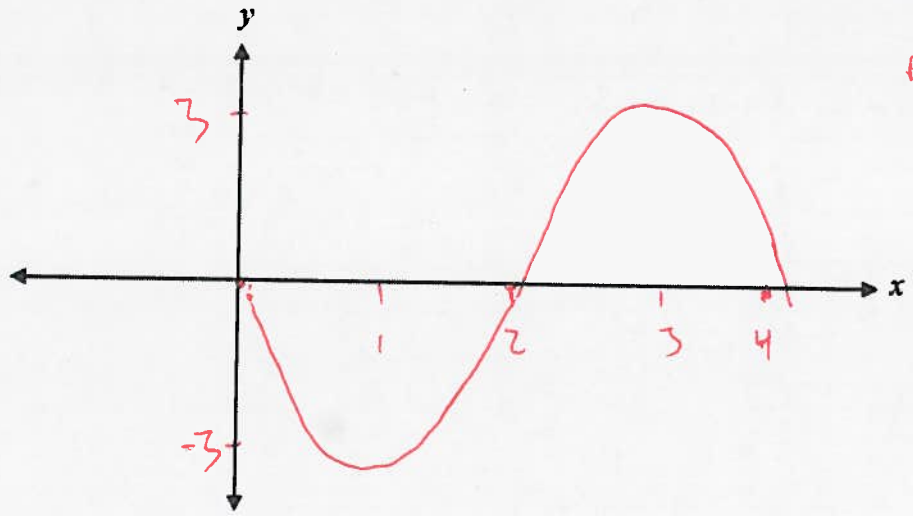
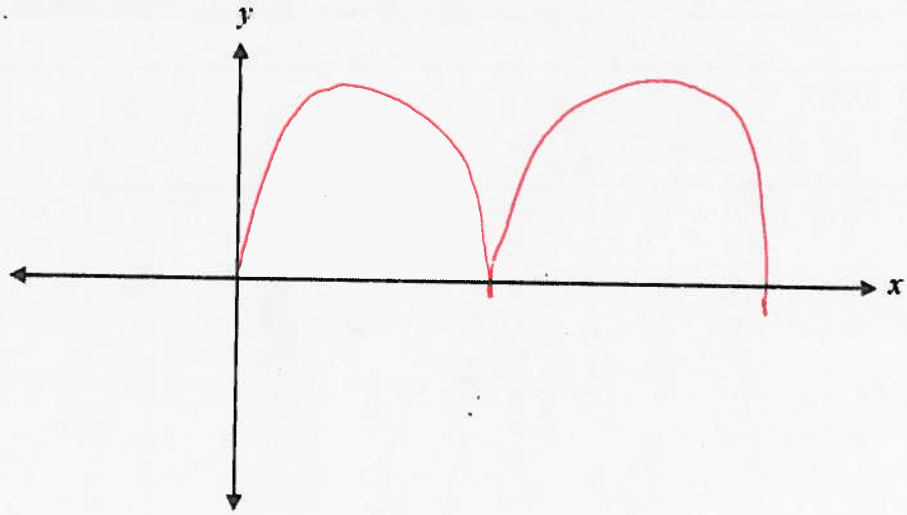
Provincial Exam Review Quiz #1 key

1. Solve the equation $\sec\theta = -4$, where $\theta \in \Re$. State your answer(s) in radian measure, correct to 3 decimal places. [3 marks]

2. a) Sketch a clearly labeled graph of at least one period of $f(x) = 3\cos\left(\frac{\pi}{2}(x+1)\right)$. [3 marks]



b) Sketch $y = |f(x)|$. [1 mark]



3. Prove the following identity: $\frac{\tan\theta\csc^2\theta}{\tan^2\theta+1} = \cot\theta$. [3 marks]

4. Solve for x: $e^x = 8^{1-x}$. Express your answer correct to 3 decimal places. [4 marks]

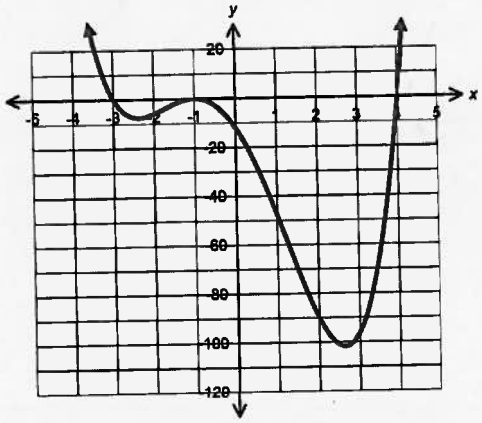
5. Using the letters from the word PORTAGE:

a) How many 5 letter arrangements are possible? Express your answer as a whole number. [1 mark]

b) How many 7 letter arrangements are possible if "P" must be the first letter and the letters "T" and "E" must be together? Briefly explain your calculations. [2 marks]

6. Given the function $f(x)=\sqrt{-x-2}+1$ explain the transformations represented by $f(x)$ as compared to $y=\sqrt{x}$. [2 marks]

7. Write the equation for the following graph. Leave polynomials in factored form. [2 marks]



8. Given $f(x)=\sqrt{x-2}$ and given $g(x)=\frac{1}{x}$. What would be the restriction for the domain on the following operations. [3 marks]

a) f(x) + g(x)

b) $\frac{f(x)}{g(x)}$

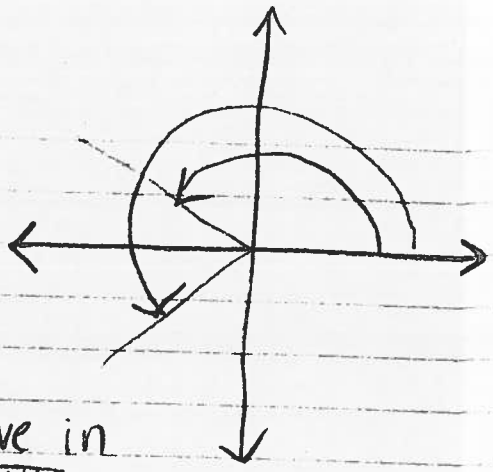
Quiz #1

1. $\sec\theta = -4$

$$\frac{1}{\cos\theta} = -4$$

$$\boxed{-\frac{1}{4} = \cos\theta}$$

$\theta_r = 1.3181\ldots$



cos -'ve in QII & III

QII: $1.8234\ldots + 2\pi k$

QIII: $4.4596\ldots + 2\pi k$

$k \in \mathbb{Z}$

$k \in \mathbb{Z}$

3. $\dfrac{\tan\theta \csc^2\theta}{\tan^2\theta + 1}$

$$\frac{\left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\sin^2\theta}\right)}{\sec^2\theta}$$

$$\frac{\frac{\sin\theta}{\sin^2\theta\cos\theta}}{\frac{1}{\cos^2\theta}}$$

$$\frac{1}{\sin\theta\cos\theta} \cdot \frac{\cos^2\theta}{1}$$

$$\frac{\cos\theta}{\sin\theta}$$

$\cot\theta$

$$\frac{\cos\theta}{\sin\theta}$$

LS = RS, identity is proved.

4. $e^x = 8^{1-x}$

$\ln e^x = \ln 8^{1-x}$

$x = (1-x)\ln 8$

$x = \ln 8 - x\ln 8$

$x + x\ln 8 = \ln 8$

$x(1 + \ln 8) = \ln 8$

$x = \frac{\ln 8}{1+\ln 8}$

$x \approx 0.6752\ldots$

5. a) ${}_7P_5 = \frac{7!}{2!}$ OR $7\times6\times5\times4\times3 =$

$= 2520$

b) P 1st : 1 way

TE together : 2!

arrange 5 "objects" : 5!

O, R, A, G, TE

$2!5! = 240$ ways.

6. $f(x) = \sqrt{-x-2} + 1$

$y - K = af(b(x-h))$

vertical shift ↑ vertical stretch ↑ reflect in x-axis if -'ve ↑ horizontal stretch $\frac{1}{|b|}$ ↑ reflect in y-axis if -ve ↑ horizontal shift

mapping: $\left(\frac{x}{b} + h,\ ay + K\right)$

ORDER OF OPERATIONS

① stretch/compress

② reflect

③ translate

$f(x) = \sqrt{-x-2} + 1 \rightarrow f(x) = \sqrt{-1(x+2)} + 1$

1. no stretch/no compressions 3. translated 2 left

7. 1. identify zeros & multiplicity
2. set up template polynomial eq'n.
3. use point of graph to solve for leading coefficient a
4. write the final poly. fxn.

① zeros: -3 multiplicity 1
-1 " 2
4 " 1

② $f(x) = a(x+3)(x+1)^2(x-4)$

③ pt: $(0,-10)$

$-10 = a(0+3)(0+1)^2(0-4)$
$-10 = -12a$
$a = \frac{5}{6}$

or

* $-90 = a(2+3)(2+1)^2(2-4)$
$-90 = a(5)(9)(-2)$
$-90 = -90a$
$a = 1$

$-50 = a(1+3)(1+1)^2(1-4)$
$-50 = a(4)(2)^2(-3)$
$-50 = -48a$
$a = 1.04$

$f(x) = \frac{5}{6}(x+1)^2(x+3)(x-4)$

8. $f(x) = \sqrt{x-2}$ $\quad g(x) = \frac{1}{x}$

a) $h(x) = \sqrt{x-2} + \frac{1}{x}$

domain of $f(x)$ is $x \geq 2$
domain of $g(x)$ is $x \neq 0$

so, domain of $h(x)$ is $x \geq 2$

b) $h(x) = \frac{\sqrt{x-2}}{\frac{1}{x}}$

$h(x) = \sqrt{x-2} \cdot x$
$h(x) = x\sqrt{x-2}$

$f(x)$ domain: $x \geq 2$
$g(x)$ domain: $x \neq 0$

$h(x)$ domain: $x \geq 2$

Provincial Exam Review Quiz #2

Q1, Q3

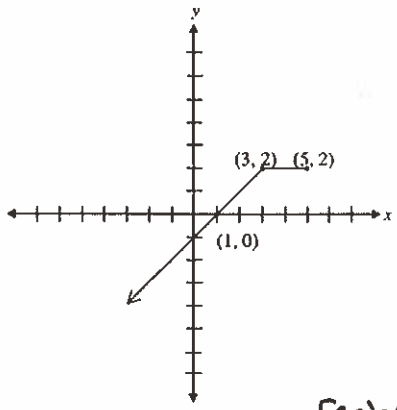
1. If $\sec\theta = -3$ and $\tan\theta > 0$, find the value of $\sin\theta$. [2 marks]

Q2, Q3

$\cos\theta = -\frac{1}{3}$ $x=-1$ $r=3$ $y = \sqrt{3^2-(-1)^2} = -\sqrt{8}$ $\sin\theta = \frac{y}{r} = \frac{-\sqrt{8}}{3}$

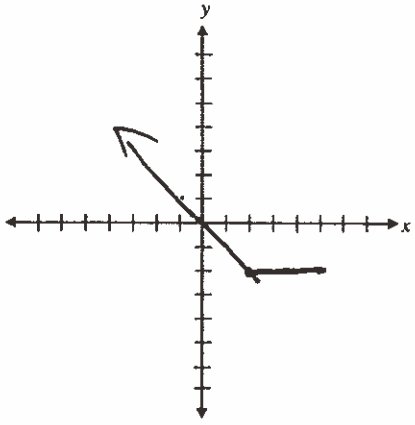
Q3

2. The graph of a function $y = f(x)$ is shown below.



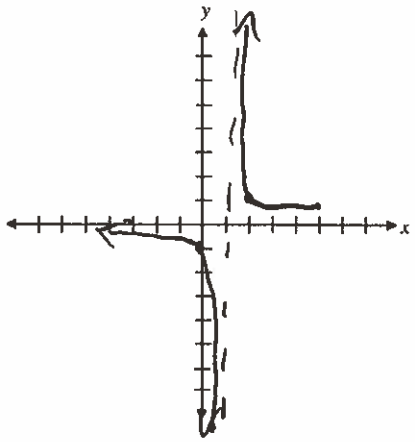
Sketch the graphs of:

a) $y = -f(x+1)$ [2 marks]



b) $y = \frac{1}{f(x)}$ [2 marks]

"reciprocal!" "$\frac{1}{y}$"



3. Find the exact value of: $\sin\frac{11\pi}{12}$ [3 marks]

$\sin\left(\frac{8\pi}{12} + \frac{3\pi}{12}\right) = \sin\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$

$= \sin\frac{2\pi}{3}\cos\frac{\pi}{4} + \cos\frac{2\pi}{3}\sin\frac{\pi}{4} = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{-1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

OR

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$ 3

4. Solve for x: $\log_9(\log_4 x) = \frac{1}{2}$. Give your answer in simplest form. [2 marks]

$9^{1/2} = \log_4 x \Rightarrow 3 = \log_4 x \Rightarrow 4^3 = x \quad x = 64$

5. Find and simplify the middle term of $(2a-b)^8$ [2 marks]

term 5

$t_5 = {}_8C_4 (2a)^4 (-b)^4$

$= 70 (16a^4)(b^4) = 1120\,a^4b^4$

6. Factor fully: $x^3 + 6x^2 + 11x + 6$ [3 marks]

Synth div! ... Try $\pm 1, 2, 3, 6$

$x = -1$ works ... ... $(x+1)(x+3)(x+2)$

7. Consider the function $f(x) = -\frac{2}{x+4}$. [4 marks]

a) Determine the key features of the function:

i) domain and range

ii) intercepts

iii) equations of any asymptotes

b) Sketch the graph of the function.

D: $(-\infty, -4) \cup (-4, \infty)$

R: $(-\infty, 0) \cup (0, \infty)$

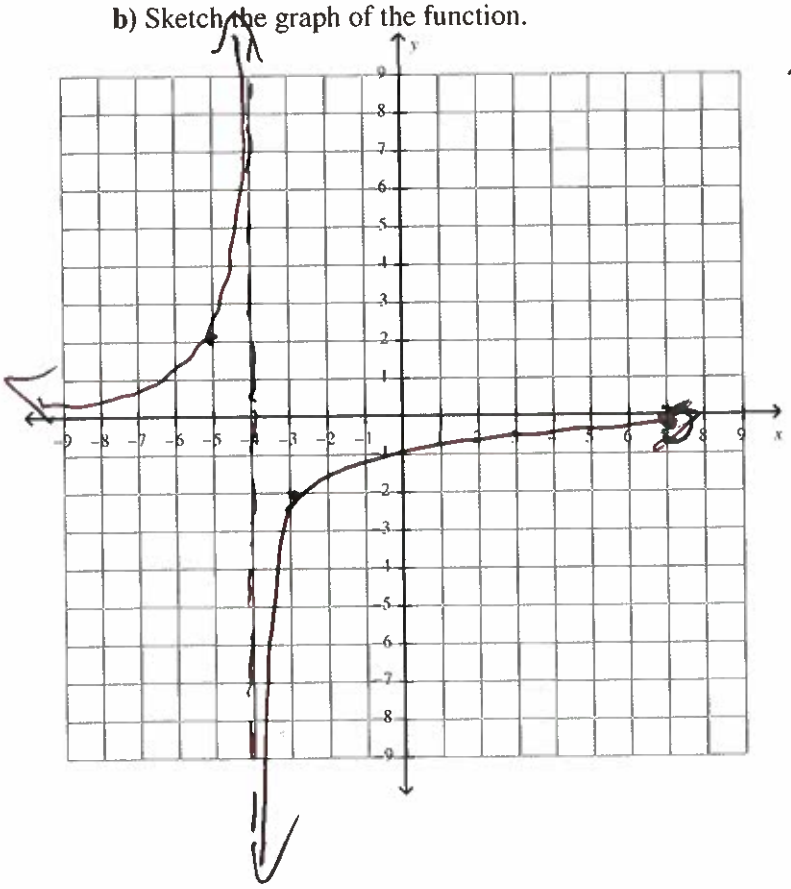
No x-int

~~y-int~~

y-int: $y = -1$

V.A @ $x = -4$

H.A @ $y = 0$



8. Given the functions $f(x) = \frac{1}{1-x^2}$ and $g(x) = \sin x$, determine the equation for $h(x) = f(g(x))$. [2 marks]

$h(x) = \frac{1}{1-\sin^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$

Provincial Exam Review Quiz #3

NO CALCULATORS!

1. The solutions to the equation $\csc\theta + 2 = 0$ in the interval $[0, 2\pi]$ are:

a) $\frac{\pi}{6}, \frac{5\pi}{6}$ b) $\frac{2\pi}{3}, \frac{4\pi}{3}$ c) $\frac{7\pi}{6}, \frac{11\pi}{6}$ d) no solution

1. C

2. An angle coterminal with 20° is:

a) -20° b) 160° c) 200° d) 380°

2. d

3. What is the period of the function $y = \cos(2x)$?

a) $\frac{\pi}{2}$ b) π c) 2π d) 4π

3. b

4. The point (4, -5) is on the graph of $y = f(x)$. On the graph $y = f(2x) - 3$, this point will be transformed to:

a) (8, -8) b) (8, -2) c) (2, -2) d) (2, -8)

4. d

5. If $\tan\theta = \frac{a}{b}$ and $\sin\theta = \frac{d}{c}$, then a possible expression for $\cos\theta$ is:

a) $\frac{bd}{ac}$ b) $\frac{ad}{bc}$ c) $\frac{ac}{bd}$ d) $1 - \left(\frac{d}{c}\right)^2$

5. a

6. The value of $\cos\frac{7\pi}{12}$ is equivalent to:

a) $\cos\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{4}\sin\frac{\pi}{3}$ b) $\cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

c) $\sin\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos\frac{\pi}{4}$ d) $\sin\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{3}\cos\frac{\pi}{4}$

6. b

7. If $\log_m 2 = r$ and $\log_m 5 = s$, then $\log_m 50$ is:

a) $2s + r$ b) $s^2 + r$ c) $s^2 r$ d) $\frac{s}{r}$

7. a

8. What is the maximum number of real roots that a cubic equation can have?

a) infinitely many b) 4 c) 3 d) 2

8. C

9. The term in the expansion of $\left(x^2 + \frac{1}{x}\right)^6$ which does not contain an x would be:

a) t_4 b) t_5 c) t_6 d) t_7

9. b

10. For a polynomial $P(x)$, if $P(-4) = 0$, then which of the following must be a factor of $P(x)$?

a) $x + 4$ b) $x^2 + 4$ c) $x - 4$ d) $x^2 - 4$

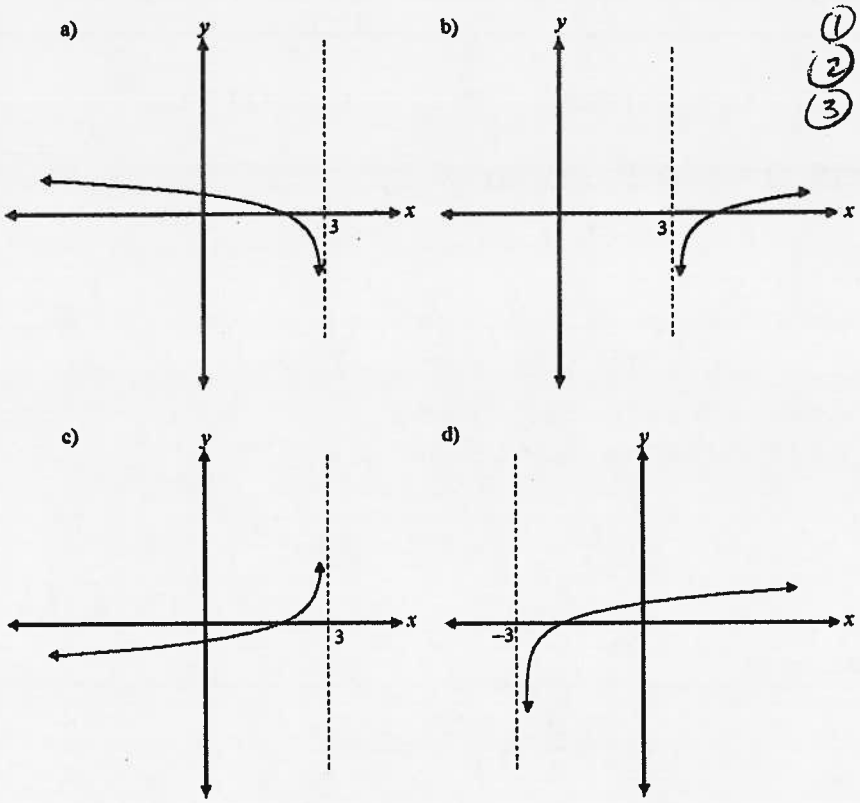
10. a

11. What are the coordinates of the invariant points(s) when the function $y = \sqrt{x} - 3$ is reflected in the y-axis?

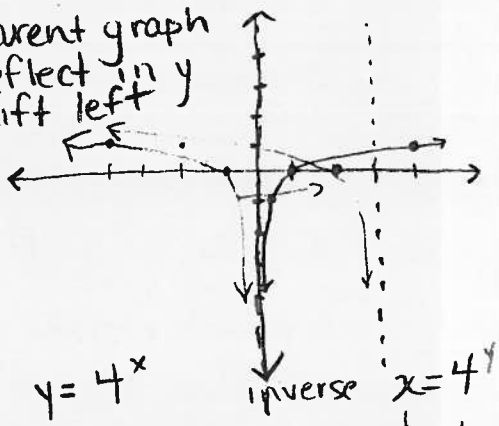
a) (9, -3) b) (-3, 0) and (9, 0) c) (0, -3) d) (0, 9)

11. C

12. The graph of the function $f(x) = \log_4[-(x-3)]$ is best described by which of the following?



① parent graph
② reflect in y
③ shift left



$y = 4^x$

x	y
-2	$\frac{1}{16}$
-1	$\frac{1}{4}$
0	1
1	4
2	16

inverse $x = 4^y$

x	y
$\frac{1}{16}$	-2
$\frac{1}{4}$	-1
1	0
4	1
16	2

12. a

13. How many x-intercepts are possible for the polynomial function $h(x) = ax^5 + bx^4 + cx^3$?

a) 4 b) 5 c) 3 d) 1

13. B

14. Determine the value of k so that $x + 2$ is a factor of $x^3 + 10x^2 + 23x + k$

a) -1 b) -14 c) 14 d) 1

14. C

15. Which function has a point of discontinuity at $x = 5$?

a) $f(x) = \dfrac{x-5}{3x^2 - 22x + 35}$ $\dfrac{x-5}{(3x-7)(x-5)}$ b) $f(x) = \dfrac{x+5}{x^2 - 10x + 35}$

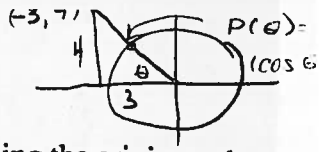
c) $f(x) = \dfrac{x-5}{x^2 - 10x + 35}$ d) $f(x) = \dfrac{x+5}{x^2 - 10x + 25}$

15. a

16. Which function has a horizontal asymptote with equation $y = \dfrac{2}{7}$?

a) $f(x) = \dfrac{-2x-3}{7x+8}$ b) $f(x) = \dfrac{7x+8}{2x-3}$ c) $f(x) = \dfrac{3x+2}{8x+7}$ d) $f(x) = \dfrac{2x-3}{7x+8}$

16. D



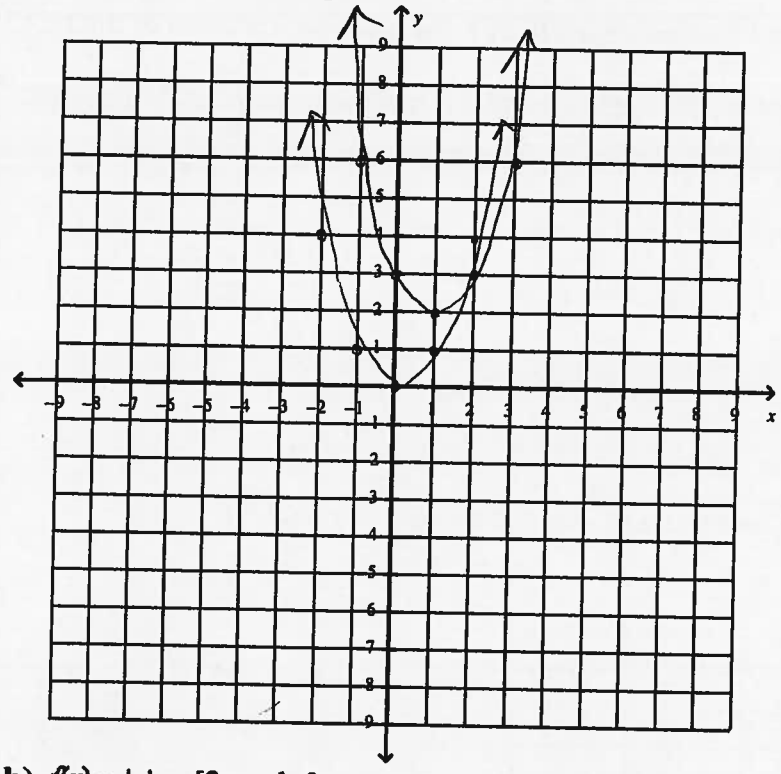
Provincial Exam Review Quiz #4

1. The point $P(\theta)$ lies on the intersection of the unit circle and the line segment joining the origin to the point (-3, 4). Find the coordinates of $P(\theta)$. [2 marks]

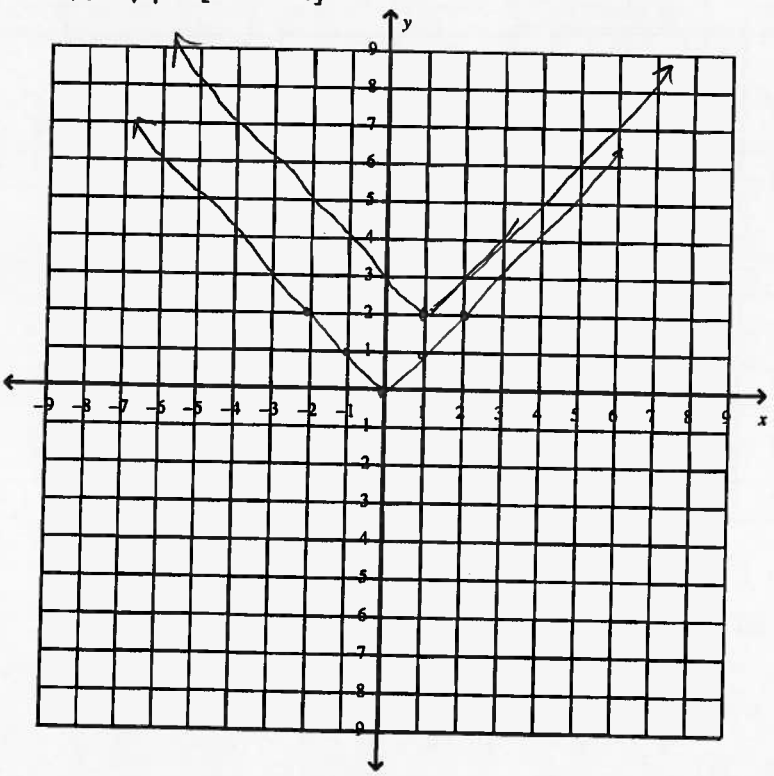
$\left(\frac{-3}{5}, \frac{4}{5}\right)$

2. Create a graph of $g(x) = f(x-1) + 2$ for each base function given, using transformations.

a) $f(x) = x^2$ [2 marks]



b) $f(x) = |x|$ [2 marks]



3. $\frac{\sin x}{\cos x + 1} + \frac{\cos x + 1}{\sin x}$

$\frac{\sin^2 x + \cos^2 x + 2\cos x + 1}{\sin x(\cos x + 1)}$

$\frac{1 + 2\cos x + 1}{\sin x(\cos x + 1)}$

$\frac{2(\cos x + 1)}{\sin x(\cos x + 1)}$

$\frac{2}{\sin x}$

2 csc

2(1/sin

2/sin

(OVER)

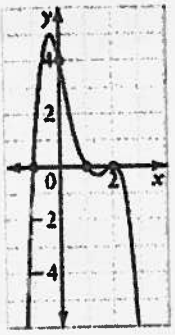
3. Prove the identity: [4 marks]

$$\frac{\sin x}{\cos x+1}+\frac{\cos x+1}{\sin x}=2\csc x$$

4. Solve for x algebraically: $\log_2(x-2)+\log_2(x+1)-2=0$. [4 marks]

5. How many numbers between 99 and 999 are divisible by 5 and have no repetition of digits? [3 marks]

6. Determine an equation in factored form for the polynomial function represented by the graph



$y = a(x-1)(x+1)(x-2)^2$

pt. $(0,4)$

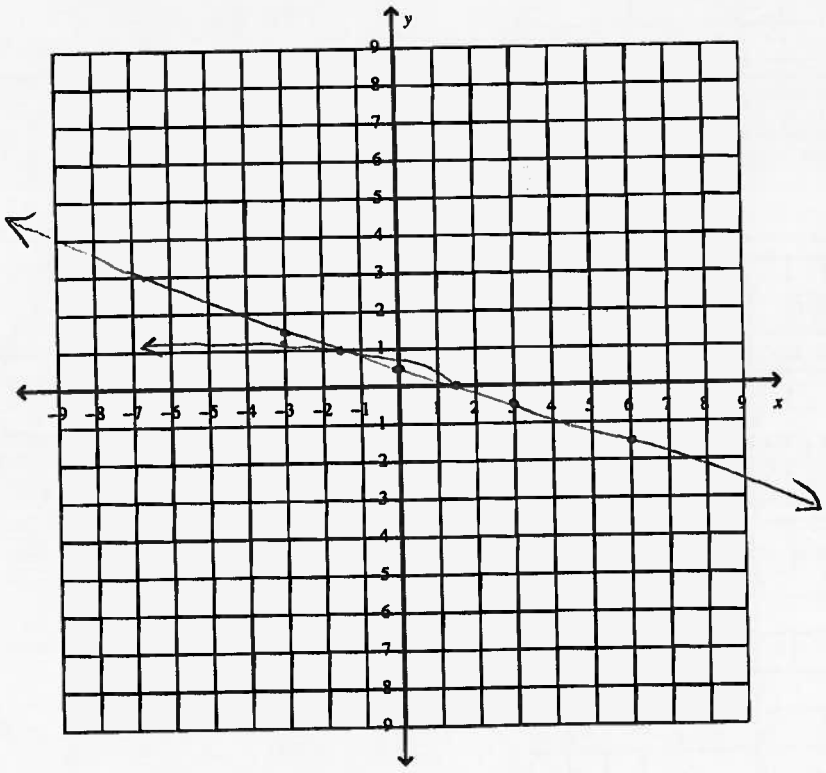
$4 = a(0-1)(0+1)(-2)^2$

$4 = -4a$

$a = -1$

$y = -(x-1)(x+1)(x-2)^2$

7. Sketch the graph of $f(x) = \frac{-2x+3}{6}$ and use it to sketch the graph of $y = \sqrt{f(x)}$.



$y = \frac{-1}{3}x + \frac{1}{2}$

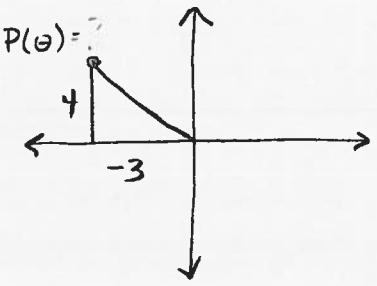
invariant pts: at $y=0$ & $y=1$

8. Given the functions $f(x) = x^2 - 7$ and $g(x) = 2 - x^3$, determine a simplified equation for $h(x) = f(x)g(x)$.

$h(x) = (x^2-7)(2-x^3)$

$= 2x^2 - 14 - x^5 + 7x^3$

$= -x^5 + 7x^3 + 2x^2 - 14$

1. 

$r = 5$ $\quad \cos\theta = \frac{-3}{5}$, $\sin\theta = \frac{4}{5}$

$$\left(\frac{-3}{5}, \frac{4}{5}\right)$$

3. $\dfrac{\text{LHS}}{\dfrac{\sin x}{\cos x+1} + \dfrac{\cos x+1}{\sin x}} \qquad \dfrac{\text{RHS}}{2\csc x}$

$$\frac{\sin x}{\cos x+1}\cdot\frac{\sin x}{\sin x} + \frac{\cos x+1}{\sin x}\cdot\frac{\cos x+1}{\cos x+1}$$

$$\frac{\sin^2 x + \cos^2 x + 2\cos x + 1}{\sin x(\cos x+1)}$$

$$\frac{1 - \cancel{\cos^2 x} + \cancel{\cos^2 x} + 2\cos x + 1}{\sin x(\cos x+1)}$$

$$\frac{2\cos x + 2}{\sin x(\cos x+1)}$$

$$\frac{2(\cos x+1)}{\sin x(\cos x+1}$$

$$\frac{2}{\sin x}$$

$$2\csc x$$

4. $\log_2(x-2) + \log_2(x+1) - 2 = 0$

$$\log_2(x-2)(x+1) = 2$$

$$2^2 = (x-2)(x+1)$$

$$4 = x^2 - x - 2$$

$$0 = x^2 - x - 6$$

$$0 = (x-3)(x+2)$$

$$x = 3, \cancel{-2}$$

$$x = 3$$

5. Case 1: ends in 0

$\underline{9} \times \underline{8} \times \underline{1}_{0} = 72$

Case 2: ends in 5

$\underline{8}_{\text{not 0}} \times \underline{8} \times \underline{1}_{5} = 64$

136

6. x-int: $-1, 1, 2$ pt: $(0, 4)$

$y = a(x+1)(x-1)(x-2)^2$

$4 = a(0+1)(0-1)(0-2)^2$

$4 = -4a$

$a = -1$

$y = -(x+1)(x-1)(x-2)^2$

8. $f(x) = x^2 - 7$

$g(x) = -x^3 + 2$

$h(x) = (x^2-7)(-x^3+2)$

$h(x) = -x^5 + 7x^3 + 2x^2 - 14$

4. $\log_2(x-2) + \log_2(x+1) - 2 = 0$

$\log_2(x-2)(x+1) = 2$

$2^2 = (x-2)(x+1)$

$4 = x^2 - x - 2$

$0 = x^2 - x - 6$

$0 = (x-3)(x+2)$

$x = 3, \cancel{-2}$

5. CASE 1: end in 0

$\frac{9}{} \times \frac{8}{} \times \frac{1}{0} = 72$

CASE 2: end in 5

$\frac{8}{} \times \frac{8}{} \times \frac{1}{5} = 64$

$72 + 64 = 136$

Quiz 5

1. $x = \begin{cases} 1.1071 + \pi k, & k \in \mathbb{Z} \\ 2.0343 + \pi k, & k \in \mathbb{Z} \end{cases}$

2. $A = 9, \quad B = \frac{\pi}{12}, \quad C = 10, \quad D = 6$

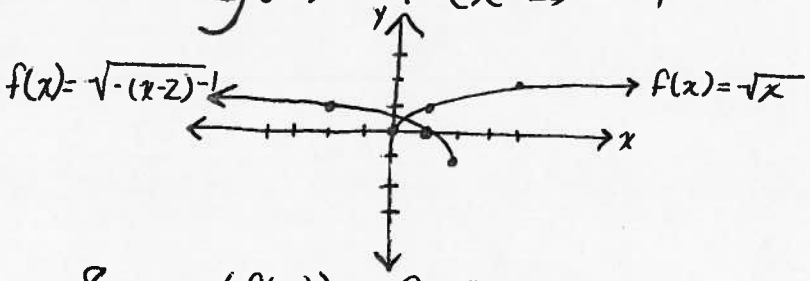
3. $x = \pi$

4. $x = 0.5631$

5. $n = 12$

6. zeros: $-1, 2, 3$

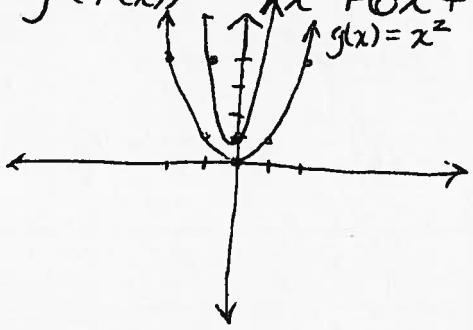
7. a) $g(x) = \sqrt{-(x-2)} - 1$



b) $g(0) = \sqrt{-(0-2)} - 1 = \sqrt{2} - 1$

c) y-int.

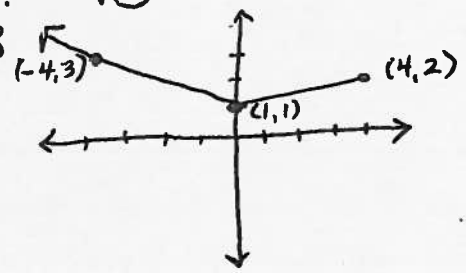
8. $g(f(x)) = 9x^2 + 6x + 1$

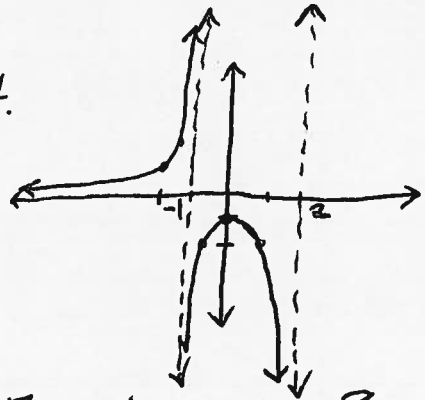


Quiz #6

1. B	5. A	9. A	13. B
2. C	6. B	10. B	14. C
3. D	7. B	11. B	15. B
4. D	8. A	12. C	16. A

Quiz #7

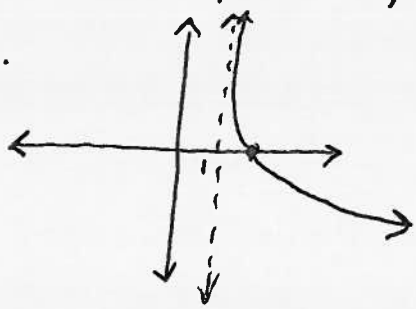
1. omit
2. $-\sqrt{3}$
3. 

4. 

5. $\frac{\sqrt{2}}{2}$
6. 1
7. $\frac{1}{2}$
8. $x=\frac{3}{2}$
9. $n=5$
10. 7
11. 4
12. $y=\frac{-1}{2}$
13. D: $-2, 2, 3$
14. D: $x \in \mathbb{R}, x \neq 1$, R: $y \in \mathbb{R}$
15. $y=\frac{(x+1)}{(x+1)(x-2)}$
16. $x=-1/2$

Quiz #8

1. $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}, 1.2490, 4.3906$

2. 

3. 1

4. 15.967 yrs.

5. 192

6.

$y = f(x)$	$y = \sqrt{f(x)}$
D: $x \in \mathbb{R}$	D: $x \geq 2$, $x \in \mathbb{R}$
R: $y \in \mathbb{R}$	R: $y \geq 0$, $y \in \mathbb{R}$

Domains differ because square root of a -'ve number is not defined.
Range is different b/c $\sqrt{f(x)}$ is always +'ve.

7. 18

8. $K(x) = x^2 + 5x + 6$

Quiz #9

1. $\theta = 72°$

2. $f(f(f(3))) = 2$

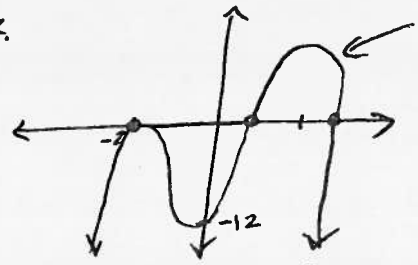
3.
$$\frac{1 - \tan^2\theta}{\sec^2\theta}$$
$$\frac{1 - \frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}$$
$$\frac{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}$$
$$\cos^2\theta - \sin^2\theta$$

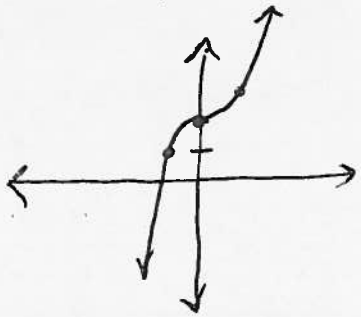
$$\cos 2\theta$$
$$\cos^2\theta - \sin^2\theta$$

4. $A = 1324000$

5. 42 (Pizza One) (48 (Pizza Zone))

6. D: $x \geq -4$, $x \in \mathbb{R}$

7. 

8. 
x-int: -1.26

Quiz #10

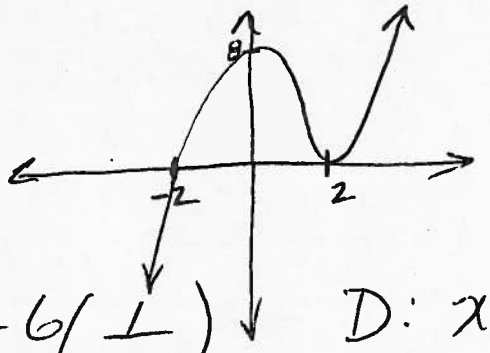
1. $\theta = \begin{cases} \pi/6 + 2\pi K, & K \in \mathbb{Z} \\ 5\pi/6 + 2\pi K, & K \in \mathbb{Z} \end{cases}$

2. $g(x) = 2(f(-2x)) + 1$

3. $\frac{33}{56}$

4. $x \approx 1.2639$

5. -3168

6. 

7.

8.a) $g(f(x)) = \left(\frac{1}{(x+5)}\right)^2 - 6\left(\frac{1}{x+5}\right)$ D: $x \in \mathbb{R}$, $x \neq -5$

b) $f(g(x)) = \frac{1}{(x-2)(x-3)}$ D: $x \in \mathbb{R}$, $x \neq 3, 2$

Quiz #5

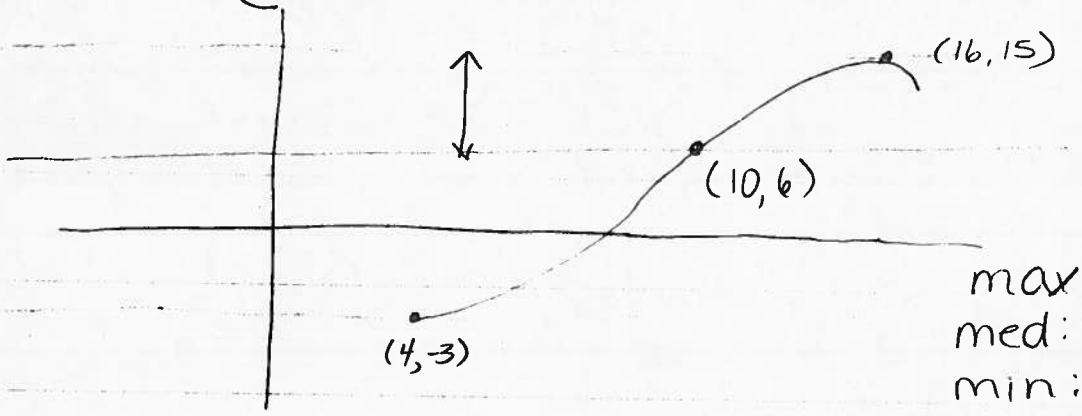
1. $\cos(2x) = -\frac{3}{5}$ Q: II & III

$x_r = 0.9272...$

$2x = \begin{cases} 2.2143... + 2\pi K, & K \in \mathbb{Z} \\ 4.0687... + 2\pi K, & K \in \mathbb{Z} \end{cases}$

$x = \begin{cases} 1.1071 + \pi K, & K \in \mathbb{Z} \\ 2.0343 + \pi K, & K \in \mathbb{Z} \end{cases}$

2.



max : 15
med : $\frac{16+(-3)}{2} = 6$
min : -3

$y = a \sin[b(x-c)] + d$

$12 = \frac{1}{2}$ period

$24 =$ period

$b = \frac{2\pi}{24}$

$= \frac{\pi}{12}$

using sine $c = 10$

$a = \frac{15-(-3)}{2} = \frac{18}{2} = 9$

$\therefore \quad y = 9 \sin\left[\frac{\pi}{12}(x-10)\right] + 6$

$d = 6$

3. $\sin^2 x - 3\cos x = 3$

$1 - \cos^2 x - 3\cos x - 3 = 0$

$\cos^2 x + 3\cos x + 2 = 0$

$(\cos x + 1)(\cos x + 2) = 0$

$\cos x = -1$ OR $\cos x = -2$ ~~-2~~

$x = \pi$

4. $3^{x+1} = 4(5^x)$

$\log(3^{x+1}) = \log(4 \cdot 5^x)$

$(x+1)\log 3 = \log 4 + x\log 5$

$x\log 3 + \log 3 = \log 4 + x\log 5$

$x\log 3 - x\log 5 = \log 4 - \log 3$

$x(\log 3 - \log 5) = \log 4 - \log 3$

$\frac{x(\log 3 - \log 5)}{\log 3 - \log 5} = \frac{\log 4 - \log 3}{\log 3 - \log 5}$

$x = -0.5631\ldots$

5. $\left(x^3 - \frac{4}{x}\right)^n$

$t_{K+1} = {}_nC_K\, a^{n-K} b^K$ $\quad n = ?,\ K = 5$

$x^{16} = (x^3)^{n-5}\left(\frac{4}{x}\right)^5$ ← this affect the coefficient only

$x^{16} = x^{3n-15}\left(\frac{4^5}{x^5}\right)$

$x^{16} = \frac{x^{3n-15}}{x^5}$

$x^{16} = x^{3n-20}$

$16 = 3n - 20$

$n = 12$

Quiz #6

1. $\tan\theta = \frac{2}{3}$ $\sin\theta < 0$ & III

$y = 2$
$x = 3$

$r = \sqrt{x^2+y^2}$
$r = \sqrt{4+9}$
$r = \pm\sqrt{13}$
$r = -\sqrt{13}$

$\sec\theta = \frac{1}{\cos\theta}$
$= \frac{1}{\frac{3}{-\sqrt{13}}}$
$= -\frac{\sqrt{13}}{3}$

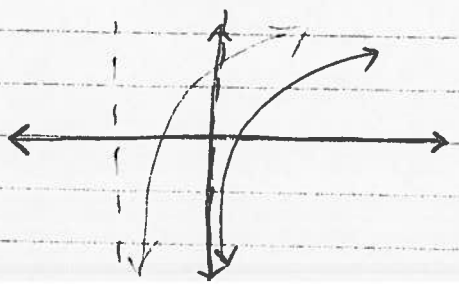
2. $a = 2(1) = 2$

3. $(9, -12) \rightarrow (3x, -y-5)$
$\rightarrow (27, 7)$
inverse $(7, 27)$ d

5. $\cos(\pi + 2A) = \cos\pi\cos 2A - \sin\pi\sin 2A$
$= -1(\cos 2A) - 0$
$= -\cos 2A$

6. $\sin 4x \cos 3x - \cos 4x \sin 3x$
$= \sin(4x - 3x)$
$= \sin x$

7.



8. $\log m - \log n - 3\log K$

$\log \frac{m}{nK^3}$

9. $t_{7+1} = {}_{11}C_7 (2x)^4 (-y)^7$

$= \frac{11!}{4!7!} (16x^4)(-y^7)$

$= \frac{11 \cdot 10 \cdot 9 \cdot 8}{4 \cdot 3 \cdot 2}$

$= -5280 x^4 y^7$

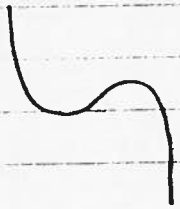
10. BALLOON

$\frac{7!}{2!2!} = \frac{5040}{4} = 1260$

* 11.

-6	1	0	-8	9	8
	↓	-6	36	-168	954
	1	-6	28	-159	962

12.



14. a) $0 = -6x - 2$

$\frac{2}{-6} = \frac{-6x}{-6}$

$x = -\frac{1}{3}$

b) $0 = 5x - 3$

$3 = 5x$

$x = \frac{3}{5}$

c) $0 = 6x - 2$

$\frac{2}{6} = \frac{6x}{6}$

$x = \frac{1}{3}$

15. $y = \dfrac{x^3 - 81x}{x+9}$

$y = \dfrac{x(x^2-81)}{x+9}$

$y = \dfrac{x(x+9)(x-9)}{x+9}$

$y = x(x-9) \qquad x \neq -9$

16. $h(x) = \dfrac{-7\sqrt{x} - 5}{\sqrt{x} + 9} \qquad x \geq 0.$

* 1. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$

2. $\cot\left(\frac{11\pi}{6}\right)$ $\qquad \frac{11\pi}{6} \cdot \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$$\cot\left(\frac{11\pi}{6}\right) = \frac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}}$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{-2}{1}$$

$$= -\sqrt{3}$$

5 $\cos 2\alpha = 2\cos^2\alpha - 1$

$$\cos\left(2 \cdot \frac{\pi}{8}\right) = \cos\frac{\pi}{4}$$

$$= \frac{\sqrt{2}}{2}$$

5. $$\frac{\left(\cos x + \frac{\sin^2 x}{\cos x}\right)}{\sec x}$$

$$\frac{\frac{\cos^2 x + \sin^2 x}{\cos x}}{\frac{1}{\cos x}}$$

$$\frac{\frac{1}{\cos x}}{\frac{\cos x}{1}} = 1$$

7. $\log_5 \sqrt{5}$

$= \log_5 5^{1/2}$

$= \frac{1}{2} \log_5 5$

$= \frac{1}{2}$

8. $5^{x-1} = \sqrt{5}$

$5^{x-1} = 5^{1/2}$

$x - 1 = 1/2$

$2x - 2 = 1$

$2x = 3$

$x = \frac{3}{2}$

9. $\frac{(n-1)!}{n!} = \frac{1}{5}$

$\frac{(n-1)!}{n(n-1)!} = \frac{1}{5}$

$\frac{1}{n} = \frac{1}{5}$

$n = 5$

10. 7

11. $P(1) = (1)^{10} - 15(1) + 18$

$= 1 - 15 + 18$

$= 4$

12. $f(x) = 2x + 3$

$y = 2x + 3$

$x = 2y + 3$

$x - 3 = 2y$

$y = \frac{x-3}{2}$

$y = \frac{2-3}{2}$

$y = \frac{-1}{2}$

13. $f(x) = \{(1, -2), (9, 2), (3, 4)\}$

D:

14. $h(x) = \frac{-2}{x-1}$

D: $x \in \mathbb{R}, x \neq 1$

R: $y \in \mathbb{R}$

15. $y = \frac{(x+1)}{(x+1)(x-2)}$

16. $h(x) = f(2x)$
$= \sqrt{2x+1}$

$2x + 1 \geq 0$
$x \geq -\frac{1}{2}$

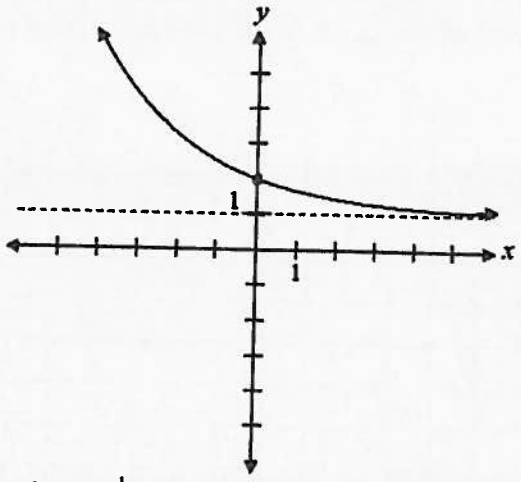
D: $x \geq 0$



Provincial Exam Review Quiz #8

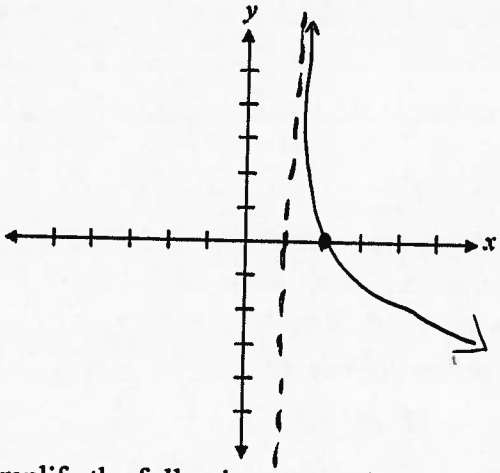
1. Solve the following equation on the interval $[0, 2\pi]$. Express your answers as exact values or correct to 3 decimal places. $3\cot^2\theta+2\cot\theta-1=0$ [4 marks]

2. The graph of $y = f(x)$ is shown below.



a) Sketch a clearly labeled graph of $y = f^{-1}(x)$. [2 marks]

b) State the domain of $y = f^{-1}(x)$. [1 mark]

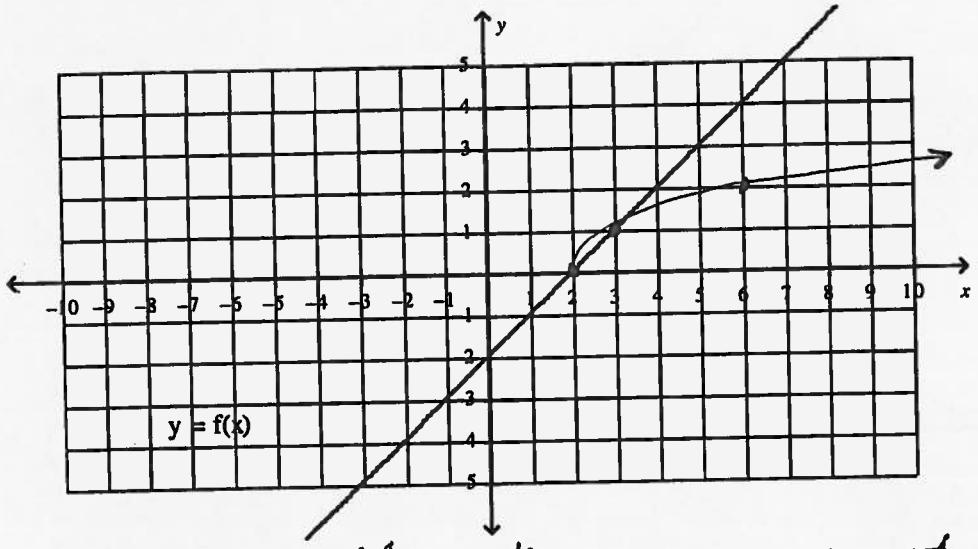


3. Simplify the following expression as much as possible: $(\sin x + \cos x)^2 - \sin 2x$. [3 marks]

4. An investment earns interest at an annual rate of 7% compounded semi-annually. How long will it take, in years, for the investment to triple? [3 marks]

5. How many different 4-letter arrangements can be formed from the 6 letters REPEAT? [3 marks]

6. For the graph of $y = f(x)$ shown below, sketch the graph of $y = \sqrt{f(x)}$.
State the domain and range of each function. If the domains and ranges are different, explain why.



$y = f(x)$
D: $x \in \mathbb{R}$
R: $y \in \mathbb{R}$

$y = \sqrt{f(x)}$
D: $x \geq 2$
R: $y \geq 0$

Domains differ b/c square root of a # is not defined.
Range differs b/c $\sqrt{f(x)}$ is always +ve

7. Brandon, Brenda, and Brad are triplets. They were born 3 years after their sister, Janice. This year, the product of their four ages is 60 687 greater than the sum of their ages. How old is Janice? Show your work.

Janice = $x+3$
Triplets = x

$(x+3)(x)(x)(x) = x+3+x+x+x+60\,687$
$x^3(x+3) = 4x + 60\,690$
$x^4 + 3x^3 - 4x - 60\,690 = 0$
$P(15) = 15^4 + 3(15)^3 - 4(15) - 60\,690$
$= 50625 + 10125 + 60 - 60690$
$= 0$
$x = 15$

Janice is 18.

8. Given the functions $f(x) = x^2 + 3x + 2$ and $g(x) = x + 1$, determine a simplified equation for $k(x) = f(g(x))$.

$K(x) = f(x+1)$
$= (x+1)^2 + 3(x+1) + 2$
$= x^2 + 2x + 1 + 3x + 3 + 2$
$= x^2 + 5x + 6$

Quiz #8

1. $3\cot^2\theta + 2\cot\theta - 1 = 0$

$(3\cot\theta - 1)(\cot\theta + 1) = 0$

$\cot\theta = \frac{1}{3}$, $\cot\theta = -1$

$\theta_r = 0.7854$

$\tan\theta = 3$, $\tan\theta = -1$

$\theta = \begin{cases} \frac{3\pi}{4}, \frac{7\pi}{4} \\ 1.2490, 4.3906 \end{cases}$

3. $(\sin x + \cos x)^2 - \sin 2x$

$= \sin^2 x + 2\sin x\cos x + \cos^2 x - 2\sin x\cos x$

$= \sin^2 x + \cos^2 x$

$= 1$

4. $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$3x = x\left(1 + \frac{0.07}{2}\right)^{2t}$

$3 = (1.035)^{2t}$

$\log 3 = 2t \log 1.035$

$\frac{\log 3}{2\log 1.035} = t$

$15.9675 \text{ yrs} = t$

$r = 7\% = 0.07$

$n = 2$

$P = x$

5. comb & perm

Case 1: no E's $4! = 24$

Case 2: 1 E ${}_4C_3 \cdot 4! = 96$

Case 3: 2 E's ${}_4C_2 \cdot \frac{4!}{2!} = 72$

192

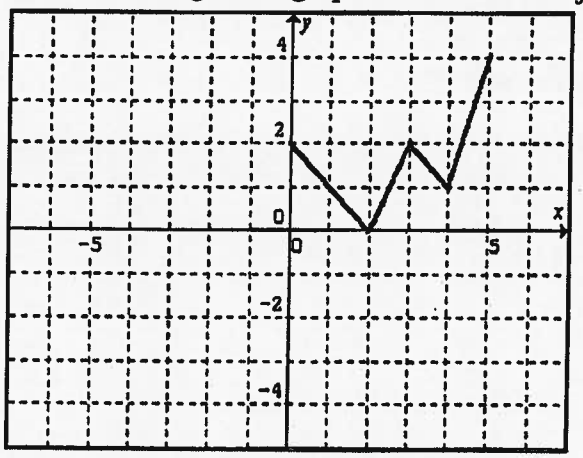
Provincial Exam Review Quiz #9

1. A circle has a radius of 10 m. A wire 4π metres long is stretched along the circumference. What is the central angle subtended by the arc in degrees?

$r = 10\text{ m}$
$S = 4\pi$

$$\theta = \frac{S}{r} = \frac{4\pi}{10} = \frac{2\pi}{5}\left(\frac{180}{\pi}\right) = 72^\circ$$

2. The following is the graph of the function $f(x)$. Find $f(f(f(3)))$.



3. Prove the identity: $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$.

$$\frac{1-\tan^2\theta}{\sec^2\theta}$$
$$\frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}$$
$$\frac{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}$$

$\cos^2\theta - \sin^2\theta$ | $\cos^2\theta - \sin^2\theta$

4. The approximate population of Noland in 1972 was 680,000. In 1984, it was 905,000. If the population is growing exponentially modeling the formula, $A = Pe^{rt}$, where A is the population after t time in years, P is the initial population, and r is the growth rate. What would the population be in the year 2000? Give your answer to the nearest thousand.

$A = Pe^{rt}$
$905\,000 = 680\,000\,e^{12r}$
$\frac{905}{680} = e^{12r}$
$\ln\frac{905}{680} = \ln e^{12r}$
$\ln\frac{905}{680} = 12r$
$r \approx 0.0238\ldots$

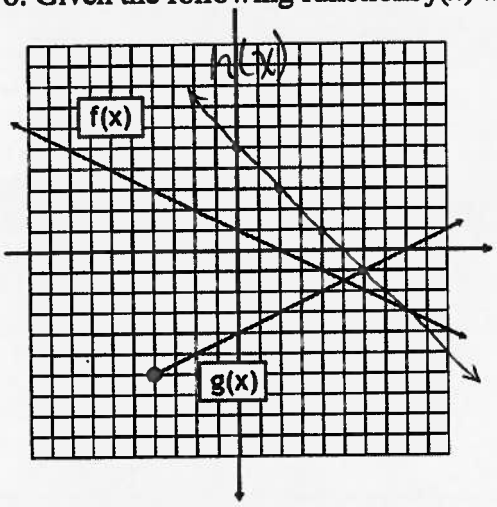
$P = 680\,000$
$A = 905\,000$
$r = ?$
$t = 12\text{ yrs}$
$A = Pe^{rt}$
$A = 680\,000\,e^{(0.0238)(28)}$
$A = 1324106$
$A = 1324000$

5. Pizza One offers 2 types of crusts, 3 types of cheeses, and 7 types of meats to make a pizza.
Pizza Zone offers 3 types of crusts, 4 types of cheeses, and 4 types of meats to make a pizza.
If a pizza consist of one type of crust, one type of cheese, and one type of meat, who offers more pizza selections? Justify.

$\frac{2}{\text{crust}} \times \frac{3}{\text{cheese}} \times \frac{7}{\text{meats}} = 42$

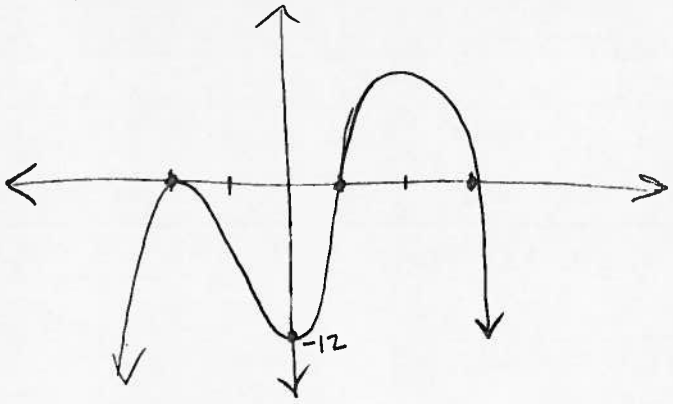
$3 \times 4 \times 4 = 48$ Pizza Zone

6. Given the following functions $f(x)$ and $g(x)$, graph and state the domain of $h(x)$ if $h(x) = f(x) - g(x)$.



D: $x \in \mathbb{R}$ $(\infty, -\infty)$

7. Sketch the graph of the function $f(x) = (1-x)(x+2)^2(x-3)$. Label all intercept(s).



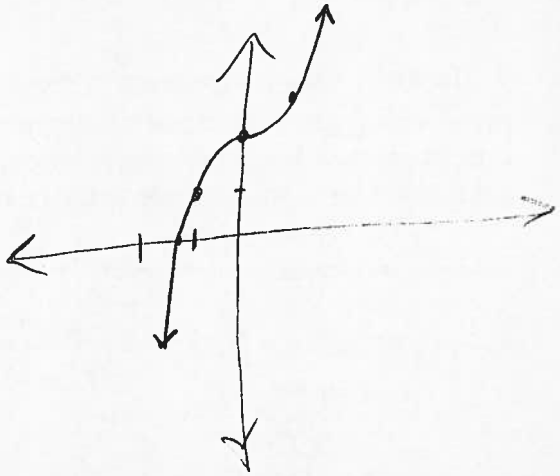
$f(x) = (-x+1)(x+2)^2(x-3)$
$= -(x-1)(x+2)^2(x-3)$

$y = -(0-1)(0+2)^2(0-3)$
$y = -(-1)(4)(-3)$
$y = -12$

8. Given $f(x) = x + 2$ and $g(x) = x^3$, sketch $f(g(x))$.

$f(g(x)) = f(x^3)$
$= (x^3) + 2$
$= x^3 + 2$

$0 = x^3 + 2$
$-2 = x^3$
$x = -1.26$



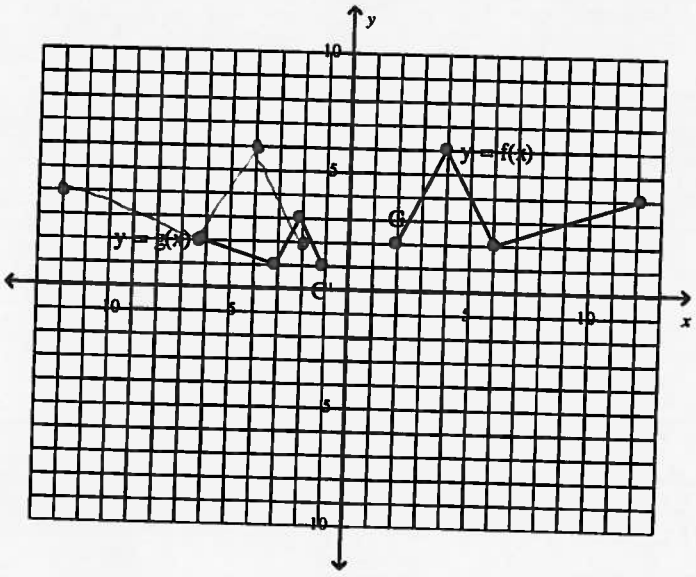
Provincial Exam Review Quiz #10

1. Solve for θ where $0 \le \theta \le 2\pi$: $\cos 2\theta = \frac{1}{2}$.

$2\theta = \begin{cases} \frac{\pi}{3} \\ \frac{5\pi}{3} \end{cases}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

2. The graph of $y = g(x)$ is the image of the graph of $y = f(x)$ after a combination of transformations. What is an equation of the image graph in terms of the function f?



$-2(2x)$

$y = \frac{1}{2} f(-2x)$

3. If α is in Quadrant II with $\sin\alpha = \frac{3}{5}$ and β is in Quadrant III with $\tan\beta = \frac{12}{5}$, find the exact value of $\tan(\alpha+\beta)$.

$\tan\alpha = \frac{-3}{4}$

$\tan\beta = 12/5$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \frac{-\frac{3}{4} + \frac{12}{5}}{1 - (-3/4)(12/5)}$

$= \frac{\frac{-15+48}{20}}{1 + 36/20}$

$= \frac{\frac{33}{20}}{\frac{56}{20}} = \frac{33}{56}$

4. Solve for x: $e^{3x-2} = 6$.

$\ln e^{3x-2} = \ln 6$

$3x - 2 = \ln 6$

$x = \frac{\ln 6 + 2}{3} \approx 1.2639\ldots$

5. Find the numerical coefficient of the term in simplified form containing x^{25} in $\left(2x^5 - \frac{1}{2x^2}\right)^{12}$.

$t_{K+1} = {}_nC_K\, a^{n-K} b^K$

$x^{25} = (2x^5)^{12-K}\left(\frac{-1}{2x^2}\right)^K$

$x^{25} = \frac{x^{60-5K}}{x^{2K}}$

$x^{25} = x^{60-7K} \quad K = 5$

$25 = 60 - 7K$

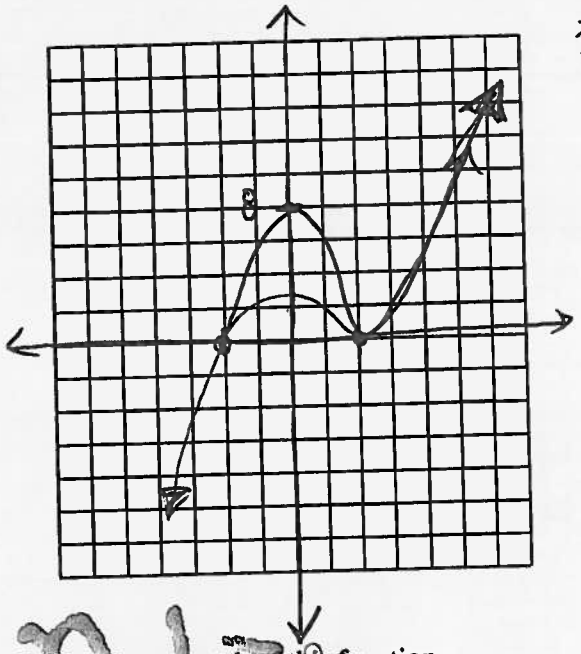
$t_{5+1} = {}_{12}C_5\,(2x^5)^7\left(\frac{-1}{2x^2}\right)^5$

$t_6 = \frac{12!}{7!5!}(128x^{35})\left(\frac{-1}{32x^{10}}\right)$

$= 792(4x^{35})\left(\frac{-1}{x^{10}}\right)$

$= -3168x^{25}$

6. Sketch the graph of the polynomial function $f(x) = x^3 - 2x^2 - 4x + 8$.



$x-2$

$$\begin{array}{c|cccc} 2 & 1 & -2 & -4 & 8 \\ & \downarrow & 2 & 0 & -8 \\ \hline & 1 & 0 & -4 & 0 \end{array}$$

$= (x-2)(x^2-4)$

$= (x-2)(x+2)(x-2)$

$= (x-2)^2(x+2)$

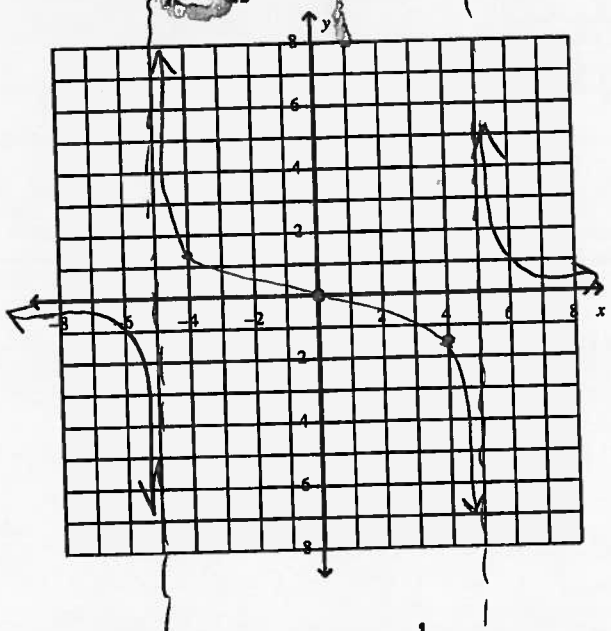
$y = 0 - 0 - 0 + 8$

$y = 8$

7. Sketch the graph of this function.

$y = \dfrac{3x}{x^2-25}$

$y = \dfrac{3x}{(x+5)(x-5)}$



$\dfrac{12}{9} = -1\frac{3}{9}$ $\quad \dfrac{6}{-21}$

$\dfrac{-12}{-9} = 1\frac{3}{9}$ $\quad \dfrac{-18}{11}$

$\dfrac{18}{11} = 1\frac{7}{11}$

8. Given the functions $f(x) = \dfrac{1}{x+5}$ and $g(x) = x^2 - 6x$, determine an explicit equation for each composite function, then state its domain. Show your work.

a) $g(f(x))$

b) $f(g(x))$

a) $g\left(\frac{1}{x+5}\right)$

$= \left(\frac{1}{x+5}\right)^2 - 6\left(\frac{1}{x+5}\right)$

$= \frac{1}{\quad} - \frac{6}{x+5}$

b) $f(g(x))$

$= f(x^2-6x)$

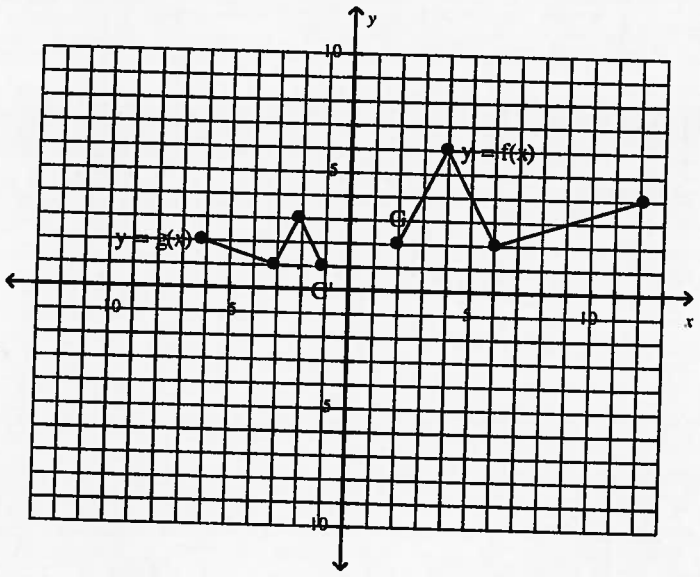
$= \dfrac{1}{x^2-6x+5}$

$= \dfrac{1}{(x-1)(x-5)}$

Provincial Exam Review Quiz #10

1. Solve for θ where $0 \leq \theta \leq 2\pi$: $\cos 2\theta = \frac{1}{2}$.

2. The graph of $y = g(x)$ is the image of the graph of $y = f(x)$ after a combination of transformations. What is an equation of the image graph in terms of the function f?



a = 2
b = -2
c =
up 1

$g(x) = 2(f(-2x)) + 1$

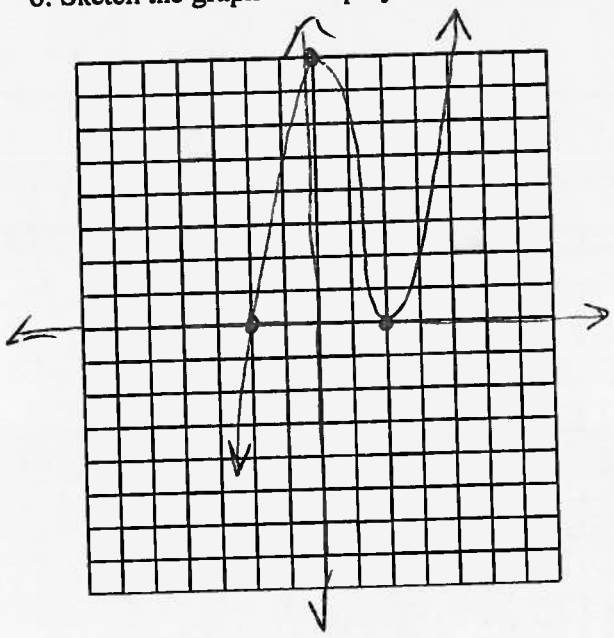
3. If α is in Quadrant II with $\sin\alpha = \frac{3}{5}$ and β is in Quadrant III with $\tan\beta = \frac{12}{5}$, find the exact value of $\tan(\alpha + \beta)$.

4. Solve for x: $e^{3x-2} = 6$.

5. Find the numerical coefficient of the term in simplified form containing x^{25} in $\left(2x^5 - \frac{1}{2x^2}\right)^{12}$.

y-int: 8

6. Sketch the graph of the polynomial function $f(x) = x^3 - 2x^2 - 4x + 8$.



$f(-1) = (-1)^3 - 2(-1)^2 - 4(-1) + 8$
$= -1 - 2 + 4 + 8$

$f(1) = 1^3 - 2(1)^2 - 4(1) + 8$
$= 1 - 2 - 4 + 8$

$f(2) = 2^3 - 2(2)^2 - 4(2) + 8$
$= 8 - 8 - 8 + 8$

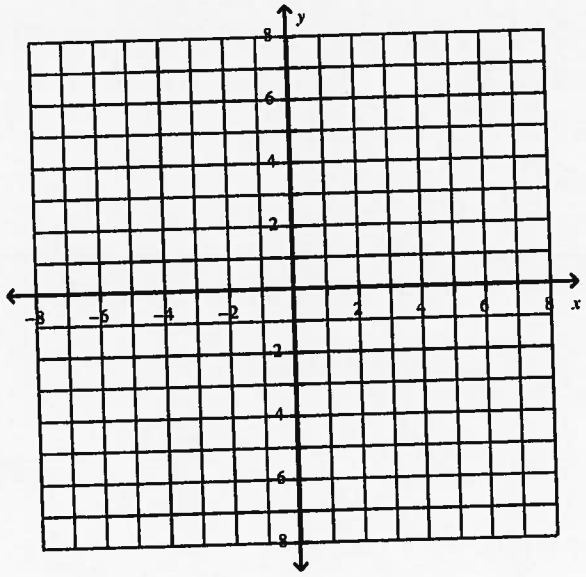
$x - 2$

2	1	-2	-4	8
	↓	2	0	-8
	1	0	-4	0

$(x-2)(x^2-4)$
$= (x-2)(x-2)(x+2)$
$= (x-2)^2(x+2)$

7. Sketch the graph of this function.

$$y = \frac{3x}{x^2 - 25}$$



$$y = \frac{3x}{(x-5)(x+5)}$$

8. Given the functions $f(x) = \frac{1}{x+5}$ and $g(x) = x^2 - 6x$, determine an explicit equation for each composite function, then state its domain. Show your work.

a) $g(f(x))$

b) $f(g(x))$

$g\left(\frac{1}{x+5}\right) = \left(\frac{1}{x+5}\right)^2 - 6\left(\frac{1}{x+5}\right)$

$=$

a) $x \in \mathbb{R},\ x \neq -5$

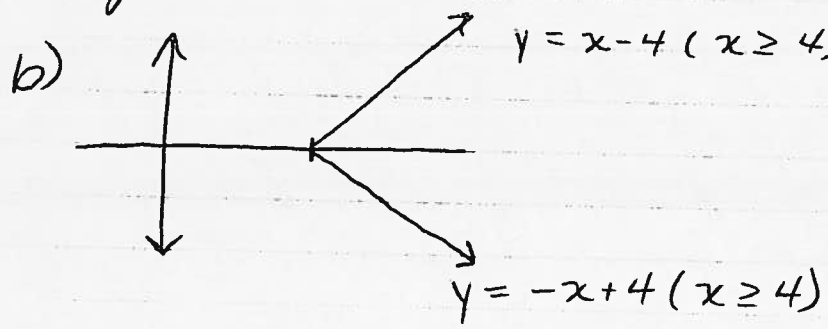
$x \neq 3, 2$

b) $f(g(x)) = f(x^2 - 6x)$

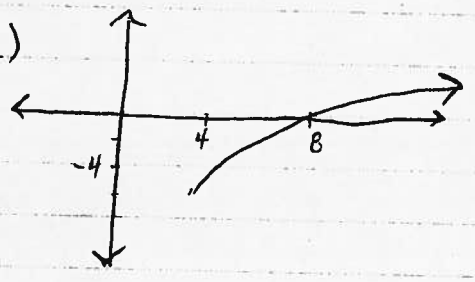
$(x-2)(x^2$

Quiz 12 Answers

1. a) yes

b) 

c) No; restrict domain of $y = |x| + 4$ to $\{x \mid x \geq 0, x \in \mathbb{R}\}$

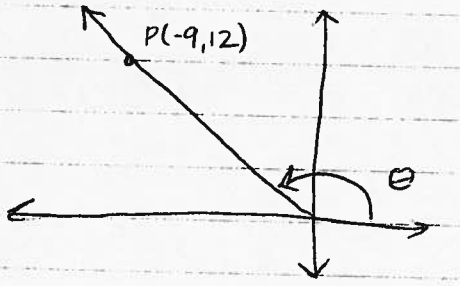
2. a) 

b) x-int: 8

b) $x = 8$

c) they are the same

3. $c = -3$; $P(x) = (x+3)(x+2)(x-1)^2$

4. a) 

b) $\sin\theta = \frac{4}{5}$ $\cos\theta = \frac{-3}{5}$

$\tan\theta = \frac{-4}{3}$ $\csc\theta = \frac{5}{4}$

$\sec\theta = \frac{-5}{3}$ $\cot\theta = \frac{3}{4}$

5. $h(x) = -25\cos\frac{2\pi}{11}x + 26$ b) $x = 3.0$ min

6. a) $x = \frac{5\pi}{6} + \pi n, n \in \mathbb{Z}$,

$x = \frac{\pi}{6} + \pi n, n \in \mathbb{Z}$

(OVER)

b) $x = \frac{\pi}{2} + \pi n, \quad n \in \mathbb{Z}$

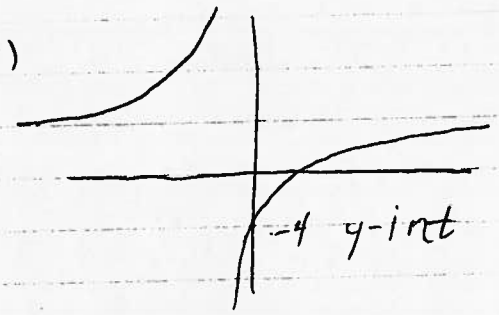
$x = \frac{7\pi}{6} + 2\pi n, \quad n \in \mathbb{Z}$

$x = \frac{11\pi}{6} + 2\pi n, \quad n \in \mathbb{Z}$

7. a) -1 b) $\frac{1}{8}$

8. a) 0.53 b) 9 c) 3 d) 2

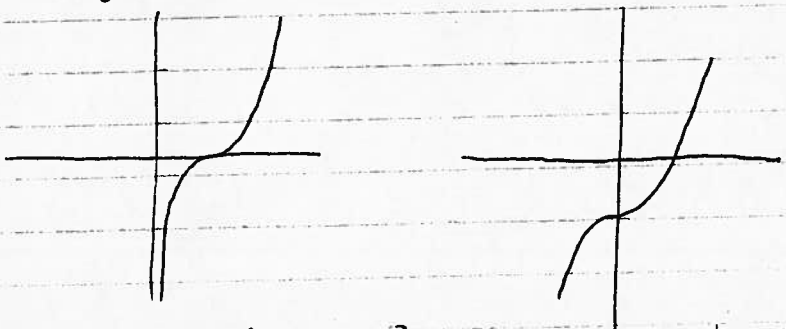
9. a)



b) D: $x \neq -1$, $x \in \mathbb{R}$
R: $y \neq 3$, $y \in \mathbb{R}$
x-int: $\frac{4}{3}$
y-int: -4
HA: $y = 3$
VA: $x = -1$

10. a) $f(g(x)) = (x-3)^3 \neq g(f(x)) = x^3 - 3$

b)



c) $f(g(x)) = (x-3)^3$ is translated 3 units right. $f(g(x)) = x^3 - 3$ is translated 3 units down

11 a) 250 b) -56