)$

f) $(-9u^2 - 5) - (-3u^2 - 9)$

g) $(3a^2 + 5ab - 7b^2) + (3b^2 - 10ab - 7a^2)$

h) $(10xy - 3y^2 + 2x) - (5y - 4x^2 + xy)$

16. The sum of two polynomials is $15c + 6$. One polynomial is $3c - 7$. What is the other polynomial? Explain how you found it.

17. Match each sum or difference of polynomials with its answer. Justify your choices.

A	$(5x^2 - 2) + (2x^2 + 4)$	P	$4x^2 + 2x - 1$
B	$(x^2 - 3x) - (4x^2 - x)$	Q	$7x^2 + 2$
C	$(x^2 + 2x + 3) + (3x^2 - 4)$	R	$x^2 + 2x - 1$
D	$(3x^2 - x + 2) - (2x^2 - 3x + 3)$	S	$-3x^2 - 2x$
E	$(-3x - 2) - (3x - 2)$	T	$-6x$

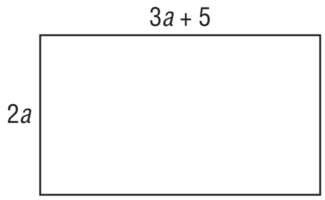
18. The difference of two polynomials is $3d^2 - 7d + 4$.
One polynomial is $-8d^2 - 5d + 1$.

a) What is the other polynomial? Explain how you found it.

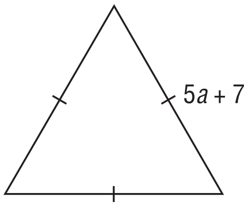
b) How many different answers can you find?

19. Write a polynomial for the perimeter of each shape. Simplify the polynomial. Determine each perimeter when $a = 3$ cm.

a)



b)



5.5

20. Write the multiplication sentence modelled by each set of algebra tiles.

a)

b)

21. For each set of algebra tiles in question 20, write a division sentence.

22. Determine each product or quotient. Use any strategy you wish.

a) $10k \div 2$ **b)** $5(-4x^2)$

c) $2(-3m + 4)$ **d)** $\frac{-6n^2}{3}$

e) $-3(4s - 1)$ **f)** $\frac{9 - 12m}{3}$

g) $5(-7 + 2x)$

h) $-2(1 - 2n + 3n^2)$

i) $2(x + 3x^2)$

j) $(-6p^2 - 6p + 4) \div (-2)$

k) $\frac{15 - 21q + 6q^2}{-3}$

l) $(2 + 5n - 7n^2)(-6)$

23. Determine each product or quotient.

a) $(xy - x^2 + y^2)(-2)$

b) $(12m^2 - 6n + 8m) \div (-2)$

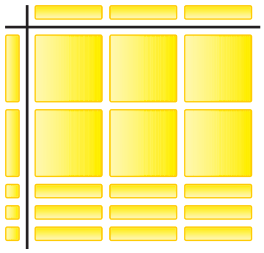
c) $\frac{-18pq + 3p^2 - 9q}{3}$

d) $4(2r^2 - 3r + 4s - 5s^2)$

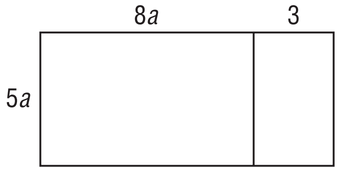
5.6

24. Write the multiplication sentence modelled by each diagram.

a)



b)



25. Write a division sentence for each diagram in question 24.

26. Determine each product.

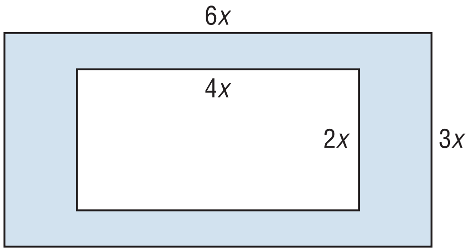
a) $(7s)(2s)$ **b)** $(-3g)(-5g)$

c) $m(3m + 2)$ **d)** $-5t(t - 3)$

e) $7z(-4z - 1)$ **f)** $(-3f - 5)(-2f)$

g) $-5k(3 - k)$ **h)** $y(1 - y)$

27. This diagram shows one rectangle inside another.



a) Determine the area of each rectangle.

b) Determine the area of the shaded region. Explain your strategy.

28. Determine each quotient.

a) $24j \div (-6j)$ **b)** $\frac{24x}{3x}$

c) $\frac{-36x^2}{-9x}$ **d)** $(-8a^2 - 12a) \div 4a$

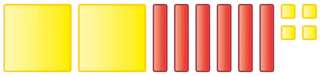
e) $(-8c + 4c^2) \div 4c$ **f)** $\frac{14y^2 - 21y}{-7y}$

29. a) The area of a rectangular deck is $(8d^2 + 20d)$ square metres. The deck is $4d$ metres long. Determine a polynomial that represents the width of the deck.

b) What are the dimensions and area of the deck when d is 4 metres?

Practice Test

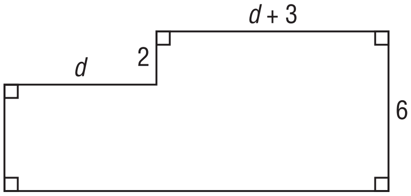
1. **a)** Which polynomial in t do these tiles represent?



b) Classify the polynomial by degree and by the number of terms.

c) Identify the constant term and the coefficient of the t^2-term.

2. **a)** Write a polynomial for the perimeter of this shape. Simplify the polynomial.



b) Determine the perimeter of the shape when $d = 5$ m.

3. Sketch algebra tiles to explain why:

a) $3x + 2x$ equals $5x$ **b)** $(3x)(2x)$ equals $6x^2$

4. A student determined the product $3r(r + 4)$.
The student's answer was $3r^2 + 4$.
Use a model to explain whether the student's answer is correct.

5. Add or subtract as indicated. What strategy will you use each time?

a) $(15 - 3d) + (3 - 15d)$ **b)** $(9h + 3) - (9 - 3h^2)$

c) $(2y^2 + 5y - 6) + (-7y^2 + 2y - 6)$ **d)** $(7y^2 + y) - (3y - y^2)$

6. Multiply or divide as indicated. What strategy will you use each time?

a) $25m(3m - 2)$ **b)** $-5(3v^2 - 2v - 1)$

c) $(8x^2 - 4x) \div 2x$ **d)** $\dfrac{-6 + 3g^2 - 15g}{-3}$

7. Determine two polynomials with:

a) a sum of $3x^2 - 4x - 2$

b) a difference of $3x^2 - 4x - 2$

8. A rectangle has dimensions $5s$ and $3s + 8$.

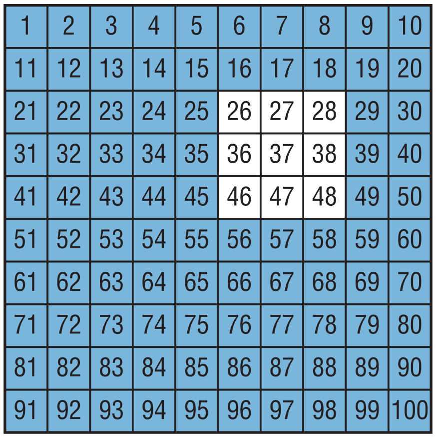
a) Sketch the rectangle and label it with its dimensions.

b) What is the area of the rectangle?

c) What is the perimeter of the rectangle?

Unit Problem

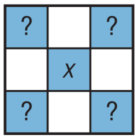
Algebra Patterns on a 100-Chart



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

You will need a copy of a 100-chart.

- Choose any 3 by 3 square of numbers on the chart. Add the numbers in each diagonal. What do you notice?
- Choose a different 3 by 3 square. Add the numbers in each diagonal. How do your results compare?
- Determine a relationship between the number at the centre of any 3 by 3 square and the sum of the numbers in a diagonal.
- Let x represent the number at the centre of any 3 by 3 square. Write a polynomial, in terms of x, for each number at the four corners of the square.



?		?
	x	
?		?

- Add the polynomials in each diagonal. What is the sum? How does this explain the relationship you found earlier?
- Suppose you know the sum of the numbers in a diagonal of a 3 by 3 square. How could you determine the number at the centre of the square?
- What do you think is the relationship between the number at the centre of a 5 by 5 square and the sum of the numbers in a diagonal? What about a 7 by 7 square? Make a prediction, then use polynomials to check.

Your work should show:

- each 3 by 3 square and the related calculations
- a relationship between the number at the centre and the sum
- how this relationship changes as the size of the square changes

Reflect on Your Learning

What did you find easy about polynomials? What did you find difficult? What strategies might you use to overcome these difficulties?

UNIT 6

Linear Equations and Inequalities

The Pep Club promotes school spirit at athletic events and school activities. The members of the club need new uniforms. They are thinking of selling healthy snacks at lunch time to raise the money needed. What information does the Pep Club need to gather? What math might the members use?

What You'll Learn

- Model and solve problems using linear equations.
- Explain and illustrate strategies to solve linear inequalities.

Why It's Important

Linear equations and inequalities occur in everyday situations involving ratios and rates, geometry formulas, scientific contexts, and financial applications. Using an equation or inequality to solve a problem is an important problem-solving strategy.

Key Words

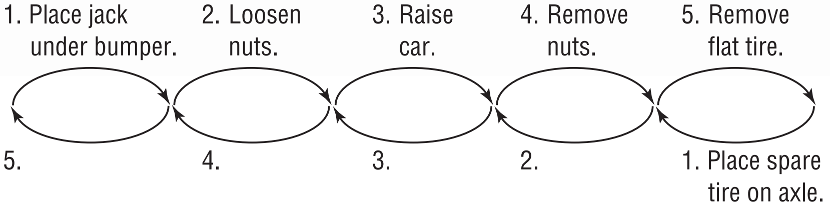
- inverse operations
- inequality

6.1 Solving Equations by Using Inverse Operations

FOCUS

- Model a problem with a linear equation, use an arrow diagram to solve the equation pictorially, and record the process symbolically.

The top row of the arrow diagram shows the steps to remove a flat tire on a car. What steps are needed to put on a new tire?
How are these steps related to the steps to remove the flat tire?



Investigate

➤ This arrow diagram shows the operations applied to the start equation $x = -7$ to build the end equation $3x + 8 = -13$.

Start equation

$$\begin{tikzcd} \boxed{x=-7} \arrow[r, shift left, "\text{Step 1. Multiply by 3.}"] & \boxed{3x=-21} \arrow[l, shift left, "\text{Step 2.}"] \arrow[r, shift left, "\text{Step 2. Add 8.}"] & \boxed{3x+8=-13} \arrow[l, shift left, "\text{Step 1.}"] \end{tikzcd}$$

End equation

Copy and complete the diagram. What are Steps 1 and 2 in the bottom row?
What operations must be applied to the end equation to return to the start equation?

➤ Choose a rational number to complete your own start equation: $x = \square$
Multiply or divide each side of the equation by the same number.
Write the resulting equation.
Add or subtract the same number from each side of the equation.
Write the resulting equation. This is the end equation.
Trade end equations with your partner.
Determine your partner's start equation. Record the steps in your solution.

Reflect & Share

Share your end equations with another pair of classmates.
Determine each other's start equations. What strategies did you use?
How are the steps used to get from the start equation to the end equation related to the steps used to reverse the process?

Connect

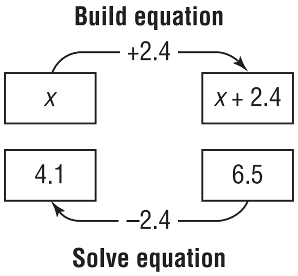
Inverse operations "undo" or reverse each other's results.
Addition and subtraction are inverse operations.
Multiplication and division are also inverse operations.

We can use inverse operations to solve many types of equations. To do this, we determine the operations that were applied to the variable to build the equation. We then use inverse operations to isolate the variable by "undoing" these operations.

For example, to solve $x + 2.4 = 6.5$:

➤ Start with x.
Identify the operation applied to x to produce the expression $x + 2.4$; that is, add 2.4 to get:
$x + 2.4$

➤ Since $x + 2.4$ is equal to 6.5, apply the inverse operation on 6.5 to isolate x; that is, subtract 2.4 to get:
$x + 2.4 - 2.4 = 6.5 - 2.4$
So, $x = 4.1$



Example 1 Writing Then Solving One-Step Equations

For each statement below, write then solve an equation to determine each number. Verify the solution.

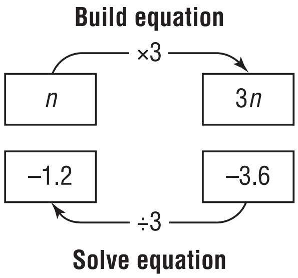
a) Three times a number is −3.6.

b) A number divided by 4 is 1.5.

A Solution

a) Let n represent the number. Then, 3 times n is −3.6.
The equation is: $3n = -3.6$

Inverse Operations



Algebraic Solution

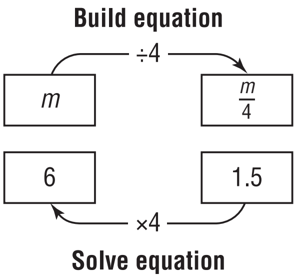
$3n = -3.6$
Undo the multiplication.
Divide each side by 3.

$$\frac{3n}{3} = \frac{-3.6}{3}$$
$$n = -1.2$$

Verify the solution: $3 \times (-1.2) = -3.6$, so the solution is correct.

b) Let m represent the number. Then, m divided by 4 is 1.5.
The equation is: $\frac{m}{4} = 1.5$

Inverse Operations



Algebraic Solution

$$\frac{m}{4} = 1.5$$

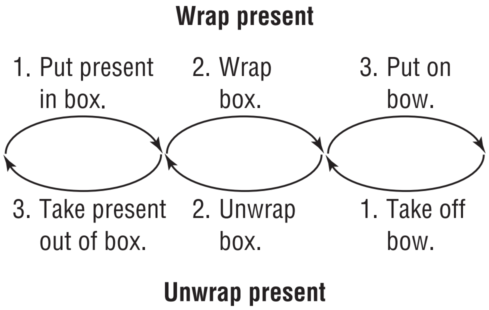
Undo the division.
Multiply each side by 4.

$$4 \times \frac{m}{4} = 4 \times 1.5$$

$$m = 6$$

Verify the solution: $\frac{6}{4} = 1.5$, so the solution is correct.

To "undo" a sequence of operations, we perform the inverse operations in the reverse order. For example, compare the steps and operations to wrap a present with the steps and operations to unwrap the present.



Example 2 Solving a Two-Step Equation

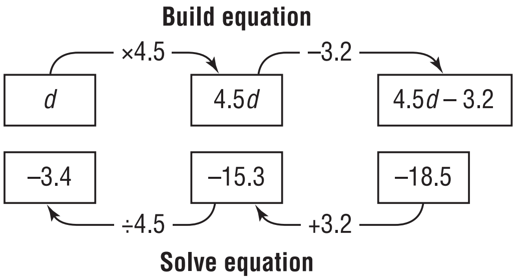
Solve, then verify each equation.

a) $4.5d - 3.2 = -18.5$ **b)** $\frac{r}{4} + 3 = 7.2$

A Solution

a) $4.5d - 3.2 = -18.5$

Inverse Operations



Algebraic Solution

$$4.5d - 3.2 = -18.5$$

Add 3.2 to each side.

$$4.5d - 3.2 + 3.2 = -18.5 + 3.2$$

$$4.5d = -15.3$$

Divide each side by 4.5.

$$\frac{4.5d}{4.5} = \frac{-15.3}{4.5}$$

$$d = -3.4$$

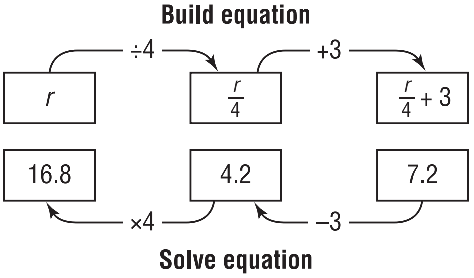
To verify the solution, substitute $d = -3.4$ into $4.5d - 3.2 = -18.5$.

$$\begin{aligned} \text{Left side} &= 4.5d - 3.2 \\ &= 4.5 \times (-3.4) - 3.2 \\ &= -15.3 - 3.2 \\ &= -18.5 \end{aligned} \qquad \text{Right side} = -18.5$$

Since the left side equals the right side, $d = -3.4$ is correct.

b) $\frac{r}{4} + 3 = 7.2$

Inverse Operations



Algebraic Solution

$$\frac{r}{4} + 3 = 7.2$$

Subtract 3 from each side.

$$\frac{r}{4} + 3 - 3 = 7.2 - 3$$

$$\frac{r}{4} = 4.2$$

Multiply each side by 4.

$$4 \times \frac{r}{4} = 4 \times 4.2$$

$$r = 16.8$$

To verify the solution, substitute $r = 16.8$ into $\frac{r}{4} + 3 = 7.2$.

$$\begin{aligned} \text{Left side} &= \frac{r}{4} + 3 \\ &= \frac{16.8}{4} + 3 \\ &= 4.2 + 3 \\ &= 7.2 \end{aligned} \qquad \text{Right side} = 7.2$$

Since the left side equals the right side, $r = 16.8$ is correct.

We can use equations to model and solve problems. With practice, you can determine the inverse operations required to solve the equation mentally. In many situations, there may be more than one way to solve the equation.

Math Link

Science

When a freighter unloads its cargo, it replaces the mass of cargo with an equal mass of sea water. This mass of water will keep the ship stable. The volume of sea water added is measured in litres. To relate the volume of water to its mass, we use this formula for density, D:

$D = \frac{M}{V}$, where M = mass, and V = volume

Once a freighter has been unloaded, it is filled with 5 million litres of water. The density of sea water is 1.030 kg/L. What mass of water was added? Solve the equation $1.030 = \frac{M}{5\ 000\ 000}$ to find out.

Example 3 Using an Equation to Model and Solve a Problem

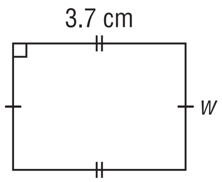
A rectangle has length 3.7 cm and perimeter 13.2 cm.

a) Write an equation that can be used to determine the width of the rectangle.

b) Solve the equation.

c) Verify the solution.

Solutions



a) Let w centimetres represent the width of the rectangle. The perimeter of a rectangle is twice the sum of the length and width. So, the equation is: $13.2 = 2(3.7 + w)$

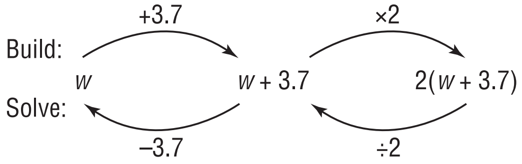
b) Solve the equation.

Method 1

Use inverse operations.

$13.2 = 2(3.7 + w)$

Think:



$$13.2 = 2(3.7 + w) \quad \text{Divide each side by 2.}$$

$$\frac{13.2}{2} = \frac{2(3.7 + w)}{2}$$

$$6.6 = 3.7 + w \quad \text{Subtract 3.7 from each side.}$$

$$6.6 - 3.7 = 3.7 + w - 3.7$$

$$2.9 = w$$

Method 2

Use the distributive property, then inverse operations.

$$13.2 = 2(3.7 + w) \quad \text{Use the distributive property to expand } 2(3.7 + w).$$

$$13.2 = 2(3.7) + 2(w)$$

$$13.2 = 7.4 + 2w$$

$$13.2 - 7.4 = 7.4 + 2w - 7.4 \quad \text{Subtract 7.4 from each side.}$$

$$5.8 = 2w \quad \text{Divide each side by 2.}$$

$$\frac{5.8}{2} = \frac{2w}{2}$$

$$2.9 = w$$

c) Check: The perimeter of a rectangle with length 3.7 cm and width 2.9 cm is:

$$\begin{aligned} 2(3.7 \text{ cm} + 2.9 \text{ cm}) &= 2(6.6 \text{ cm}) \\ &= 13.2 \text{ cm} \end{aligned}$$

The solution is correct. The width of the rectangle is 2.9 cm.

Example 4 Using an Equation to Solve a Percent Problem

Seven percent of a number is 56.7.

a) Write, then solve an equation to determine the number.

b) Check the solution.

A Solution

a) Let n represent the number. Then, 7% of the number is $7\% \times n$, or $0.07n$.

An equation is: $0.07n = 56.7$

$0.07n = 56.7$ Divide each side by 0.07.

$\frac{0.07n}{0.07} = \frac{56.7}{0.07}$ Use a calculator.

$n = 810$

The number is 810.

b) $7\% \text{ of } 810 = 0.07 \times 810$

$= 56.7$

So, the solution is correct.

Discuss the ideas

1. How are inverse operations used to solve an equation?
How can you verify your solution of an equation?

2. When you build or solve an equation, why must you apply the operations or inverse operations to both sides of the equation?

3. When you verify the solution to an equation, why should you substitute the solution in the original equation?

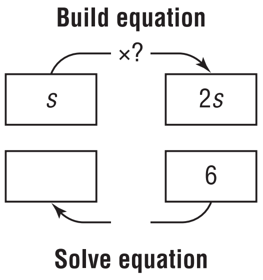
4. When you solve a two-step equation using inverse operations, how is the order in which you apply the inverse operations related to the order in which you would build the end equation?

Practice

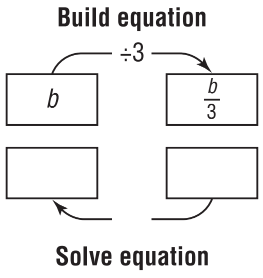
Check

5. Solve each equation by copying and completing the arrow diagram.
How do you know that your solution is correct?

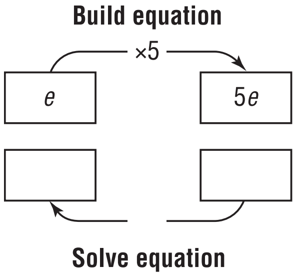
a) $2s = 6$



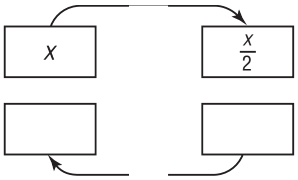
b) $\frac{b}{3} = 5$



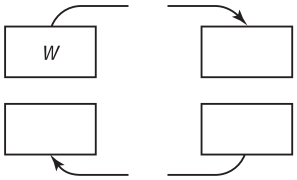
c) $5e = -35$



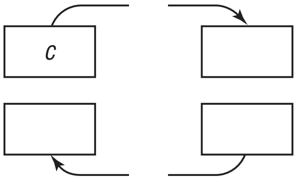
d) $\frac{x}{2} = -7$



e) $-9w = 2.7$

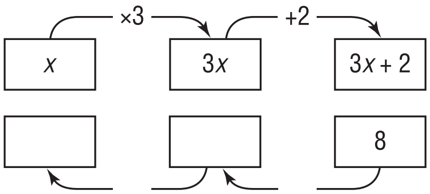


f) $\frac{c}{5} = -1.2$

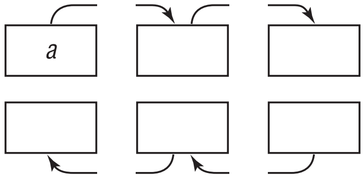


6. Solve each equation by copying and completing the arrow diagram. How do you know that your solution is correct?

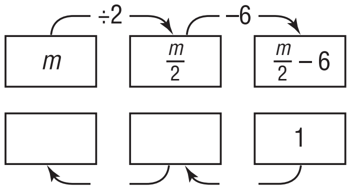
a) $3x + 2 = 8$



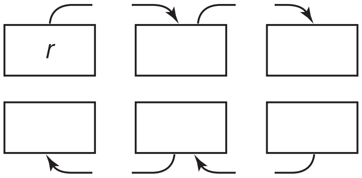
b) $-5a - 6 = 7$



c) $\frac{m}{2} - 6 = 1$



d) $\frac{r}{8} + 5.5 = 2$



7. A student tried to solve the equation $-5m = 15$ by adding 5 to each side. Explain what is wrong with the student's method.
Show the correct way to solve the equation.

Apply

8. Solve each equation.
Which strategy did you use?
Verify the solution.

a) $4x = 9.6$

b) $10 = 3b - 12.5$

c) $-5.25x = -210$

d) $-0.5 = -2x + 8.1$

e) $250 + 3.5n = 670$

f) $-22.5 = -2c - 30.5$

9. For each statement below, write then solve an equation to determine the number. Verify the solution.

a) Two times a number is −10.

b) Three times a number, plus 6.4, is 13.9.

c) Four times a number is −8.8.

d) Ten is equal to two times a number, plus 3.6.

10. Solve each equation. Verify the solution.

a) $\frac{c}{3} = 15$ **b)** $\frac{m}{6} - 1.5 = -7$

c) $-1.5 = \frac{n}{4}$ **d)** $5 = \frac{q}{-2} - 5$

e) $\frac{2c}{5} = 1.2$ **f)** $1.2 = \frac{2a}{3} + 5.1$

11. For each statement below, write then solve an equation to determine the number. Verify the solution.

a) A number divided by 4 is −7.

b) Three, plus a number divided by 5 is 6.

c) One-half of a number is 2.5.

d) One-third of a number, minus 4, is 2.

12. Jenna says that, to build the equation $-2b + 4 = -\frac{3}{4}$, she multiplied each side of the start equation by −2, then added 4 to each side. Can Jenna's partner solve this equation by dividing each side by −2, then subtracting 4 from each side? Explain why or why not.

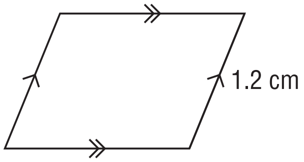
13. Erica is thinking of a number. If you divide her number by 3 then subtract 13.5, the result is 2.8.

a) Let b represent Erica's number. Write an equation to determine this number.

b) Solve the equation.

c) Verify the solution.

14. A parallelogram has one shorter side of length 1.2 cm and perimeter 6.6 cm.



a) Write an equation that can be used to determine the length of the longer side.

b) Solve the equation.

c) Verify the solution.

15. Twelve percent of a number is 39.48.

a) Write, then solve an equation to determine the number.

b) Check the solution.

16. Stephanie has a job in sales. She earns a monthly salary of $2500, plus a commission of 8% of her sales. One month, Stephanie earns a total of $2780. This can be represented by the equation
$2780 = 2500 + 0.08s$,
where s is Stephanie's sales in dollars.

a) Solve the equation to determine Stephanie's sales for that month.

b) Verify the solution.

17. Steve works in a clothing store. He earns $1925 a month, plus a commission of 10% of his sales. One month, Steve earned $2725.

a) Choose a variable to represent Steve's sales in dollars, then write an equation to determine Steve's sales that month.

b) Solve the equation. What were Steve's sales?

18. Solve each equation. Verify the solution.

a) $5(x - 7) = -15$

b) $2(m + 4) = 11$

c) $-3(t - 2.7) = 1.8$

d) $7.6 = -2(-3 - y)$

e) $8.4 = -6(a + 2.4)$

19. **Assessment Focus** Vianne took 4 bottles of water and 6 bottles of juice to a family picnic. Each bottle of juice contained 0.5 L. The total volume of water and juice was 4.42 L. What was the volume of 1 bottle of water?

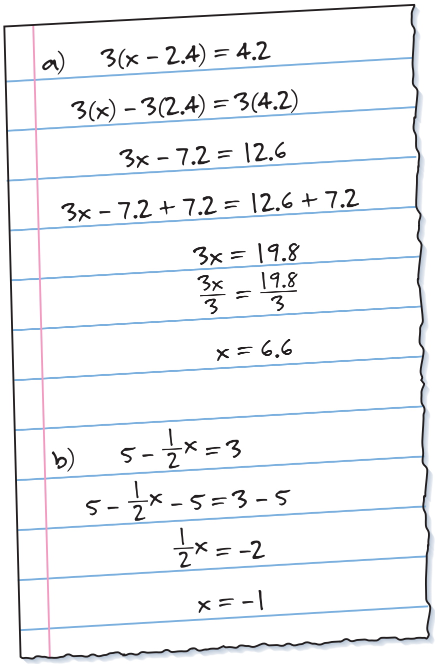
a) Choose a variable and write an equation for this situation.

b) Solve the equation.

c) Verify the solution.

Show your work.

20. On a test, a student solved these equations:



a) $3(x - 2.4) = 4.2$

$3(x) - 3(2.4) = 3(4.2)$

$3x - 7.2 = 12.6$

$3x - 7.2 + 7.2 = 12.6 + 7.2$

$3x = 19.8$

$\frac{3x}{3} = \frac{19.8}{3}$

$x = 6.6$

b) $5 - \frac{1}{2}x = 3$

$5 - \frac{1}{2}x - 5 = 3 - 5$

$\frac{1}{2}x = -2$

$x = -1$

What mistakes did the student make? Write a correct solution for each equation.

21. A large pizza with tomato sauce and cheese costs $7.50, plus $1.50 for each additional topping. A customer orders a large pizza and is charged $16.50. How many toppings did the customer order?

a) Write an equation to solve the problem.

b) Solve the problem. Verify the solution.

22. An item increased in price by $4.95. This is a 9% increase. What did the item cost before the price increase?

a) Write an equation to solve the problem.

b) Solve the equation. Verify your solution.

Take It Further

23. The expression $180(n - 2)$ represents the sum of the interior angles in a polygon with n sides. Suppose the sum of its interior angles is 1080°. How many sides does the polygon have?

a) Write an equation to solve the problem.

b) Kyler solves the equation after using the distributive property to simplify $180(n - 2)$. Show the steps in Kyler's solution.

c) Esta solves the equation by undoing the operations that were used to build the equation. Show the steps in Esta's solution.

d) Whose method do you prefer? Explain.

24. Solve each equation. Verify the solution.

a) $4x + \frac{37}{5} = -17$ **b)** $8m - \frac{6}{7} = \frac{176}{7}$

c) $\frac{3}{4} - 5p = \frac{67}{6}$ **d)** $\frac{22}{8} + 10g = \frac{62}{5}$

Reflect

Choose a reversible routine from daily life. Explain why reversing the routine means undoing each step in the reverse order. Explain how this idea can be used to solve an equation. Include an example.

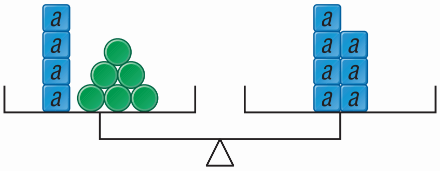
6.2

Solving Equations by Using Balance Strategies

FOCUS

- Model a problem with a linear equation, use balance strategies to solve the equation pictorially, and record the process symbolically.

Tracey has to solve the equation $4a + 6 = 7a$.
Could she use an arrow diagram to model this equation?
How could these balance scales help Tracey?



Each ● has a mass of 1 g.

Investigate

Use a model or strategy of your choice to solve this equation: $5a + 7 = 2a + 1$
Record the solution algebraically.
How do you know that your solution is correct?

Reflect & Share

Compare your strategies and solutions with those of another pair of classmates.
If you used different strategies, explain your strategy and choice of strategy.
What are the advantages and disadvantages of each strategy?
Which strategies did not work?

Connect

To solve an equation, we need to isolate the variable on one side of the equation.

In Lesson 6.1, we isolated the variable by reversing the operations acting on the variable. However, this strategy can only be used when the variable occurs once in the equation.

Another way to isolate the variable is to use a balance strategy modelled by balance scales. The scales remain balanced when we do the same thing to each side.

Example 1 Modelling Equations with Variables on Both Sides

a) Solve: $6x + 2 = 10 + 4x$

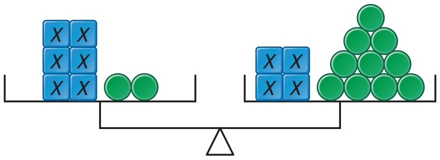
b) Verify the solution.

A Solution

a) Rearrange the equation so that both terms containing the variable are on the same side of the equation. Then isolate the variable to solve the equation.

Pictorial Solution **Algebraic Solution**

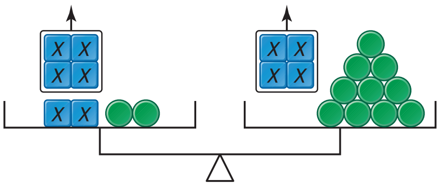
Draw balance scales. On the left pan, draw masses to represent $6x + 2$. On the right pan, draw masses to represent $10 + 4x$.



Each ● has a mass of 1 g.

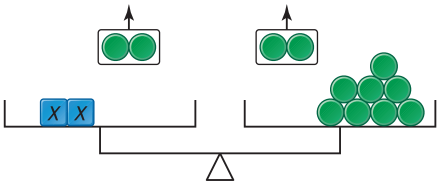
$$6x + 2 = 10 + 4x$$

Remove four x masses from each pan.



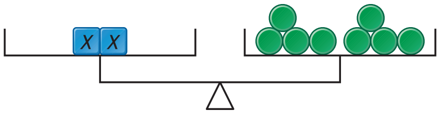
$$6x - 4x + 2 = 10 + 4x - 4x$$
$$2x + 2 = 10$$

Remove two 1-g masses from each pan.



$$2x + 2 - 2 = 10 - 2$$
$$2x = 8$$

Divide the masses in each pan into 2 equal groups. Each x-mass in the left pan corresponds to a group of 4 g in the right pan.



$$\frac{2x}{2} = \frac{8}{2}$$
$$x = 4$$

b) Check: Substitute $x = 4$ in each side of the equation.

$$\begin{aligned} \text{Left side} &= 6x + 2 \\ &= 6(4) + 2 \\ &= 24 + 2 \\ &= 26 \end{aligned} \qquad \begin{aligned} \text{Right side} &= 10 + 4x \\ &= 10 + 4(4) \\ &= 10 + 16 \\ &= 26 \end{aligned}$$

Since the left side equals the right side, $x = 4$ is correct.

We cannot easily use a balance scales model when any term in an equation is negative. But we can use algebra tiles to model and solve the equation. We use the principle of balance by adding the same tiles to each side or subtracting the same tiles from each side.

Example 2 Using Algebra Tiles to Solve an Equation

Solve: $-3c + 7 = 2c - 8$

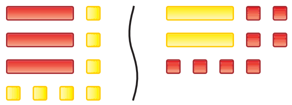
A Solution

Algebra Tile Model

Algebraic Solution

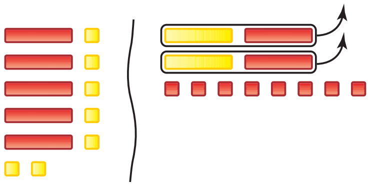
Model the equation with tiles.

$$-3c + 7 = 2c - 8$$



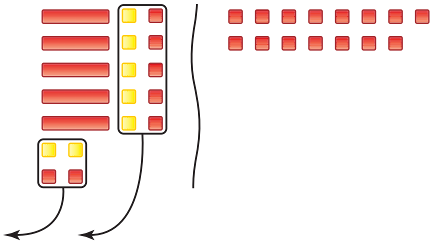
Add two $-c$-tiles to each side to get the terms containing c on the same side. Remove zero pairs.

$$-3c + 7 - 2c = 2c - 8 - 2c$$
$$-5c + 7 = -8$$

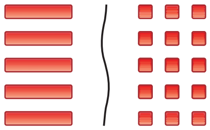


Add seven -1-tiles to each side to get the constant terms on the same side. Remove zero pairs.

$$-5c + 7 - 7 = -8 - 7$$
$$-5c = -15$$



Arrange the remaining tiles on each side into 5 groups.



One $-c$-tile is equal to -3.

$$-1c = -3$$

Flip the tiles. One c-tile is equal to 3.

$$c = 3$$

Equations with rational numbers in fraction or decimal form cannot be modelled easily with balance scales. However, we can solve these equations by doing the same thing to each side of the equation to isolate the variable. We may:

- Add the same quantity to each side.
- Subtract the same quantity from each side.
- Multiply or divide each side by the same non-zero quantity.

Example 3 Solving Equations with Rational Coefficients

Solve each equation, then verify the solution.

a) $\frac{122}{r} = 3, r \neq 0$ **b)** $\frac{2a}{3} = \frac{4a}{5} + 7$

A Solution

Create an equivalent equation without fractions.

a) To clear the fraction, multiply each side by the denominator.

$$\frac{122}{r} = 3$$

Multiply each side by r.

$$r \times \frac{122}{r} = 3 \times r$$

Think: $\frac{r}{1} \times \frac{122}{r} = \frac{122}{1}$

$$122 = 3r$$

Divide each side by 3.

$$\frac{122}{3} = \frac{3r}{3}$$

$$\frac{122}{3} = r$$

So, $r = \frac{122}{3}$, or $40\frac{2}{3}$, or $40.\overline{6}$

Check: Substitute $r = \frac{122}{3}$ in the original equation.

Left side $= \frac{122}{r}$ Right side $= 3$

$$= \frac{122}{\frac{122}{3}}$$

$$= \frac{122}{1} \times \frac{3}{122}$$

$$= 3$$

Since the left side equals the right side, $r = \frac{122}{3}$ is correct.

b) To clear the fractions, multiply each side by the common denominator.

$$\frac{2a}{3} = \frac{4a}{5} + 7$$

Multiply each side by the common denominator 15.

$$15 \times \frac{2a}{3} = 15\left(\frac{4a}{5} + 7\right)$$

Use the distributive property.

$$\overset{5}{\cancel{15}} \times \frac{2a}{\cancel{3}_1} = \overset{3}{\cancel{15}} \times \frac{4a}{\cancel{5}_1} + 15 \times 7$$

$$10a = 12a + 105$$

Subtract $12a$ from each side.

$$10a - 12a = 12a + 105 - 12a$$

$$-2a = 105$$

Divide each side by -2.

$$\frac{-2a}{-2} = \frac{105}{-2}$$

$$a = -52\frac{1}{2}, \text{ or } -52.5$$

Check: Substitute $a = -52.5$ in each side of the original equation.

Left side $= \frac{2a}{3}$	Right side $= \frac{4a}{5} + 7$
$= \frac{2(-52.5)}{3}$	$= \frac{4(-52.5)}{5} + 7$
$= -35$	$= -42 + 7$
	$= -35$

Since the left side equals the right side, $a = -52.5$ is the correct solution.

Example 4 Using an Equation to Model and Solve a Problem

A cell phone company offers two plans.

Plan A: 120 free minutes, \$0.75 per additional minute

Plan B: 30 free minutes, \$0.25 per additional minute

Which time for calls will result in the same cost for both plans?

a) Model the problem with an equation.

b) Solve the problem.

c) Verify the solution.

A Solution

a) Let t minutes represent the time for calls.

For Plan A, you pay only for the time that is greater than 120 min.

So, the time you pay for is $(t - 120)$ min.

Each minute costs \$0.75, so the cost in dollars is: $0.75(t - 120)$

For Plan B, you pay only for the time that is greater than 30 min.

So, the time you pay for is $(t - 30)$ min.

Each minute costs \$0.25, so the cost in dollars is: $0.25(t - 30)$

When these two costs are equal, the equation is:

$0.75(t - 120) = 0.25(t - 30)$

b)

$$0.75(t - 120) = 0.25(t - 30) \qquad \text{Use the distributive property.}$$
$$0.75(t) + 0.75(-120) = 0.25(t) + 0.25(-30)$$
$$0.75t - 90 = 0.25t - 7.5 \qquad \text{Subtract } 0.25t \text{ from each side.}$$
$$0.75t - 0.25t - 90 = 0.25t - 0.25t - 7.5$$
$$0.50t - 90 = -7.5 \qquad \text{Add 90 to each side.}$$
$$0.50t - 90 + 90 = -7.5 + 90$$
$$0.50t = 82.5 \qquad \text{Divide each side by 0.50.}$$
$$\frac{0.50t}{0.50} = \frac{82.5}{0.50}$$
$$t = 165$$

The cost is the same for both plans when the time for calls is 165 min.

c) For Plan A, you pay for: 165 − 120, or 45 min
The cost is: 45 × \$0.75 = \$33.75
For Plan B, you pay for: 165 − 30, or 135 min
The cost is: 135 × \$0.25 = \$33.75
These costs are equal, so the solution is correct.

Discuss the ideas

1. Why can we not use an arrow diagram to solve an equation with a variable term on each side?

2. When you solve an equation with variables on both sides of the equation, does it matter if you isolate the variable on the left side or the right side of the equation? Explain.

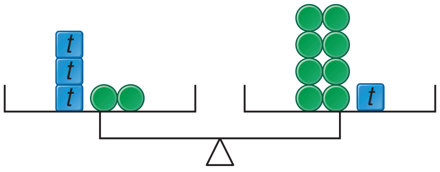
3. For an equation such as $\frac{122}{r} = 3$, why do we include the statement that $r \neq 0$?

Practice

Check

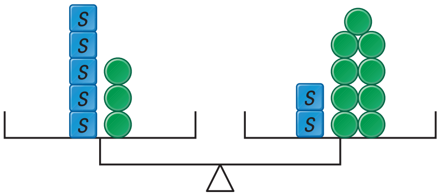
4. Write the equation represented by each picture. Solve the equation. Record the steps algebraically.

a)



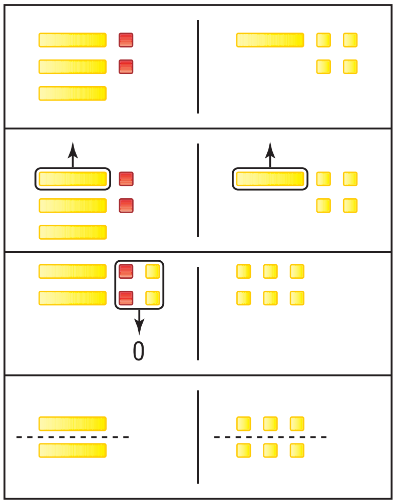
Each ● has a mass of 1 g.

b)



Each ● has a mass of 1 g.

5. Hammy uses algebra tiles to solve the equation $3f - 2 = f + 4$. These pictures show the steps in the solution:



a) Explain the action taken in each step.
b) Record each step algebraically.

6. Use algebra tiles to solve each equation. Record the steps.

a) $4g = 7 - 3g$

b) $4k + 4 = -2k - 8$

c) $-4a - 3 = 3 - a$

d) $3h - 5 = 7 - 3h$

Apply

7. a) Solve each equation.

i) $\frac{6}{h} = 2, h \neq 0$ **ii)** $\frac{-6}{h} = 2, h \neq 0$

iii) $-2 = \frac{6}{h}, h \neq 0$ **iv)** $\frac{6}{-h} = 2, h \neq 0$

v) $-2 = \frac{-6}{h}, h \neq 0$ **vi)** $\frac{6}{-h} = -2, h \neq 0$

b) Explain why there are only 2 solutions to all the equations in part a.

8. Solve each equation.
What strategy did you use?
Verify the solution.

a) $2.4 = \frac{4.8}{s}, s \neq 0$

b) $\frac{-5.4}{t} = 1.8, t \neq 0$

c) $-6.5 = \frac{-1.3}{w}, w \neq 0$

9. Ten divided by a number is -3. Write, then solve an equation to determine the number. Verify the solution.

10. Solve each equation.
What strategy did you use?
Verify the solution.

a) $-12a = 15 - 15a$

b) $10.6y = 2.1y - 27.2$

c) $-10.8 + 7z = 5z$

d) $6u - 11.34 = 4.2u$

e) $-20.5 - 2.2b = -7.2b$

f) $-5.3p = -9 - 8.9p$

11. Solve each equation. Verify the solution.

a) $2 - 3n = 2n + 7$

b) $13 - 3q = 4 - 2q$

c) $-2.4a + 3.7 = -16.1 + 3.1a$

d) $8.8v + 2.1 = 2.3v - 16.1$

e) $-2.5x - 2 = -5.7x + 6$

f) $6.4 - 9.3b = 25.3 - 3.9b$

12. Two rental halls are considered for a wedding.

Hall A costs \$50 per person.
Hall B costs \$2000, plus \$40 per person.

Determine the number of people for which the halls will cost the same to rent.

a) Model this problem with an equation.

b) Solve the problem.

c) Verify the solution.

13. Five subtract 3 times a number is equal to 3.5 times the same number, subtract 8. Write, then solve an equation to determine the number. Verify the solution.

14. A part-time sales clerk at a store is offered two methods of payment.

Plan A: \$1500 per month with a commission of 4% on his sales
Plan B: \$1700 per month with a commission of 2% on his sales

Let s represent the sales in dollars.

a) Write an expression to represent the total earnings under Plan A.

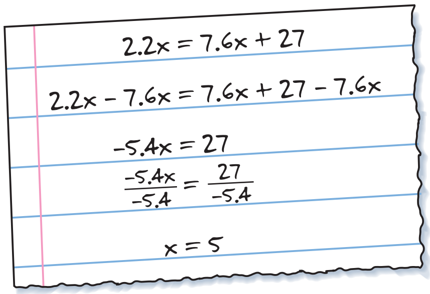
b) Write an expression to represent the total earnings under Plan B.

c) Write an equation to determine the sales that result in the same total earnings from both plans.

d) Solve the equation. Explain what the answer represents.

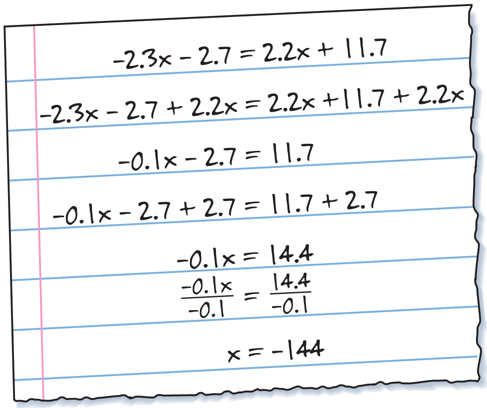
15. Verify each student's work.
If the solution is incorrect, write a correct and complete solution.

a) Student A:



$$2.2x = 7.6x + 27$$
$$2.2x - 7.6x = 7.6x + 27 - 7.6x$$
$$-5.4x = 27$$
$$\frac{-5.4x}{-5.4} = \frac{27}{-5.4}$$
$$x = 5$$

b) Student B:



$$-2.3x - 2.7 = 2.2x + 11.7$$
$$-2.3x - 2.7 + 2.2x = 2.2x + 11.7 + 2.2x$$
$$-0.1x - 2.7 = 11.7$$
$$-0.1x - 2.7 + 2.7 = 11.7 + 2.7$$
$$-0.1x = 14.4$$
$$\frac{-0.1x}{-0.1} = \frac{14.4}{-0.1}$$
$$x = -144$$

16. a) Solve each pair of equations.

i) $\frac{x}{27} = 3; \frac{27}{x} = 3, x \neq 0$

ii) $\frac{a}{36} = 12; \frac{36}{a} = 12, a \neq 0$

b) How are the steps to solve for a variable in the denominator of a fraction similar to the steps used to solve for a variable in the numerator? How are they different? Explain.

17. Solve each equation. Verify the solution.

a) $4(g + 5) = 5(g - 3)$

b) $3(4j + 5) = 2(-10 + 5j)$

c) $2.2(h - 5.3) = 0.2(-32.9 + h)$

d) $0.04(5 - s) = 0.05(6 - s)$

18. Assessment Focus Hendrik has a choice of 2 companies to rent a car.
Company A charges $199 per week, plus $0.20 per kilometre driven.
Company B charges $149 per week, plus $0.25 per kilometre driven.
Determine the distance that Hendrik must drive for the two rental costs to be the same.

a) Model this problem with an equation.

b) Solve the problem.

c) Verify the solution.

Show your work.

19. Solve each equation.

a) $\frac{7}{2}(m + 12) = \frac{5}{2}(20 + m)$

b) $\frac{1}{3}(5 - 3t) = \frac{5}{6}(t - 2)$

c) $\frac{3}{2}(1 + 3r) = \frac{2}{3}(2 - 3r)$

d) $\frac{2}{3}(6x + 5) = \frac{4}{5}(20x - 7)$

20. Both Dembe and Bianca solve the equation:
$\frac{x}{3} + \frac{x}{4} = x - \frac{1}{6}$
Dembe clears the fractions by multiplying each side by 12. Bianca clears the fractions by multiplying each side by 24.

a) Solve the equation using each student's method. Compare the solutions.

b) When you solve an equation involving fractions, why is it a good idea to multiply each side by the least common denominator?

21. Solve each equation. Verify the solution.

a) $\frac{x}{4} + \frac{7}{4} = \frac{5}{6}$

b) $\frac{5x}{16} - \frac{5}{4} = \frac{x}{4}$

c) $2 - \frac{x}{24} = \frac{5x}{24} + 1$

d) $\frac{25}{9} + \frac{x}{9} = \frac{7x}{6} - \frac{5}{2}$

Take It Further

22. In volleyball, statistics are kept about players. The equation $B = M + \frac{1}{2}A$ can be used to calculate the total blocks made by a player. In the equation, B is the total blocks, M is the number of solo blocks, and A is the number of assisted blocks. Marlene has 9 total blocks and 4 solo blocks. How many assisted blocks did Marlene make? How do you know that your answer is correct?

23. A cell phone company offers two different plans.

Plan A
Monthly fee of $28
30 free minutes
$0.45 per additional minute

Plan B
Monthly fee of $40
No free minutes
$0.25 per minute

a) Write an equation to determine the time in minutes that results in the same monthly cost for both plans.

b) Solve the equation.

c) Verify the solution.

Reflect

List some strategies for solving an equation. For each strategy, provide an example of an equation and its solution.

Math Link

Science

Ohm's Law relates the resistance, R ohms, in an electrical circuit to the voltage, V volts, across the circuit and the current, I amperes, through the circuit: $R = \frac{V}{I}$

For a light bulb, when the voltage is 120 V and the resistance is 192 Ω, the current in amperes can be determined by solving this equation: $192 = \frac{120}{I}$

Start Where You Are

How Can I Use My Problem-Solving Skills?

Suppose I have to solve this problem:

The sale price of a jacket is \$41.49.
The original price has been
reduced by 17%.
What was the original price?

➤ What do I know?

- The sale price is \$41.49.
- This is 17% less than the original price.

➤ What strategy could I use to solve the problem?

- I could write, then solve an equation.
  Let d dollars represent the original price.
  17% of d is $0.17d$.
  A word equation is:
  (original price) − (17% of original price) is \$41.49
  An algebraic equation is:

$$\begin{aligned} 1d - 0.17d &= 41.49 && \text{Combine the terms in } d. \\ 0.83d &= 41.49 && \text{Divide each side by 0.83.} \\ \frac{0.83d}{0.83} &= \frac{41.49}{0.83} \\ d &\doteq 49.99 \end{aligned}$$

- I could write, then solve a proportion.
  Let d dollars represent the original price, which is 100%.
  Since the price has been reduced by 17%,
  the sale price is 100% − 17%, or 83% of the original price.
  So, the ratio of sale price to original price is equal
  to the ratio of 83% to 100%.

As a proportion: $\frac{41.49}{d} = \frac{83}{100}$

$$\begin{aligned} \frac{41.49}{d} &= \frac{83}{100} && \text{Multiply each side by 100.} \\ 100 \times \frac{41.49}{d} &= 100 \times \frac{83}{100} \\ \frac{4149}{d} &= 83 && \text{Multiply each side by } d. \\ d \times \frac{4149}{d} &= 83 \times d \\ 4149 &= 83d && \text{Divide each side by 83.} \\ \frac{4149}{83} &= \frac{83d}{83} \\ 49.99 &\doteq d \end{aligned}$$

- I could draw a diagram to help me reason the answer.

The original price is 100%.
Since the price has been reduced by 17%,
the sale price is 100% − 17%,
or 83% of the original price.
83% of the original price is $41.49
So, 1% of the original price is $\frac{\$41.49}{83}$

And, 100%, which is the original price $= \frac{\$41.49}{83} \times 100$

$$= \frac{\$4149}{83}$$

$$\doteq \$49.99$$

The original price was $49.99.

Check: The discount is:
17% of \$49.99 = 0.17 × \$49.99
= \$8.50
Sale price = original price − discount
= \$49.99 − \$8.50
= \$41.49
This is the same as the given sale price, so the answer is correct.

➤ Look back.
Which method do you find easiest? Why?
Would you have solved the problem a different way?
If your answer is yes, show your method.

Check

1. The price of gasoline increased by 6%. The new price is $1.36/L.
What was the price of gasoline before it increased?

2. Make up your own percent problem. Solve your problem.
Trade problems with a classmate, then solve your classmate's problem.
Compare your strategies for answering both problems.

Mid-Unit Review

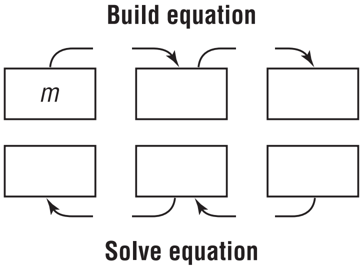
6.1

1. For each equation, write the first operation you would use to isolate the variable. Justify your choice of operation.

a) $-3j = 9.6$ **b)** $\frac{1}{4}r - 2 = 4$

c) $2(-3x + 1.5) = 6$ **d)** $3 = -2n + 9$

2. Marshall creates this arrow diagram to show the steps in the solution of $\frac{m}{10} + 20.3 = 45.5$.



a) Copy and complete the arrow diagram.

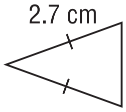
b) Record the solution algebraically.

3. Sheila is charged a fare of $27.70 for a cab ride to her friend's house. The fare is calculated using a flat fee of $2.50, plus $1.20 per kilometre. What distance did Sheila travel?

a) Let *k* kilometres represent the distance travelled. Write an equation to solve the problem. Solve the problem.

b) Verify the solution.

4. An isosceles triangle has two equal sides of length 2.7 cm and perimeter 7.3 cm.



a) Write an equation that can be used to determine the length of the third side.

b) Solve the equation.

c) Verify the solution.

5. Solve each equation. Verify the solution.

a) $\frac{k}{3} = -1.5$

b) $10.5 = 3b - 12.5$

c) $5(x - 7.2) = 14.5$

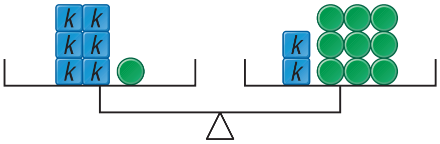
d) $8.4 = 1.2b$

e) $2 + \frac{n}{3} = 2.8$

f) $-8 = 0.4(3.2 + h)$

6.2

6. Write the equation modelled by these balance scales. Solve the equation. Verify the solution.



Each ● has a mass of 1 g.

7. Solve each equation. Verify the solution.

a) $\frac{56}{a} = -3.5, a \neq 0$

b) $8w - 12.8 = 6w$

c) $-8z + 11 = -10 - 5.5z$

d) $\frac{5x}{2} = 11 + \frac{2x}{3}$

e) $0.2(5 - 2r) = 0.3(1 - r)$

f) $12.9 + 2.3y = 4.5y + 19.5$

g) $\frac{2}{5}(m + 4) = \frac{1}{5}(3m + 9)$

8. Skateboards can be rented from two shops in a park.

Shop Y charges $15 plus $3 per hour
Shop Z charges $12 plus $4 per hour

Determine the time in hours for which the rental charges in both shops are equal.

a) Write an equation to determine the time.

b) Solve the equation.

c) Verify the solution.

GAME

Equation Persuasion

You will need

- a deck of playing cards

Number of Players

- 2

Goal of the Game

- To solve and verify your partner's end equation

How to Play

1. Each player picks a secret integer between -9 and 9.
Use the secret integer to write an equation of the form:
$n =$ secret integer

2. Remove the face cards from the deck.
Each player draws 3 cards from the deck, one at a time.

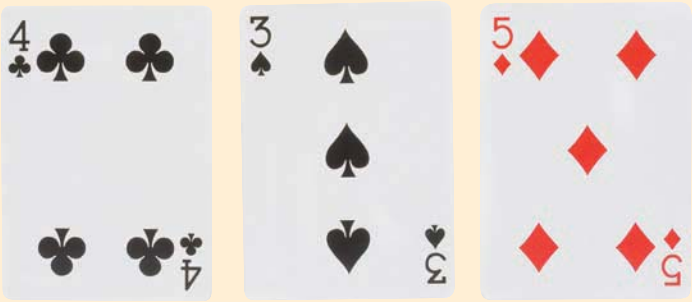
Suit	Meaning
♣	Add the number on the card to each side of the equation.
♠	Add the indicated number of *n*s to each side of the equation.
♦	Multiply each side of the equation by the number on the card.
♥	Subtract the number on the card from each side of the equation.

For example:

- Suppose you choose a secret number of -2 and draw these three cards in the given order: 4 of ♣, 3 of ♠, and 5 of ♦.
- Secretly perform the operations indicated by the cards on the start equation: $n = -2$

Add 4 to each side of the equation: $n + 4 = 2$
Add $3n$ to each side of the equation: $4n + 4 = 2 + 3n$
Multiply each side by 5: $5(4n + 4) = 5(2 + 3n)$
This is the end equation.

3. Trade end equations with your partner.
Solve each other's end equation.



4. If you solved your partner's equation correctly, you receive 5 points. You get an additional 5 points if you verify the solution. But, if you gave your partner an incorrect end equation, you get 0 points.

5. The first player to get 50 points wins.

6.3 Introduction to Linear Inequalities

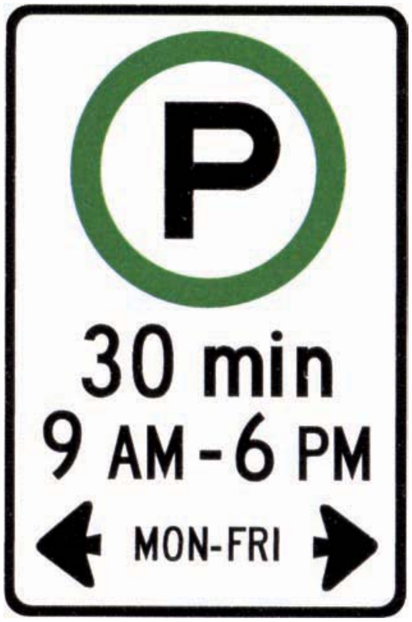
FOCUS

- Write and graph inequalities.

We use an **inequality** to model a situation that can be described by a range of numbers instead of a single number.
When one quantity is less than or equal to another quantity,
we use this symbol: $\leq$
When one quantity is greater than or equal to another quantity,
we use this symbol: $\geq$
Which of these inequalities describes the time, *t* minutes, for which a car could be legally parked?

$t > 30$
$t \geq 30$
$t < 30$
$t \leq 30$

Inequality signs
$<$ less than
$>$ greater than



Investigate

Define a variable and write an inequality for each situation.

a)

b)

Height Restriction
You must be
at least 102 cm
to go on this ride.

c)

d)

Reflect & Share

Compare your inequalities with those of another pair of classmates.
If the inequalities are different, how can you find out which is correct?
Work together to describe three other situations that involve inequalities.
Write an inequality for each situation.

Connect

Here are some examples of inequality statements:

- One expression is less than another; a is less than 3: $a < 3$
- One expression is greater than another; b is greater than -4: $b > -4$
- One expression is less than or equal to another;
  c is less than or equal to $\frac{3}{4}$: $c \leq \frac{3}{4}$
- One expression is greater than or equal to another;
  d is greater than or equal to -5.4: $d \geq -5.4$

Many real-world situations can be modelled by inequalities.

Example 1 Writing an Inequality to Describe a Situation

Define a variable and write an inequality to describe each situation.

a) Contest entrants must be at least 18 years old.

b) The temperature has been below −5°C for the last week.

c) You must have 7 items or less to use the express checkout line at a grocery store.

d) Scientists have identified over 400 species of dinosaurs.

A Solution

a) Let a represent the age of a contest entrant.
"At least 18" means that entrants must be 18, or 19, or 20, and so on.
So, a can equal 18 or be greater than 18.
The inequality is $a \geq 18$.

b) Let t represent the temperature in degrees Celsius.
For the temperature to be "below −5°C", it must be less than −5°C.
The inequality is $t < -5$.

c) Let n represent the number of items.
The number of items must be 7 or less than 7.
The inequality is $n \leq 7$.

d) Let s represent the number of species of dinosaurs.
"Over 400" means greater than 400.
The inequality is $s > 400$.

A linear equation is true for only one value of the variable.
A linear inequality may be true for many values of the variable.
The solution of an inequality is any value of the variable that makes the inequality true.
There are usually too many numbers to list, so we may show them on a number line.

Example 2 Determining Whether a Number Is a Solution of an Inequality

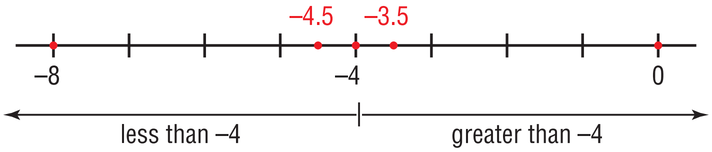
Is each number a solution of the inequality $b \geq -4$? Justify the answers.

a) -8 **b)** -3.5 **c)** -4 **d)** -4.5 **e)** 0

Solutions

Method 1

Use a number line. Show all the numbers on a line.
The solution of $b \geq -4$ is all numbers that are greater than or equal to -4.



For a number to be greater than -4, it must lie to the right of -4.

a) -8 is to the left of -4, so -8 is not a solution.

b) -3.5 is to the right of -4, so -3.5 is a solution.

c) -4 is equal to itself, so it is a solution.

d) -4.5 is to the left of -4, so -4.5 is not a solution.

e) 0 is to the right of -4, so 0 is a solution.

Method 2

Use substitution. Substitute each number for b in the inequality $b \geq -4$.
Determine whether the resulting inequality is true or false.

a) Since $-8 \geq -4$ is false, -8 is not a solution.

b) Since $-3.5 \geq -4$ is true, -3.5 is a solution.

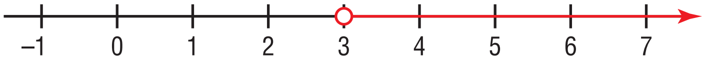
c) Since $-4 = -4$, -4 is a solution.

d) Since $-4.5 \geq -4$ is false, -4.5 is not a solution.

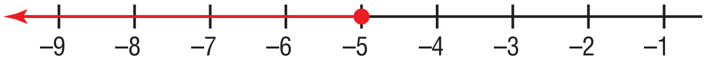
e) Since $0 \geq -4$ is true, 0 is a solution.

We can illustrate the solutions of an inequality by graphing them on a number line.

For $a > 3$, the solution is all numbers greater than 3. Since 3 is not part of the solution, we draw an open circle at 3 to indicate this.



For $b \leq -5$, the solution is all numbers less than or equal to -5. Since -5 is part of the solution, we draw a shaded circle at -5 to indicate this.



Example 3 Graphing Inequalities on a Number Line

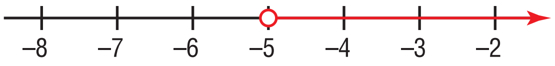
Graph each inequality on a number line.
Write 4 numbers that are solutions of the inequality.

a) $t > -5$ **b)** $-2 \geq x$ **c)** $0.5 \leq a$ **d)** $p < -\frac{25}{3}$

A Solution

a) $t > -5$

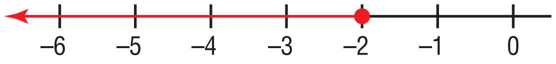
Any number greater than -5 satisfies the inequality.



Four possible solutions are:

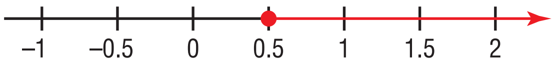
$-4, -2.1, 0, \frac{1}{2}$

b) $-2 \geq x$ means that -2 is greater than or equal to x, or x is less than or equal to -2; that is, $x \leq -2$



Four possible solutions are:
$-2, -4\frac{1}{4}, -6.8, -100$

c) $0.5 \leq a$ means that 0.5 is less than or equal to a, or a is greater than or equal to 0.5; that is, $a \geq 0.5$

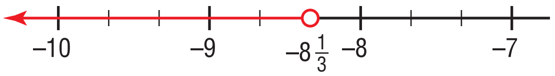


Four possible solutions are:
$0.5, 2, 3\frac{3}{4}, 1000$

d) $p < -\frac{25}{3}$

$-\frac{25}{3}$ is $-8\frac{1}{3}$.

The solution is all numbers that are less than $-8\frac{1}{3}$.



Four possible solutions are:
$-9, -15.8, -20\frac{2}{5}, -99$

Discuss the ideas

1. How is the solution of an inequality different from the solution of an equation?

2. How do you know whether to use an open circle or a shaded circle in the graph of an inequality?

Practice

Check

3. Is each inequality true or false?
Explain your reasoning.

a) $5 < 8$
b) $-5 < -8$
c) $5 < -8$
d) $5 < 5$
e) $5 \leq 5$
f) $0 \geq -5$
g) $5.01 < 5.1$
h) $\frac{1}{5} < \frac{1}{8}$

4. Use a symbol to write an inequality that corresponds to each statement.

a) x is less than -2.
b) p is greater than or equal to 6.
c) y is negative.
d) m is positive.

5. Is each number a solution of $x < -2$?
How do you know?

a) 0
b) -6.9
c) -2.001
d) -3
e) -2
f) $-\frac{1}{2}$

6. Write 4 numbers that are solutions of each inequality.

a) $b > 5$
b) $7 > x$
c) $-2 \leq v$
d) $w \leq -12$

Apply

7. Determine whether the given number is a solution of the inequality.
If the number is not a solution, write an inequality for which the number is a solution.

a) $w < 3$; 3
b) $-3.5 < y$; 0
c) $m \geq 5\frac{1}{2}$; 5.05
d) $a \leq -2$; -15

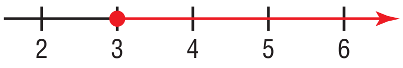
8. Define a variable and write an inequality to model each situation.

a) A coffee maker can hold no more than 12 cups of water.
b) You must be at least 15 years old to obtain a learner's permit to drive in Nunavut.
c) The maximum seating capacity of a school bus is 48 students.
d) Over 2500 people participate in the charity bike-a-thon each year.
e) The shoe store sells sizes no larger than 13.

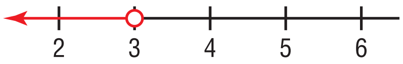
9. Match each equation or inequality with the graph of its solution below.
Justify your choice.

a) $m > 3$
b) $p = 3$
c) $k \leq 3$
d) $t < 3$
e) $v \geq 3$
f) $3 < n$
g) $3 \geq h$
h) $3 \leq s$

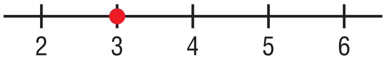
i)



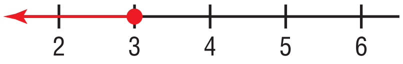
ii)



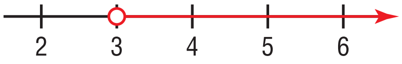
iii)



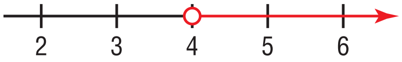
iv)



v)



10. Tom and Stevie write the inequality whose solution is shown on this graph:



Tom writes $a > 4$. Stevie writes $4 < b$. Can both of them be correct? Explain.

11. Assessment Focus

a) For each situation, define a variable and write an inequality to describe the situation.

  i) In Canada, a child under 23 kg must ride in a car seat.

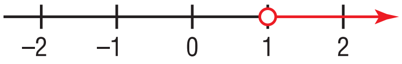
  ii) A silicone oven mitt is heat resistant to temperatures up to 485°C.

  iii) The minimum wage in Alberta is $8.40 an hour.

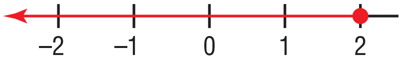
b) Graph the solution of each inequality on a number line.

12. Write an inequality whose solution is graphed on the number line. In each case, are 1 and -3 solutions of the inequality? Explain.

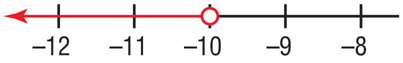
a)



b)



c)



13. Graph the solution of each inequality on a number line.

a) $w > 5.5$ **b)** $x \leq -2$

c) $z > -6$ **d)** $a < 6.8$

e) $b \leq 6.8$ **f)** $c > \frac{2}{3}$

g) $d \leq -\frac{2}{3}$ **h)** $x \leq \frac{18}{5}$

Take It Further

14. Joel is producing a one-hour TV show. An advertiser wants at least 12 min of commercials, and the station will not allow more than 20 min of commercials. Graph the possible show times on a number line. Write two inequalities to describe the situation.

15. The words "over," "under," "maximum," "minimum," "at least," and "no more than" can describe inequalities.

a) Which symbol describes each word?

b) Give a real-world situation that could be described by each word. Write the situation as an inequality.

16. Use a symbol to write an inequality for this statement: y is not negative.
Justify your inequality.

Reflect

An inequality can be described with words, symbols, or a graph. Which representation do you find easiest to understand? Explain. Include an example in your explanation.

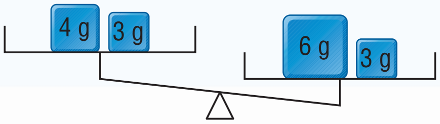
6.4 Solving Linear Inequalities by Using Addition and Subtraction

FOCUS

- Use addition and subtraction to solve inequalities.

Jamina places masses on balance scales.
Why is the right pan lower than the left pan?
Will the right pan remain lower than the left pan in each situation?

- Jamina places 2 g on each pan.
- Jamina removes 3 g from each pan.



Investigate

2

- ➤ Write two different numbers.
  Write the symbol < or > between the numbers to make an inequality.
- ➤ Choose another number. Add that number to each side of the inequality.
  Is the resulting inequality still true?
- ➤ Repeat the preceding step 3 more times with different numbers.
- ➤ Subtract the same number from each side of the original inequality.
  Is the inequality still true?
- ➤ Repeat the preceding step 3 more times with different numbers.

Reflect & Share

Compare your results with those of another pair of classmates.
When the same number is added to or subtracted from each side of an inequality, is the resulting inequality still true? Explain.
How could you use this property to solve the inequality $x + 5 \geq 11$?
Work together to solve the inequality.
How do you know that your solution is correct?

Connect

We can use a number line to investigate the effect of adding to and subtracting from each side of an inequality.

-2 is less than 4 because -2 is to the left of 4 on a number line.

- Adding the same number to each side of an inequality

$-2 < 4$ Add 2 to each side.

$-2 + 2 < 4 + 2$

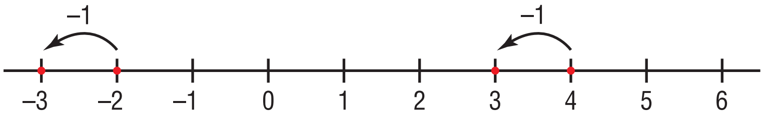
$0 < 6$ This resulting inequality is true.

- Subtracting the same number from each side of an inequality

$-2 < 4$ Subtract 1 from each side.

$-2 - 1 < 4 - 1$

$-3 < 3$ This resulting inequality is true.



When we add the same number to, or subtract the same number from, each side of an inequality, the points move left or right, but their relative positions do not change.

The examples above illustrate this property of inequalities:

- When the same number is added to or subtracted from each side of an inequality, the resulting inequality is still true.

To solve an inequality, we use the same strategy as for solving an equation: isolate the variable by adding to or subtracting from each side of the inequality. Compare the following solutions of an equation and a related inequality.

Equation	**Inequality**
$h + 3 = 5$	$h + 3 < 5$
$h + 3 - 3 = 5 - 3$	$h + 3 - 3 < 5 - 3$
$h = 2$	$h < 2$
There is only one solution: $h = 2$	There are many solutions; too many to list. Any number that is less than 2 is a solution; for example, 0, -5.7, -3452, and so on

Example 1 Solving an Inequality

a) Solve the inequality: $6.2 \leq x - 4.5$ **b)** Verify the solution. **c)** Graph the solution.

A Solution

a) $6.2 \leq x - 4.5$ Add 4.5 to each side.

$6.2 + 4.5 \leq x - 4.5 + 4.5$

$10.7 \leq x$

b) The solution of the inequality $10.7 \leq x$ is all numbers greater than or equal to 10.7.
Choose several numbers greater than 10.7; for example, 11, 20, 30
Substitute $x = 11$ in the original inequality.

Left side = 6.2 Right side $= x - 4.5$
$= 11 - 4.5$
$= 6.5$

Since $6.2 < 6.5$, the left side is less than the right side,
and $x = 11$ satisfies the inequality.
Substitute $x = 20$ in the original inequality.

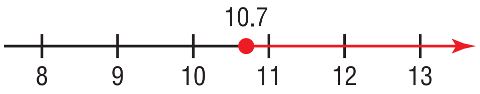
Left side = 6.2 Right side $= x - 4.5$
$= 20 - 4.5$
$= 15.5$

Since $6.2 < 15.5$, the left side is less than the right side,
and $x = 20$ satisfies the inequality.
Substitute $x = 30$ in the original inequality.

Left side = 6.2 Right side $= x - 4.5$
$= 30 - 4.5$
$= 25.5$

Since $6.2 < 25.5$, the left side is less than the right side,
and $x = 30$ satisfies the inequality.
Since all 3 substitutions verify the inequality,
it suggests that $x \geq 10.7$ is correct.

c) Graph the solution on a number line.



It is impossible to check all of the solutions of an inequality. We verified the inequality in *Example 1* by selecting several numbers from the solution and substituting them into the original inequality. Since the resulting statements were true, this suggests that the solution is correct.

A term containing a variable represents a number, so this term can be added to or subtracted from each side of an inequality.

Example 2 Using an Inequality to Model and Solve a Problem

Jake plans to board his dog while he is away on vacation.

- Boarding house A charges $90 plus $5 per day.
- Boarding house B charges $100 plus $4 per day.

For how many days must Jake board his dog for boarding house A to be less expensive than boarding house B?

a) Choose a variable and write an inequality that can be used to solve this problem.

b) Solve the problem.

c) Graph the solution.

A Solution

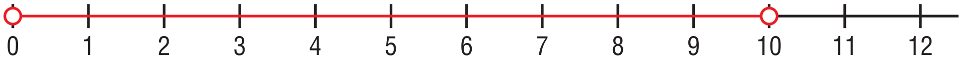
a) Let d represent the number of days Jake boards his dog. For house A: \$90 + \$5/day can be written, in dollars, as $90 + 5d$. For house B: \$100 + \$4/day can be written, in dollars, as $100 + 4d$. For house A to be less expensive than house B, $90 + 5d$ must be less than $100 + 4d$.
So, an inequality is: $90 + 5d < 100 + 4d$

b)

$$\begin{aligned} 90 + 5d &< 100 + 4d && \text{Subtract } 4d \text{ from each side.} \\ 90 + 5d - 4d &< 100 + 4d - 4d && \\ 90 + d &< 100 && \text{Subtract 90 from each side.} \\ 90 - 90 + d &< 100 - 90 && \\ d &< 10 && \end{aligned}$$

Boarding house A is less expensive if Jake leaves his dog there for less than 10 days.

c) $d < 10$



In *Example 2*, the number line begins with a circle at 0 because a dog cannot be boarded for a negative number of days.

Discuss the ideas

1. Why is it impossible to check all the solutions of an inequality?

2. When a solution of an equation is verified, we say that the solution is correct. When a solution of an inequality is verified, we can only say that this suggests the solution is correct. Why?

3. Suppose the solution of an inequality is $r \geq 5.6$. How would you choose suitable values of r to substitute to check?

Practice

Check

4. Which operation will you perform on each side of the inequality to isolate the variable?

a) $a + 4 > 3$ **b)** $0 < -\frac{2}{3} + m$

c) $r - 4 \geq -3$ **d)** $k - 4.5 \leq 5.7$

e) $s + \frac{3}{10} \leq -3$ **f)** $6.1 > 4.9 + z$

5. What must you do to the first inequality to get the second inequality?

a) $x - 2 > 8$
$x > 10$

b) $12.9 \leq y + 4.2$
$y \geq 8.7$

c) $p - \frac{1}{2} \leq \frac{1}{2}$
$p \leq 1$

6. State three values of x that satisfy each inequality: one integer, one fraction, and one decimal.

a) $x + 3 \geq 7$ **b)** $x - 3 \leq 7$

c) $x + 7 < 3$ **d)** $x - 3 > 7$

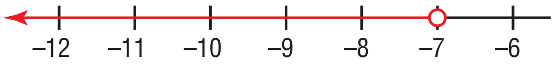
Apply

7. Match each inequality with the graph of its solution below. Is 3 a possible solution of each inequality? How can you find out?

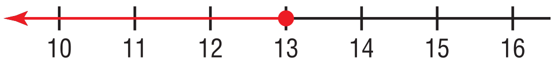
a) $c - 2 > 2$ **b)** $8 \geq -5 + w$

c) $1 > r + 8$ **d)** $7 + m \leq -2$

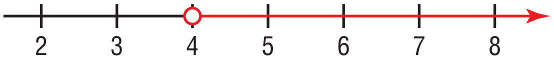
i)



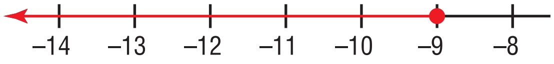
ii)



iii)



iv)



8. Solve each inequality. Graph the solution. Verify the solution.

a) $x + 5 > 2$ **b)** $-9 \geq y - 3$

c) $4 + a \leq 8$ **d)** $2 > x + 7$

e) $k + 8 < -13$ **f)** $q - 2.5 < 3.9$

9. Solve each inequality. Graph the solution. Show the steps in the solution.
Verify the solution by substituting 3 different numbers in each inequality.

a) $4t - 19 < 24 + 3t$

b) $3x < 2x - 11$

c) $5x - 7 < 4x + 4$

d) $2 + 3a \leq 2a - 5$

e) $1.7p + 2.8 \geq 0.7p - 7.6$

f) $2y + 13.3 \geq y - 24.1$

10. A student says $b \geq -9$ is the solution of $-7 \geq b + 2$ because substituting -9 into the original inequality gives the true statement $-7 \geq -7$. Do you agree? Justify your answer.

11. a) Solve the equation: $7.4 + 2p = p - 2.8$

b) Solve the inequality: $7.4 + 2p \geq p - 2.8$

c) Compare the processes in parts a and b. How is solving an inequality like solving the related equation? How is it different?

d) Compare the solutions in parts a and b. How is the solution of an inequality like the solution of the related equation? How is it different?

12. Joel currently has a balance of \$212.35 in his bank account. He must maintain a minimum balance of \$750 in the account to avoid paying a monthly fee. How much money can Joel deposit into his account to avoid paying this fee?

- **a)** Choose a variable, then write an inequality that can be used to solve this problem.
- **b)** Solve the problem.
- **c)** Graph the solution.

13. Teagan is saving money to buy a snowmobile helmet. One weekend, she earned \$20 to add to her savings, but she still did not have the \$135.99 she needed for the helmet.

- **a)** Choose a variable, then write an inequality to represent this situation.
- **b)** Solve the inequality. What does the solution represent?
- **c)** Verify the solution and graph it on a number line.

14. Assessment Focus Marie has \$4.85. She wants to buy a muffin and a cake at a bake sale. The cake is on sale for \$3.45. How much can Marie spend on a muffin?

- **a)** Choose a variable, then write an inequality to solve the problem.
- **b)** Use the inequality to solve the problem.
- **c)** Graph the solution on a number line.
- **d)** A deluxe muffin costs \$1.45.
  Can Marie afford to buy this muffin?
  Justify your answer.

Show your work.

Take It Further

15. a) Solve each inequality. Graph the solution.

- **i)** $2a - 5 \geq 2 + 3a$
- **ii)** $0.7p - 7.6 \leq 1.7p + 2.8$

b) What strategies did you use to solve the inequalities in part a?

c) Compare your solution and graphs in part a with the solutions to questions 9d and 9e. Explain the differences.

16. a) Graph each inequality. Describe the solution in words.

- **i)** $x < -2.57$
- **ii)** $b \geq -10.25$
- **iii)** $p \leq 1.005$

b) Explain how the graphs of these inequalities are different from those that you have graphed before.

c) Which is a more accurate way to describe a solution: using an inequality or using a graph? Explain.

Reflect

How is solving an inequality by using addition or subtraction similar to solving an equation by using addition or subtraction? How is it different?
Use an example in your explanation.

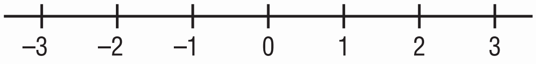
6.5

Solving Linear Inequalities by Using Multiplication and Division

FOCUS

- Use multiplication and division to solve inequalities.

How does the position of a number on a number line determine whether it is greater than or less than another number?
How does this explain why $2 < 3$ but $-2 > -3$?



Investigate

In the patterns below, each side of the inequality $12 > 6$ is multiplied or divided by the same non-zero number.

Multiplication Pattern	**Division Pattern**
$12 > 6$	$12 > 6$
$12(-3) \square 6(-3)$	$12 \div (-3) \square 6 \div (-3)$
$12(-2) \square 6(-2)$	$12 \div (-2) \square 6 \div (-2)$
$12(-1) \square 6(-1)$	$12 \div (-1) \square 6 \div (-1)$
$12(1) \square 6(1)$	$12 \div 1 \square 6 \div 1$
$12(2) \square 6(2)$	$12 \div 2 \square 6 \div 2$
$12(3) \square 6(3)$	$12 \div 3 \square 6 \div 3$

- Copy and simplify each expression in the patterns.
- Replace each ☐ with $<$ or $>$ to create a true statement.
- Compare the inequality signs in the pattern with the inequality sign in $12 > 6$.
  When did the inequality sign stay the same?
  When did the inequality sign change?

Reflect & Share

Share your results with another pair of classmates.
What happens to an inequality when you multiply or divide each side by:

- a positive number?
- a negative number?

Work together to explain these results.