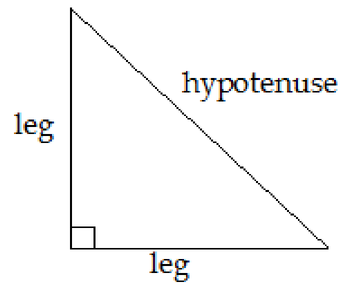
Lesson 1: Intro to Trigonometry

The word **trigonometry** means 'triangle measurement'. Trigonometry allows us to find the length of missing sides and the measure of missing angles in triangles. In this unit, we will be dealing with **right triangles.**

Labelling Right Triangles 90°

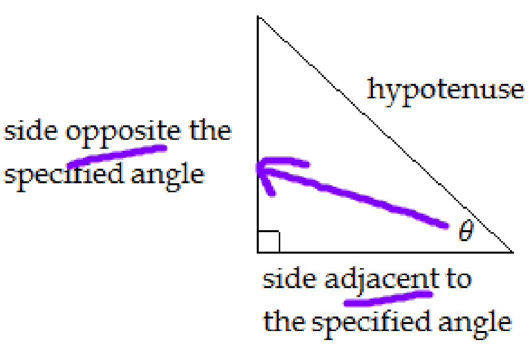
The longest side in a right triangle is *always* opposite the right angle, and it is called the hypotenuse. The two sides that form the right angle are called the legs.

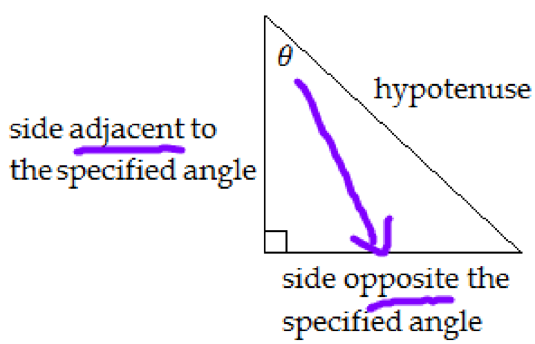


The two legs in a right triangle may be labelled with different names depending upon which angle is being referred to.

- The leg that meets the hypotenuse to create the specified angle is called the adjacent side.
- The leg that is across from the specified angle is called the opposite side.

The specified angle can be identified with a symbol like θ (called theta), x, or by a given value.

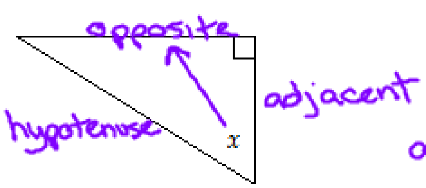




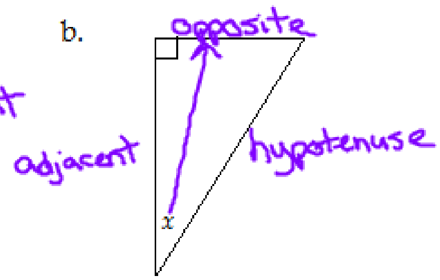
Example 1
Label the sides in these right triangles (hypotenuse, adjacent and opposite) given the specified angle, x.

1 3 2

a.

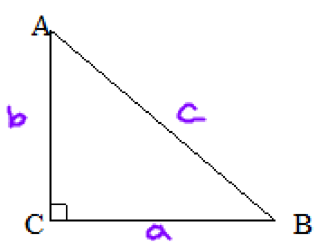


b.



Capital letters are often used to label the vertices (i.e. where the line segments meet) of a triangle. The same lowercase letter is used to label the side opposite a vertex.

Example 2
Label the sides of the triangle *a, b,* and *c*. Then, label the hypotenuse, adjacent, and opposite sides in relation to ∠A and ∠B.



hyp = c

opp ∠B = b

adj ∠B = a

opp ∠A = a

adj ∠A = b

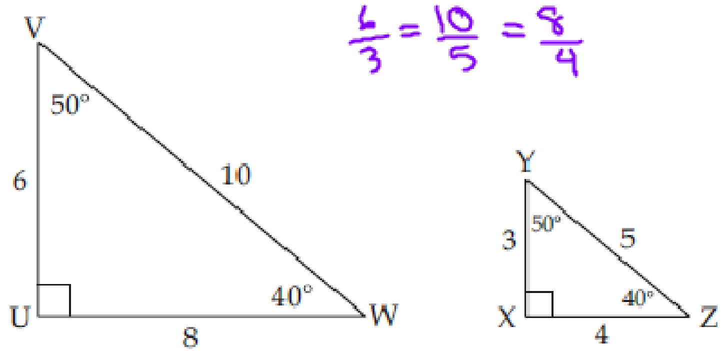
Similar Triangles

Similar triangles are triangles that have the following properties

- Angles are congruent (same)
- Sides are proportional

$$\frac{6}{3} = \frac{10}{5} = \frac{8}{4}$$

ΔUVW ~ ΔXYZ



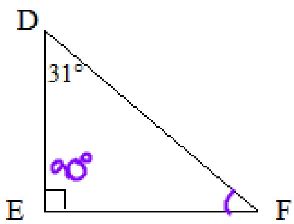
Angle-Sum Property

Recall: The Angle-Sum Property states that all angles in a triangle add up to 180°.

Example 3

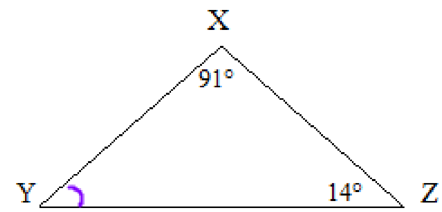
Determine the missing angle measurement in each triangle.

a.



$\angle F = 180° - 90° - 31°$
$= 59°$

b.



$\angle Y = 180° - 91° - 14°$
$= 75°$

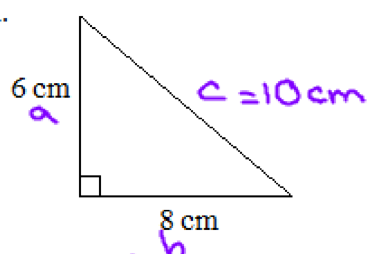
Pythagorean Theorem Review

The Pythagorean Theorem states that in any right triangle, the sum of the squares of the two legs is equal to the square of the hypotenuse. If you label the legs of the hypotenuse with the letters *a* and *b* and the hypotenuse *c*, then $\boxed{a^2 + b^2 = c^2.}$

Example 4

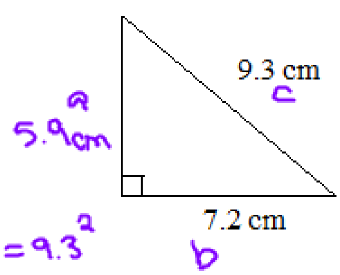
Determine the length of the missing side of each triangle.

a.

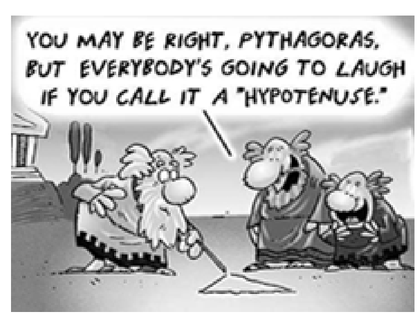


$6^2 + 8^2 = c^2$
$36 + 64 = c^2$
$100 = c^2$
$\therefore c = 10$

b.



$a^2 + 7.2^2 = 9.3^2$
$a^2 = 86.49 - 51.84$
$a^2 = 34.65$
$a = 5.9$ cm



Lesson 2: The Tangent Ratio

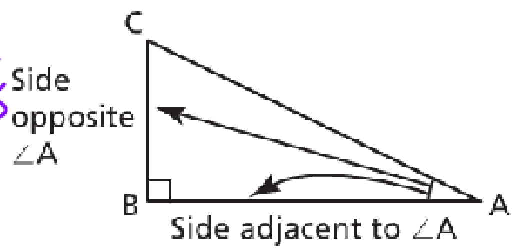
In any right triangle there are three different ratios of side lengths. These ratios of side lengths will always be the same no matter what the size of the triangles are, as long as the angles are the same measure (similar triangles). One such ratio is the length of the side Opposite an acute angle to the length of the side adjacent to the same acute angle. This is called the **tangent ratio**.

For example, in the triangle below, the tangent of $\angle A$ (written as **tan *A***) is the ratio of the side opposite angle A (a) to the side adjacent to $\angle A$ (b). In other words,

$$\boxed{\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}.}$$

Angle

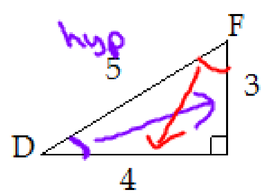
side lengths



If $\angle A$ is an acute angle in a right triangle, then

$$\tan A = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}$$

Example 1: Finding the Tangent Ratio



Determine the ratios tan D and tan F.

$$\tan D = \frac{\text{side opposite D}}{\text{side adjacent to D}} = \frac{3}{4}$$

$$\tan F = \frac{\text{side opposite F}}{\text{side adjacent to F}} = \frac{4}{3}$$

When you are using trigonometry, you need to follow these three steps:

- 1 Label the triangle fully. → hyp, opp, adj
- 2 Choose a trigonometric ratio based on the information you have about the triangle
- Solve for the unknown using a calculator.

If you are given the tangent ratio of an angle you can find the actual angle measure using a calculator. To do this, you need to work backwards by using **inverse tangent** (**$\tan^{-1}$**).

Example 2: Using Inverse Tangent

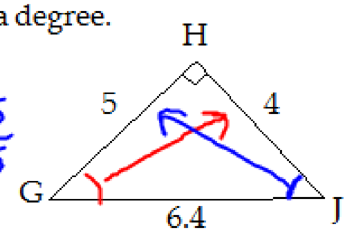
Find angle θ if $\tan\theta = 4.356$.

Use $\tan^{-1}$ to find θ.

$\tan^{-1}(4.356) = 77.1°$

Example 3: Using the Tangent Ratio to Find Angle Measurements

Determine the measures of $\angle G$ and $\angle J$ to the nearest tenth of a degree.



hyp

$\angle G$

$\tan\angle G = \dfrac{\text{side opposite } \angle G}{\text{side adjacent } \angle G}$

$\tan\angle G = \dfrac{4}{5}$

$\angle G = \tan^{-1}\left(\dfrac{4}{5}\right)$

$\angle G = 39°$

$\angle J$

$\tan\angle J = \dfrac{\text{side opp } \angle J}{\text{side adj. } \angle J}$

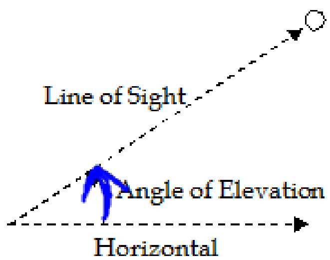
$\tan\angle J = \dfrac{5}{4}$

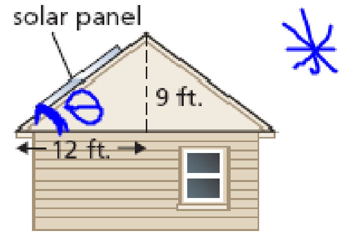
$\angle J = \tan^{-1}\left(\dfrac{5}{4}\right)$

$\angle J = 51°$

Example 4: Using the Tangent Ratio to Determine an Angle of Elevation

South-facing solar panels on a roof work best when the **angle of elevation** of the roof is approximately equal to the latitude of the house. The **angle of elevation** is the angle formed by the horizontal (level of sight) and the line of sight to an object above the horizontal.





The latitude of Fort Smith, NWT, is approximately 60°. Determine whether this design for a solar panel is the best for Fort Smith. Justify your answer.

$$\tan\theta = \frac{\text{opp.}}{\text{adj.}} = \frac{O}{A}$$

$$\tan\theta = \frac{9}{12}$$

$$\theta = \tan^{-1}\left(\frac{9}{12}\right)$$

$$\theta = 36.9^\circ$$

∴ Not close to 60°, design doesn't work.

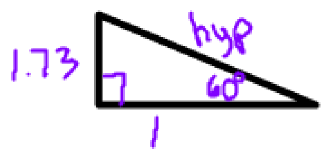
Lesson 3: Using the Tangent Ratio to Calculate Lengths

In the previous lesson, you learned how to find a missing angle in a right triangle using the tangent ratio. You can also use the tangent ratio to find missing sides of a triangle.

Example 1: Determining Tangent Ratios $\tan\theta = \frac{opp}{adj}$

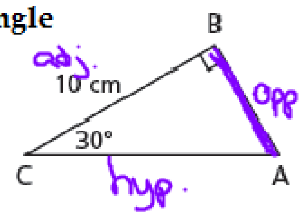
Determine the tangent of 60°, and explain what this means using a diagram.

$\tan 60 = 1.73$ $\frac{1.73}{1} = \frac{opp}{adj}$



Example 2: Determining the Length of a Side Opposite a Given Angle

Determine the length of AB to the nearest tenth of a centimetre.



Step 1: Label your triangle.

Step 2: Which information do you have? adj.

Which information do you want to find? opp.

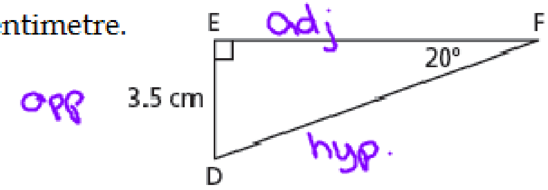
$\tan\theta = \frac{opp}{adj}$ $(10)\tan 30° = \frac{AB}{\cancel{10}}(\cancel{10})$

$10\tan 30° = AB$

$\boxed{AB = 5.8\text{ cm}}$

Example 3: Determining the Length of a Side Adjacent to a Given Angle

Determine the length of EF to the nearest tenth of a centimetre.



$$\tan\theta = \frac{opp}{adj}$$

$$(EF)\tan 20° = \frac{3.5}{\cancel{EF}}(\cancel{EF})$$

$$\frac{(EF)\cancel{\tan 20°}}{\cancel{\tan 20°}} = \frac{3.5}{\tan 20°}$$

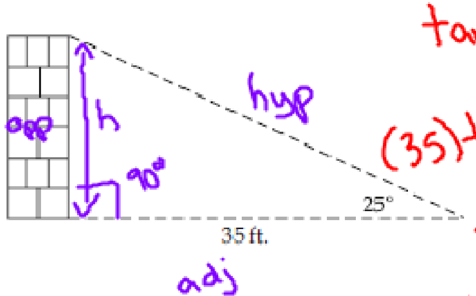
$$EF = \frac{3.5}{\tan 20°}$$

$$EF = 9.6 \text{ cm}$$

Calc: 3.5/(tan20)

Example 4: Using Tangent to Solve a Problem

What is the height of the building in the diagram below?



$$\tan\theta = \frac{opp}{adj}$$

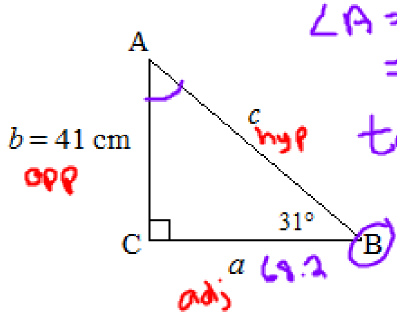
$$(35)\tan 25° = \frac{h}{35}(35)$$

$$35\tan 25° = h$$

$$\boxed{16.3 \text{ ft} = h}$$

Example 5: Using Tangent to Solve a Triangle

In a right triangle, the side opposite a 31° angle is 41 cm. Solve the triangle (find the values for all 3 sides and all 3 angles). Include a diagram.



$\angle A = 180° - 90° - 31°$
$= 59°$

$\tan \angle B = \frac{opp}{adj}$

$\tan 31° = \frac{41}{a}$

$a = \frac{41}{\tan 31°}$

$\boxed{a = 68.2 \text{ cm}}$

Find c

$a^2 + b^2 = c^2$

$(68.2)^2 + 41^2 = c^2$

$4651.24 + 1681 = c^2$

$6332.24 = c^2$

$\sqrt{6332.24} = c$

$\boxed{79.6 \text{ cm} = c}$

HWk pg 82
#3-8, 9a, 12

Lesson 4: The Sine and Cosine Ratios

The sine of $\angle A$ (written as **sin** $\boldsymbol{A}$) is the ratio of the side opposite angle A (side a) to the hypotenuse (side c).

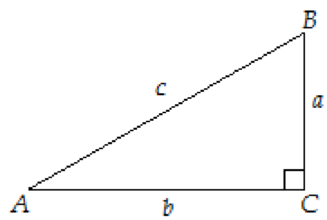
~~What do you think the ratio for cosine is?~~

The cosine of $\angle A$ (written as **cos** $\boldsymbol{A}$) is the ratio of the side adjacent to angle A (side b) to the hypotenuse (c).

In other words,

$$\sin\theta = \frac{opp}{hyp} = \frac{a}{c}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{b}{c}$$



The three ratios – tangent, sine, and cosine, are referred to as the **primary trigonometric ratios**. To remember these three ratios, we can use the acronym "SOH-CAH-TOA".

S – **s**in equals
O – **o**pposite over
H – **h**ypotenuse

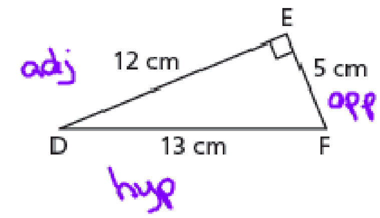
C – **c**os equals
A – **a**djacent over
H – **h**ypotenuse

T – **t**an equals
O – **o**pposite over
A – **a**djacent

Example 1: Determining the Sine and Cosine of an Angle

a. In $\triangle DEF$, identify the side opposite $\angle D$ and the side adjacent to $\angle D$.

b. Determine $\sin D$ and $\cos D$. → Find the ratio.



$\sin D = \frac{opp}{hyp} = \boxed{\frac{5}{13}}$

$\cos D = \frac{adj}{hyp} = \boxed{\frac{12}{13}}$

Example 2: Using Inverse Sine and Cosine

Find the angle θ for each of the following.

a) $\sin\theta = 0.8976$

$\theta = \sin^{-1}(0.8976)$

$\theta = 63.8°$

b) $\cos\theta = \frac{3}{4}$

$\theta = \cos^{-1}(\frac{3}{4})$

$\theta = 41.4°$

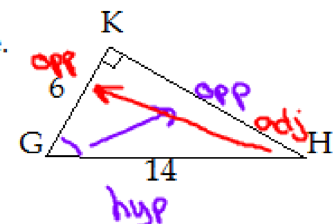
c) $\tan\theta = 1$

$\theta = \tan^{-1}(1)$

$\theta = 45°$

Example 3: Using Sine or Cosine to Determine the Measure of an Angle

Determine the measures of $\angle G$ and $\angle H$ to the nearest tenth of a degree.



$\cos\angle G = \frac{adj}{hyp} = \frac{6}{14}$

$\cos G = \frac{6}{14}$

$G = \cos^{-1}(\frac{6}{14})$

$G = 65°$

$\sin H = \frac{opp}{hyp}$

$\sin H = \frac{6}{14}$

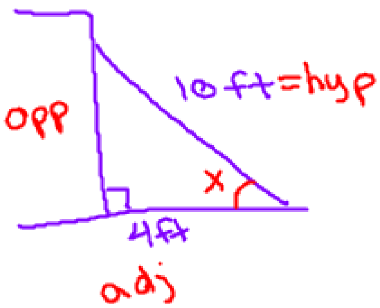
$H = \sin^{-1}(\frac{6}{14})$

$H = 25°$

$65 + 25 + 90 \stackrel{?}{=} 180$ ✓

Example 4: Using the Sine or Cosine Ratio to Solve a Problem

A 10 ft. ladder leans against the side of a building with its base 4 ft. from the wall. What angle, to the nearest degree, does the ladder make with the ground?



$\cos x = \frac{adj}{hyp}$

$\cos x = \frac{4}{10}$

$x = \cos^{-1}\left(\frac{4}{10}\right)$

$x = 66°$

$\therefore$ The ladder makes a 66° angle with the ground.

HWK pg 95 #4, 6-8, 10-13

Lesson 5: Using Sine and Cosine Ratios to Calculate Lengths

You can use the sine ratio or the cosine ratio to calculate the length of a leg in a right triangle if you know the measure of one acute angle and the length of the hypotenuse. Alternatively, you can use the sine ratio or the cosine ratio to calculate the length of the hypotenuse if you know the measure of one acute angle and the length of either leg of the triangle.

To find missing sides in right triangles using sine or cosine ratios, we can use the following steps:

1. Set up a trigonometric equation.
2. Solve for the unknown side.

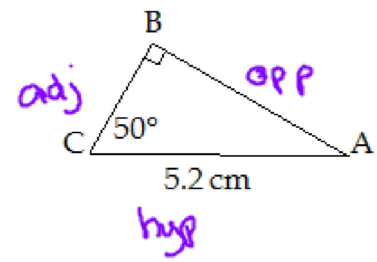
Example 1: Using the Sine or Cosine Ratio to Determine the Length of a Leg

a. Determine the length of BC to the nearest tenth of a centimetre.

$\cos\theta = \frac{adj}{hyp}$

$(5.2)\cos 50° = \frac{BC}{\cancel{5.2}}(\cancel{5.2})$

$3.3\text{ cm} = BC$



b. Determine the length of AB to the nearest tenth of a centimetre.

$\sin\theta = \frac{opp}{hyp}$

$(5.2)\sin 50 = \frac{AB}{\cancel{5.2}}(\cancel{5.2})$

$4.0\text{ cm} = AB$

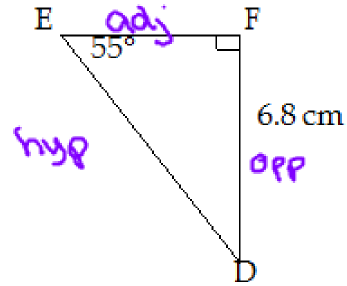
Example 2: Using the Sine or Cosine Ratio to Determine the Length of the Hypotenuse

a. Determine the length of DE to the nearest tenth of a centimetre.

$$\sin\theta = \frac{opp}{hyp}$$

$$\sin 55^\circ = \frac{6.8}{DE}$$

$$DE = \frac{6.8}{\sin 55^\circ} = 8.3\text{cm}$$

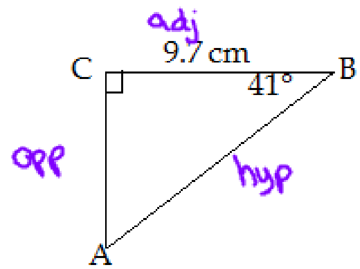


b. Determine the length of the hypotenuse.

$$\cos\theta = \frac{adj}{hyp}$$

$$\cos 41^\circ = \frac{9.7}{hyp}$$

$$hyp = \frac{9.7}{\cos 41^\circ} = 12.9\text{cm}$$



Example 3: Solving an Indirect Measurement Problem

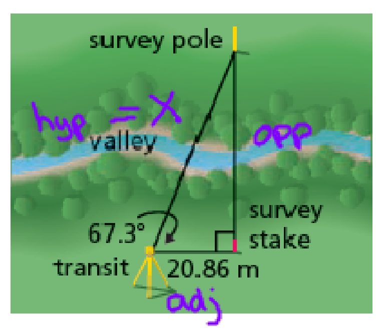
A surveyor made the measurements shown in the diagram. How could the surveyor determine the distance from the transit to the survey pole to the nearest hundredth of a metre?

$$\cos\theta = \frac{adj}{hyp}$$

$$\cos 67.3^\circ = \frac{20.86}{x}$$

$$x = \frac{20.86}{\cos 67.3^\circ}$$

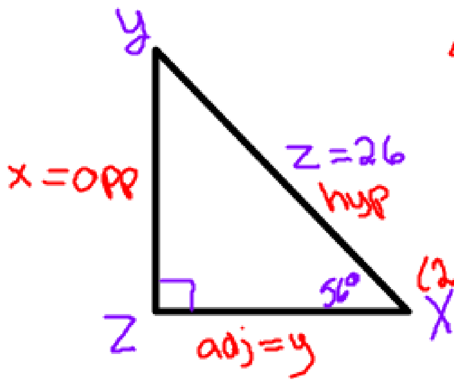
$$x = 54.05\text{m}$$



The distance from the transit to the survey pole is 54.05m.

Example 4: Using Sine and Cosine to Solve a Triangle

Given triangle XYZ with $\angle Z = 90°$, $z = 26$, $\angle X = 56°$, solve the triangle.



$\angle Y = 180° - 90° - 56°$
$= 34°$

$\sin 56° = \frac{opp}{hyp}$

$(26)\sin 56° = \frac{x}{26}(26)$

$x = 21.6$

$\cos 56° = \frac{adj}{hyp}$

$(26)\cos 56° = \frac{y}{\cancel{26}}(\cancel{26})$

$14.5 = y$

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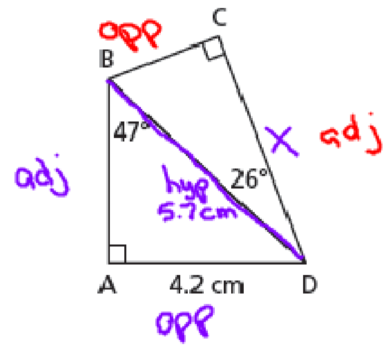
Lesson 7: What Happens if You Have More Than One Right Triangle?

To solve problems that have more than one triangle, you use the same steps you used to solve one right triangle. In general, when solving a problem that involves more than one right triangle, we will have to find a missing angle or side in one right triangle and use it to find another missing angle or side in the other right triangle.

Example 1: Calculating a Side Length Using More than One Triangle

Calculate the length of CD to the nearest tenth of a centimetre.

x



$$\sin\theta = \frac{opp}{hyp}$$

$$\sin 47° = \frac{4.2}{hyp}$$

$$hyp = \frac{4.2}{\sin 47°} = 5.7\text{ cm}$$

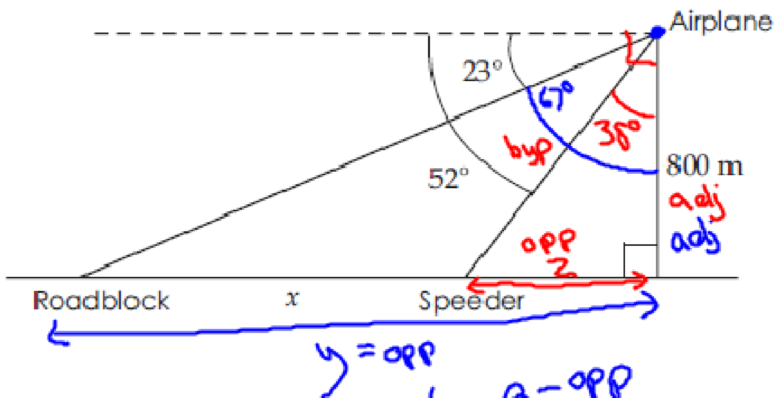
$$\cos\theta = \frac{adj}{hyp}$$

$$(5.7)\cos 26° = \frac{x}{\cancel{5.7}}(\cancel{5.7})$$

$$\boxed{5.1\text{cm} = x = CD}$$

Example 2: Solving a Problem with Two Superimposed Triangles

A police airplane, flying at an altitude of 800 m, spots a speeding vehicle at an angle of depression of 52°. If a road block is set up along the same highway at an angle of depression of 23°, find the distance the vehicle is from the road block (to the nearest hundredth of a kilometre).



$\tan\theta = \frac{opp}{adj}$

$(800)\tan 38° = \frac{z}{800}(800)$

$625\text{m} = z$

$\therefore x = y - z$

$= 1885 - 625$

$\boxed{= 1260\text{m}}$

$\tan\theta = \frac{opp}{adj}$

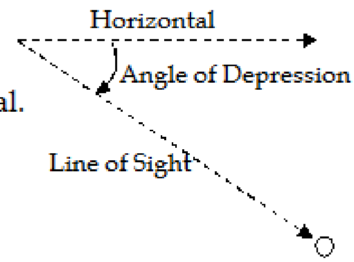
$(800)\tan 67° = \frac{y}{800}(800)$

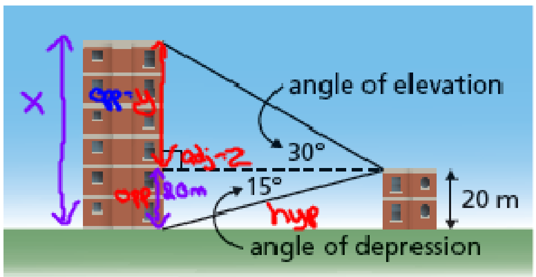
$1885\text{m} = y$

Example 3: Solving a Problem with Triangles in the Same Plane

From the top of a 20 m high building, a surveyor measured the angle of elevation to the top of another building and the angle of depression to the base of that building. The surveyor sketched this plan of her measurements. Determine the height of the taller building to the nearest tenth of a metre.

The **angle of depression** is the angle formed by the horizontal (level of sight) and the line of sight to an object below the horizontal.





$\tan\theta = \frac{opp}{adj}$

$\tan 15° = \frac{20}{z}$

$z = \frac{20}{\tan 15°}$

$z = 74.6\text{ m}$

$\tan\theta = \frac{opp}{adj}$

$(74.6)\tan 30° = \frac{y}{74.6}(74.6)$

$43.1\text{ m} = y$

$\therefore$ The height of taller building is $43.1 + 20 = 63.1$m.