Lesson 7.1 Developing Systems of Linear Equations Exercises (pages 401-402)

A

4. a) $2x + y = 11$
$x = 13 + y$
This is a linear system because both equations are linear.

b) $2x = 11 - y$
$4x - y = 13$
This is a linear system because both equations are linear.

c) $-\frac{1}{2}x - y = \frac{3}{4}$
$\frac{3}{2}x + 2 = -\frac{7}{8}$
This is a linear system because both equations are linear.

d) $-x^2 + y = 10$
$x + y = 5$
This is not a linear system because the first equation above is not linear; it contains an x^2-term.

5. Substitute $x = -1$ and $y = 2$ into each equation to see if the values satisfy the equation.

a) $3x + 2y = -1$ ①
$2x - y = 1$ ②
Consider equation ①: $3x + 2y = -1$
Substitute: $x = -1$ and $y = 2$
L.S. $= 3x + 2y$ R.S. $= -1$
$= 3(-1) + 2(2)$
$= -3 + 4$
$= 1$
Since the left side is not equal to the right side, $x = -1$ and $y = 2$ are not a solution of equation ①, so cannot be a solution of the linear system.

b) $3x - y = -1$ ①
$-x - y = -1$ ②
Consider equation ①: $3x - y = -1$
Substitute: $x = -1$ and $y = 2$
L.S. $= 3x - y$ R.S. $= -1$
$= 3(-1) - (2)$
$= -3 - 2$
$= -5$
Since the left side is not equal to the right side, $x = -1$ and $y = 2$ are not a solution of equation ①, so cannot be a solution of the linear system.

c) $-3x + 5y = 13$ ①
$4x - 3y = -10$ ②
Consider equation ①: $-3x + 5y = 13$
Substitute: $x = -1$ and $y = 2$

L.S. = $-3x + 5y$ R.S. = 13
$= -3(-1) + 5(2)$
$= 3 + 10$
$= 13$

Since the left side is equal to the right side, $x = -1$ and $y = 2$ are a solution of equation ①.
Consider equation ②: $4x - 3y = -10$
Substitute: $x = -1$ and $y = 2$

L.S. = $4x - 3y$ R.S. = -10
$= 4(-1) - 3(2)$
$= -4 - 6$
$= -10$

Since the left side is equal to the right side, $x = -1$ and $y = 2$ are a solution of equation ②.
Since $x = -1$ and $y = 2$ satisfy both equations in the linear system, these values are a solution of the linear system.

B

6. Each linear system has the equation $2x + 2y = 228$. Look at the second equation in each system.

a) A jacket costs $44 more than a sweater; that is, the difference in costs is $44.
Suppose x dollars represents the cost of a jacket and y dollars represents the cost of a sweater.
An equation to represent these costs is: $x - y = 44$
This matches the second equation in part iii, so the system in part iii matches this situation.

b) The width of the tennis court is 42 ft. less than the length, or the length is 42 ft. more than the width.
Suppose x feet represents the length of the tennis court and y feet represents the width of the court.
An equation to represent these dimensions is: $x - y = 42$
This matches the second equation in part i, so the system in part i matches this situation.

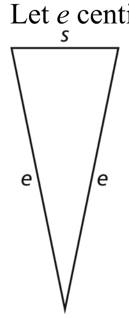
c) Forty more chapatti breads than naan breads were sold; that is, the difference in numbers of breads sold is 40.
Suppose x represents the number of chapatti breads that were sold and y represents the number of naan breads that were sold.
An equation to represent these numbers is: $x - y = 40$
This matches the second equation in part ii, so the system in part ii matches this situation.

7. a) Let s feet represent the length of a shorter piece of pipe.
Let l feet represent the length of a longer piece of pipe.
From the first diagram:
2 shorter pieces + 2 longer pieces = 20 ft.
So, one equation is: $2s + 2l = 20$
From the second diagram:
1 shorter piece + 3 longer pieces = 22 ft.
So, another equation is: $s + 3l = 22$
A linear system is:

$$2s + 2l = 20$$
$$s + 3l = 22$$

b) From the first diagram:
The length of 2 shorter pieces is: 2 × 4 ft. = 8 ft.
The length of 2 longer pieces is: 2 × 6 ft. = 12 ft.
The sum of these lengths is: 8 ft. + 12 ft. = 20 ft.
This length is the same as the length on the diagram.
From the second diagram:
The length of 1 shorter piece is: 4 ft.
The length of 3 longer pieces is: 3 × 6 ft. = 18 ft.
The sum of these lengths is: 4 ft. + 18 ft. = 22 ft.
This length is the same as the length on the diagram.
Since both lengths agree with the diagrams given, the length of each piece of pipe is correct.

8. a) Sketch a diagram.
Let s centimetres represent the length of the shorter side.
Let e centimetres represent the length of each equal side.



The perimeter, 24 cm, is the sum of the lengths of the sides.
So, one equation is: $e + e + s = 24$
Or, $2e + s = 24$
An equal side is 6 cm longer than the shorter side.
So, another equation is: $e = 6 + s$
A linear system is:
$2e + s = 24$
$e = 6 + s$

b) If the side lengths are 10 cm, 10 cm, and 4 cm, then the perimeter is:
10 cm + 10 cm + 4 cm = 24 cm
This agrees with the given information.
The difference in lengths between an equal side and a shorter side is:
10 cm – 4 cm = 6 cm
This agrees with the given information.
Since all 3 lengths agree with the given information, the lengths are correct.

9. a) The sum of the masses on the left pan of the balance scales is equal to the sum of the masses on the right pan.
From the first diagram:
$x + x + x + y = 10 + 5 + 2$
So, one equation is: $3x + y = 17$
From the second diagram:
$x = y + 1 + 1 + 1$
So, another equation is: $x = y + 3$

A linear system is:
$3x + y = 17$
$x = y + 3$

b) If a small bag of rice has a mass of 2 kg and a large bag of rice has a mass of 5 kg, then:
From the first balance scales, the mass in the left pan is:
5 kg + 5 kg + 5 kg + 2 kg = 17 kg
The mass in the right pan is: 10 kg + 5 kg + 2 kg
Since these masses are equal, the scales balance and the individual masses are correct.
From the second balance scales, the mass in the left pan is: 5 kg
The mass in the right pan is: 2 kg + 1 kg + 1 kg + 1kg = 5 kg
Since these masses are equal, the scales balance and the individual masses are correct.

c) Check that the values of x and y satisfy each equation.
Substitute $x = 5$ and $y = 2$ in $3x + y = 17$
L.S. $= 3x + y$
$= 3(5) + 2$
$= 15 + 2$
$= 17$
$=$ R.S.
Since the left side is equal to the right side, the values of x and y satisfy the equation.
Substitute $x = 5$ and $y = 2$ in $x = y + 3$

L.S. $= x$	R.S. $= y + 3$
$= 5$	$= 2 + 3$
	$= 5$

Since the left side is equal to the right side, the values of x and y satisfy the equation.

10. The total distance is 25 km.
From the diagram the total distance is: $x + y + y + 5$, or $x + 2y + 5$
So, one equation is: $x + 2y + 5 = 25$, or $x + 2y = 20$
The distance from home to cabin 2 is 13 km.
From the diagram, this distance is: $x + y$
So, another equation is: $x + y = 13$
A linear system is:
$x + 2y = 20$
$x + y = 13$
The distance from home to cabin 1 is x, so substitute $x = 7$.
The distance from cabin 1 to cabin 2 is y, so substitute $y = 6$.
Check solution A.
Substitute $x = 7$ and $y = 6$ in $x + 2y = 20$
L.S. $= x + 2y$
$= 7 + 2(6)$
$= 7 + 12$
$= 19$
$\neq$ R.S.
Since the left side is not equal to the right side, the solution is not correct.
Check solution B.
Substitute $x = 6$ and $y = 7$ in $x + 2y = 20$.
L.S. $= x + 2y$
$= 6 + 2(7)$
$= 6 + 14$

$= 20$
$= \text{R.S.}$
Since the left side is equal to the right side, $x = 6$ and $y = 7$ satisfy this equation.
Substitute $x = 6$ and $y = 7$ in $x + y = 13$.
$\text{L.S.} = x + y$
$= 6 + 7$
$= 13$
$= \text{R.S.}$
Since the left side is equal to the right side, $x = 6$ and $y = 7$ satisfy this equation.
Since $x = 6$ and $y = 7$ satisfy both equations, this solution is correct.

11. Let w minutes represent the time that Padma walked.
Let j minutes represent the time that Padma jogged.
Padma jogged for 1 h, which is 60 min.
So, one equation is: $w + j = 60$
Padma walked for 10 min more than she jogged.
So, another equation is: $w = j + 10$
A linear system is:
$w + j = 60$
$w = j + 10$
Check solution A.
Substitute $w = 35$ and $j = 25$ in $w + j = 60$.
$\text{L.S.} = w + j$
$= 35 + 25$
$= 60$
$= \text{R.S.}$
Since the left side is equal to the right side, $w = 35$ and $j = 25$ satisfy this equation.
Substitute $w = 35$ and $j = 25$ in $w = j + 10$.

$\text{L.S.} = w$	$\text{R.S.} = j + 10$
$= 35$	$= 25 + 10$
	$= 35$

Since the left side is equal to the right side, $w = 35$ and $j = 25$ satisfy this equation.
Since $w = 35$ and $j = 25$ satisfy both equations, this solution is correct.

Check solution B.
Substitute $w = 25$ and $j = 35$ in $w + j = 60$.
$\text{L.S.} = w + j$
$= 25 + 35$
$= 60$
$= \text{R.S.}$
Since the left side is equal to the right side, $w = 25$ and $j = 35$ satisfy this equation.
Substitute $w = 25$ and $j = 35$ in $w = j + 10$.

$\text{L.S.} = w$	$\text{R.S.} = j + 10$
$= 25$	$= 35 + 10$
	$= 45$

Since the left side is not equal to the right side, this solution is not correct.

12. a), b) The problem is about a collection of toonies and loonies. So, the variables could represent the number of toonies and the number of loonies in the collection.
Use the letters in the equations.
Let t represent the number of toonies and let l represent the number of loonies.

Then the first equation indicates that the total value for t toonies and l loonies is \$160.
The second equation indicates that the sum of the number of toonies and the number of loonies is 110.
Shen might have written this problem:
There are 110 coins in a collection of loonies and toonies.
The value of the collection of coins is \$160.
How many loonies and how many toonies are in the collection?

13. a), b) The problem is about the costs of tickets for a child and an adult, and about the total cost of tickets for a group of children and adults. So, the variables could represent the cost of a ticket for an adult and the cost of a ticket for a child.
Use the letters in the equations.
Let a dollars represent the cost of an adult ticket and let c dollars represent the cost of a ticket for a child.
Then the first equation indicates that the total cost for 5 adults and 2 children is \$38.
The second equation indicates that the difference in costs for an adult's ticket and a child's ticket is \$2.
Jacqui might have written this problem:
Five adults and 2 children went to a local fair.
The total cost for the tickets was \$38.
An adult's ticket cost \$2 more than a child's ticket.
What was the cost of each ticket?

14. Two variables are added and subtracted. The situation could involve the sum and the difference of two numbers.
From the first equation, the sum of two numbers is 100.
From the second equation, the difference of two numbers is 10.
Let x represent the greater number and let y represent the lesser number.
Then a problem could be:
The sum of two numbers is 100.
The difference of the two numbers is 10.
What are the two numbers?

C

15. $Ax + By = C$
$Dx + Ey = F$

a) Since the solution of the linear system is $(0, y)$, substitute $x = 0$ in each equation.
$Ax + By = C$ becomes
$By = C$ Divide each side by B.
$$y = \frac{C}{B}$$
$Dx + Ey = F$ becomes
$Ey = F$ Divide each side by E.
$$y = \frac{F}{E}$$
Since y is a solution of both equations, the two values of y are equal, so
$$\frac{C}{B} = \frac{F}{E}$$

b) Since the solution of the linear system is $(x, 0)$, substitute $y = 0$ in each equation.

$Ax + By = C$ becomes

$Ax = C$ Divide each side by A.

$x = \frac{C}{A}$

$Dx + Ey = F$ becomes

$Dx = F$ Divide each side by D.

$x = \frac{F}{D}$

Since x is a solution to both equations, the two values of x are equal, so $\frac{C}{A} = \frac{F}{D}$.

16. $\frac{-3x + 24}{x + y} = -6$ ①

$-\frac{x}{5} - \frac{y}{3} = x + y$ ②

Consider equation ①:

$\frac{-3x+24}{x + y} = -6$ Multiply each side by $(x + y)$ to remove the fraction.

$(x + y)\left(\frac{-3x + 24}{x + y}\right) = -6(x + y)$ Simplify. Remove brackets.

$-3x + 24 = -6x - 6y$ Collect like terms. Add $6x$ and $6y$ to each side.

$6x + 6y - 3x + 24 = 0$

$3x + 6y + 24 = 0$ Simplify. Divide each side by 3.

$x + 2y + 8 = 0$ This is a linear equation in x and y.

Consider equation ②:

$-\frac{x}{5} - \frac{y}{3} = x + y$ Multiply each side by the common denominator 15.

$15\left(-\frac{x}{5}\right) - 15\left(\frac{y}{3}\right) = 15(x + y)$ Simplify.

$-3x - 5y = 15x + 15y$ Collect like terms. Subtract $15x$ and $15y$ from each side.

$-3x - 5y - 15x - 15y = 0$

$-18x - 20y = 0$ Divide each side by -2.

$9x + 10y = 0$

The linear system is:

$x + 2y + 8 = 0$

$9x + 10y = 0$

17. a) I wrote two equations in x and y that have the solution $x = 1$ and $y = 1$.

I wrote this solution as the ordered pair (1, 1).

For the first equation, I chose another ordered pair (3, 5).

I used the formula for writing an equation when the coordinates of two points that satisfy the equation are known:

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

I substituted $y_1 = 1$, $x_1 = 1$, $y_2 = 5$, and $x_2 = 3$.

$$\frac{y-1}{x-1}=\frac{5-1}{3-1}$$

$$\frac{y-1}{x-1}=\frac{4}{2}$$

$$\frac{y-1}{x-1}=2$$ Multiply each side by $(x-1)$.

$$(x-1)\left(\frac{y-1}{x-1}\right)=2(x-1)$$

$$y-1=2x-2$$ Add 1 to each side.

$$y=2x-1$$

For the second equation, I chose another ordered pair (4, 2).
I used the formula:

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$$

I substituted $y_1 = 1$, $x_1 = 1$, $y_2 = 2$, and $x_2 = 4$.

$$\frac{y-1}{x-1}=\frac{2-1}{4-1}$$

$$\frac{y-1}{x-1}=\frac{1}{3}$$ Multiply each side by $(x-1)$.

$$(x-1)\left(\frac{y-1}{x-1}\right)=\frac{1}{3}(x-1)$$

$$y-1=\frac{1}{3}x-\frac{1}{3}$$ Add 1 to each side.

$$y=\frac{1}{3}x+\frac{2}{3}$$

A linear system is:

$y = 2x - 1$

$y=\frac{1}{3}x+\frac{2}{3}$

b) There is more than one linear system with the same solution because I can write many different pairs of equations that have the solution $x = 1$ and $y = 1$. I can use the formula in part a and substitute different pairs of values for x_2 and y_2.

18. a) Answers may vary. For example:
I examined the two equations of the linear system:

$x + 2y = 7$ ①
$x + 3y = 9$ ②

I added y to each side of equation ①.

$x + 2y + y = 7 + y$
$x + 3y = 7 + y$ ③

From ②, $x + 3y = 9$
So, comparing ① and ③, $7 + y = 9$, or $y = 2$
So, $y = 2$ is part of the solution of the linear system.

b) $x + 2y = 7$ ①
$x + 3y = 9$ ②
I know $y = 2$ is part of the solution.
I substitute $y = 2$ in each equation, then solve for x.
For equation ①:
$x + 2y = 7$
$x + 2(2) = 7$
$x + 4 = 7$
$x = 3$
For equation ②:
$x + 3y = 9$
$x + 3(2) = 9$
$x + 6 = 9$
$x = 3$
The solution is $x = 3$ and $y = 2$.

Lesson 7.2 Solving a System of Linear Equations Graphically Exercises (pages 409-410)

A

3. The solution is the coordinates of the point of intersection of the two lines on each graph.

a) The solution is: $x = -4$ and $y = 2$

b) The solution is: $x = 2$ and $y = 3$

c) The solution is: $x = 1$ and $y = -3$

d) The solution is: $x = -2$ and $y = -1$

B

4. a) The solution appears to be: $x = 9$ and $y = -2$
Substitute these values for x and y into each equation to check.

$2x + 3y = 12$
$$\begin{aligned}\text{L.S.} &= 2x + 3y\\ &= 2(9) + 3(-2)\\ &= 18 - 6\\ &= 12\\ &= \text{R.S.}\end{aligned}$$

$x - y = 11$
$$\begin{aligned}\text{L.S.} &= x - y\\ &= 9 - (-2)\\ &= 9 + 2\\ &= 11\\ &= \text{R.S.}\end{aligned}$$

For each equation, the left side is equal to the right side.
So, $x = 9$ and $y = -2$ is a solution of the linear system. This solution is exact.

b) The solution appears to be: $x = -1\frac{3}{4}$ and $y = 2\frac{3}{4}$
Substitute these values for x and y into each equation to check.

$-3x + y = \frac{31}{4}$
$$\begin{aligned}\text{L.S.} &= -3x + y\\ &= -3\left(-1\frac{3}{4}\right) + 2\frac{3}{4}\\ &= (-3)\left(-\frac{7}{4}\right) + \frac{11}{4}\\ &= \frac{21}{4} + \frac{11}{4}\\ &= \frac{32}{4}\end{aligned}$$

$3x - 4y = -16$
$$\begin{aligned}\text{L.S.} &= 3x - 4y\\ &= 3\left(-1\frac{3}{4}\right) - 4\left(2\frac{3}{4}\right)\\ &= 3\left(-\frac{7}{4}\right) - 4\left(\frac{11}{4}\right)\\ &= -\frac{21}{4} - \frac{44}{4}\\ &= -\frac{65}{4}, \text{ or } -16\frac{1}{4}\end{aligned}$$

For each equation, the left and right sides are not equal, but they are close in value.
So, $x = -1\frac{3}{4}$ and $y = 2\frac{3}{4}$ is an approximate solution of the linear system.

5. a) i) $x + y = 7$ ①
$3x + 4y = 24$ ②
Determine the x- and y-intercepts of the graph of each equation.
For equation ①:
$x + y = 7$

When $x = 0$,
$0 + y = 7$
$y = 7$

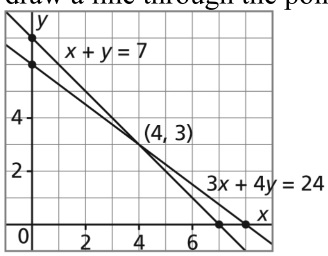
When $y = 0$,
$x + 0 = 7$
$x = 7$

On a grid, mark a point at 7 on each axis, then draw a line through the points.
For equation ②:
$3x + 4y = 24$

When $x = 0$,
$3(0) + 4y = 24$
$4y = 24$
$y = 6$

When $y = 0$,
$3x + 4(0) = 24$
$3x = 24$
$x = 8$

On the grid, mark a point at 6 on the y-axis and mark a point at 8 on the x-axis, then draw a line through the points.



The point of intersection appears to be: (4, 3)

Verify the solution.
Substitute $x = 4$ and $y = 3$ into each equation.

$x + y = 7$
L.S. $= x + y$
$= 4 + 3$
$= 7$
$=$ R.S.

$3x + 4y = 24$
L.S. $= 3x + 4y$
$= 3(4) + 4(3)$
$= 12 + 12$
$= 24$
$=$ R.S.

Since the left side is equal to the right side in each equation, then $x = 4$ and $y = 3$ is the solution of the linear system.

ii) $x - y = -1$ ①
$3x + 2y = 12$ ②

Determine the x- and y-intercepts of the graph of each equation.
For equation ①:
$x - y = -1$

When $x = 0$,
$0 - y = -1$
$y = 1$

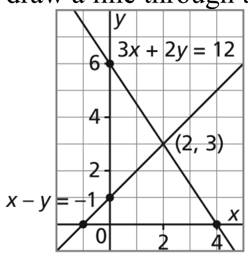
When $y = 0$,
$x - 0 = -1$
$x = -1$

On a grid, mark a point at 1 on the y-axis and mark a point at –1 on the x-axis, then draw a line through the points.
For equation ②:
$3x + 2y = 12$

When $x = 0$,
$3(0) + 2y = 12$
$2y = 12$
$y = 6$

When $y = 0$,
$3x + 2(0) = 12$
$3x = 12$
$x = 4$

On the grid, mark a point at 6 on the y-axis and mark a point at 4 on the x-axis, then draw a line through the points.



The point of intersection appears to be: (2, 3)

Verify the solution.
Substitute $x = 2$ and $y = 3$ into each equation.

$x - y = -1$
L.S. $= x - y$
$= 2 - 3$
$= -1$
$=$ R.S.

$3x + 2y = 12$
L.S. $= 3x + 2y$
$= 3(2) + 2(3)$
$= 6 + 6$
$= 12$
$=$ R.S.

Since the left side is equal to the right side in each equation, then $x = 2$ and $y = 3$ is the solution of the linear system.

iii) $5x + 4y = 10$ ①
$5x + 6y = 0$ ②
Determine the x- and y-intercepts of the graph of each equation.
For equation ①:
$5x + 4y = 10$

When $x = 0$,
$5(0) + 4y = 10$
$4y = 10$
$y = \frac{10}{4}$, or $2\frac{1}{2}$

When $y = 0$,
$5x + 4(0) = 10$
$5x = 10$
$x = 2$

Since the y-intercept is not where grid lines intersect, determine the coordinates of another point on the line.
Substitute: $x = 6$
$5(6) + 4y = 10$
$30 + 4y = 10$
$4y = -20$
$y = -5$
On a grid, mark a point at (6, –5) and mark a point at 2 on the x-axis, then draw a line through the points.
For equation ②:
$5x + 6y = 0$

When $x = 0$,
$5(0) + 6y = 0$
$6y = 0$
$y = 0$

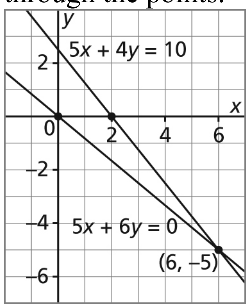
When $y = 0$,
$5x + 6(0) = 0$
$5x = 0$
$x = 0$

Since the x-and y-intercepts are equal, determine the coordinates of another point on the line.

Choose a value for x that is a multiple of 6, so the corresponding y-value is an integer.
Substitute: $x = 6$

$$\begin{aligned} 5(6) + 6y &= 0 \\ 30 + 6y &= 0 \\ 6y &= -30 \\ y &= -5 \end{aligned}$$

On the grid, mark a point at (6, –5) and mark a point at the origin, then draw a line through the points.



The point of intersection appears to be: (6, –5)
Since the coordinates of this point were determined using each equation, these coordinates satisfy each equation and $x = 6$ and $y = -5$ is the solution of the linear system.

iv) $x + 2y = -1$ ①
$2x + y = -5$ ②
Determine the x- and y-intercepts of the graph of each equation.
For equation ①:
$x + 2y = -1$

When $x = 0$,
$$\begin{aligned} 0 + 2y &= -1 \\ 2y &= -1 \\ y &= -\frac{1}{2} \end{aligned}$$

When $y = 0$,
$$\begin{aligned} x + 2(0) &= -1 \\ x &= -1 \end{aligned}$$

Since the y-intercept is not where grid lines intersect, determine the coordinates of another point on the line.
Substitute: $x = 1$

$$\begin{aligned} 1 + 2y &= -1 \\ 2y &= -2 \\ y &= -1 \end{aligned}$$

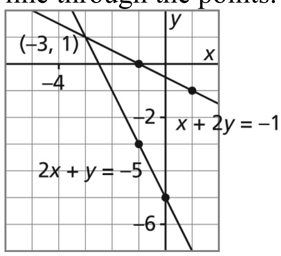
On a grid, mark a point at (1, –1) and mark a point at –1 on the x-axis, then draw a line through the points.
For equation ②:
$2x + y = -5$

When $x = 0$,
$$\begin{aligned} 2(0) + y &= -5 \\ y &= -5 \end{aligned}$$

When $y = 0$,
$$\begin{aligned} 2x + 0 &= -5 \\ 2x &= -5 \\ x &= -2\frac{1}{2} \end{aligned}$$

Since the x-intercept is not where grid lines intersect, determine the coordinates of another point on the line.

Substitute: $x = -1$
$2(-1) + y = -5$
$-2 + y = -5$
$y = -3$
On the grid, mark a point at (–1, –3) and mark a point at –5 on the y-axis, then draw a line through the points.



The point of intersection appears to be: (–3, 1)

Verify the solution.
Substitute $x = -3$ and $y = 1$ into each equation.

$x + 2y = -1$
L.S. $= -3 + 2(1)$
$= -3 + 2$
$= -1$
= R.S.

$2x + y = -5$
L.S. $= 2x + y$
$= 2(-3) + 1$
$= -6 + 1$
$= -5$
= R.S.

Since the left side is equal to the right side in each equation, then $x = -3$ and $y = 1$ is the solution of the linear system.

b) For the linear system in part a, iv:
$x + 2y = -1$ ①
$2x + y = -5$ ②
The point of intersection of the graphs is the point with coordinates (–3, 1).
This is the solution of the linear system. This point lies on both lines of the system, and its coordinates satisfy both equations of the system.

6. The linear system is:
$3x - y = 1149$
$-x + 2y = 142$
Substitute the solution (500, 300) to check whether it is exact or approximate.

$3x - y = 1149$
L.S. $= 3x - y$
$= 3(500) - 300$
$= 1500 - 300$
$= 1200$

$-x + 2y = 142$
L.S. $= -x + 2y$
$= -500 + 2(300)$
$= -500 + 600$
$= 100$

For each equation, the left and right sides are not equal, but are close in value.
So, the solution $x = 500$ and $y = 300$ is an approximate solution.

7. a) $2x + 4y = -1$ ①
$3x - y = 9$ ②
Graph each equation.
For equation ①:
$2x + 4y = -1$
Since the coefficients of x and y are even and the constant term is odd, there are no

integer coordinates that satisfy the equation.
Determine the x- and y-intercepts, then graph the equation on a grid with a larger scale.
$2x + 4y = -1$

When $x = 0$,
$2(0) + 4y = -1$
$4y = -1$
$y = -\frac{1}{4}$

When $y = 0$,
$2x + 4(0) = -1$
$2x = -1$
$x = -\frac{1}{2}$

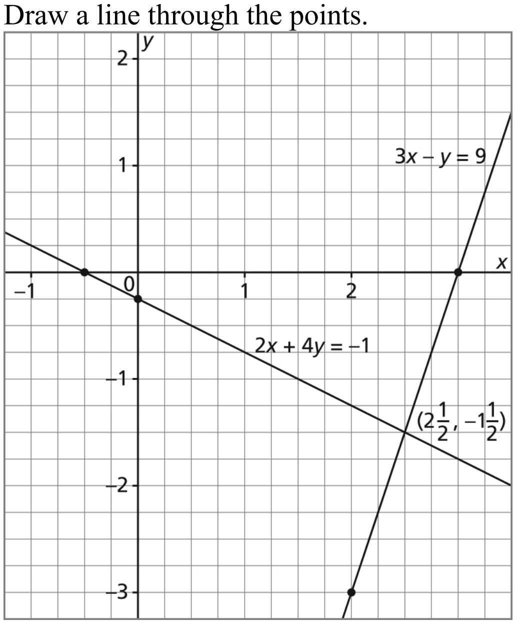
On a grid, use a scale of 4 squares represents 1 unit, mark a point at $-\frac{1}{4}$ on the y-axis and mark a point at $-\frac{1}{2}$ on the x-axis, then draw a line through the points.

For equation ②:
$3x - y = 9$

When $x = 0$,
$3(0) - y = 9$
$-y = 9$
$y = -9$

When $y = 0$,
$3x - 0 = 9$
$3x = 9$
$x = 3$

For a y-intercept of –9, the y-axis would be very long on a graph with a scale of 4 squares to 1 unit, so choose a different point to plot.
Substitute $x = 2$ in $3x - y = 9$.
$3(2) - y = 9$
$6 - y = 9$
$y = -3$
On the grid, mark a point at (2, –3) and mark a point at 3 on the x-axis.
Draw a line through the points.



The solution appears to be: $x = 2\frac{1}{2}$ and $y = -1\frac{1}{2}$
Write these numbers as decimals: $x = 2.5$ and $y = -1.5$

Substitute these values for x and y into each equation to check.

$2x + 4y = -1$
L.S. $= 2x + 4y$
$= 2(2.5) + 4(-1.5)$
$= 5 - 6$
$= -1$
$=$ R.S.

$3x - y = 9$
L.S. $= 3x - y$
$= 3(2.5) - (-1.5)$
$= 7.5 + 1.5$
$= 9$
$=$ R.S.

Since the left side is equal to the right side for each equation, the solution of the linear system is $x = 2.5$ and $y = -1.5$.

b) $5x + 5y = 17$ ①
$x - y = -1$ ②
Graph each equation.
For equation ①:
$5x + 5y = 17$
Since the coefficients of x and y are even and the constant term is odd, there are no integer coordinates that satisfy the equation.
Determine the x-and y-intercepts, then graph the equation on a grid with a larger scale.
$5x + 5y = 17$

When $x = 0$,
$5(0) + 5y = 17$
$5y = 17$
$y = \frac{17}{5}$, or $3\frac{2}{5}$

When $y = 0$,
$5x + 5(0) = 17$
$5x = 17$
$x = \frac{17}{5}$, or $3\frac{2}{5}$

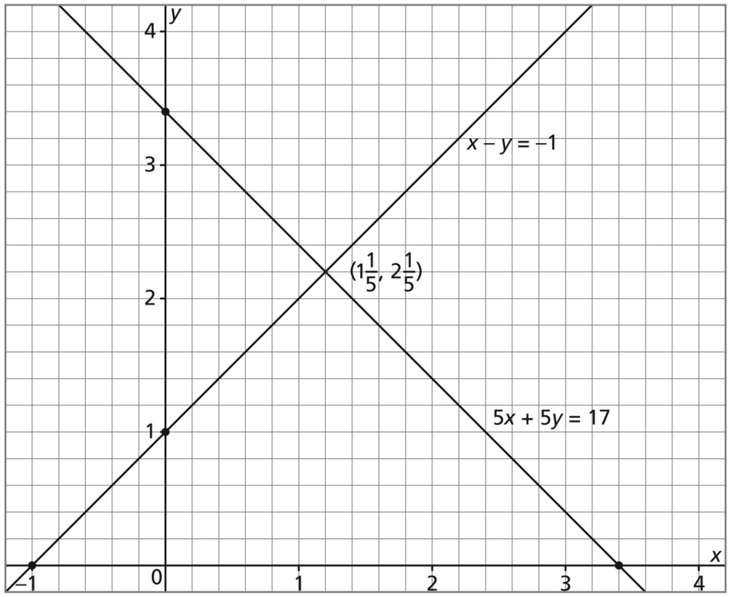
On a grid, use a scale of 5 squares represents 1 unit, mark a point at $3\frac{2}{5}$ on the y-axis and mark a point at $3\frac{2}{5}$ on the x-axis, then draw a line through the points.

For equation ②:
$x - y = -1$

When $x = 0$,
$0 - y = -1$
$y = 1$

When $y = 0$,
$x - 0 = -1$
$x = -1$

On the grid, mark a point at 1 on the y-axis and mark a point at –1 on the x-axis. Draw a line through the points.



The solution appears to be: $x = 1\frac{1}{5}$ and $y = 2\frac{1}{5}$

Write these numbers as decimals: $x = 1.2$ and $y = 2.2$

Substitute these values for x and y into each equation to check.

$5x + 5y = 17$

L.S. $= 5x + 5y$
$= 5(1.2) + 5(2.2)$
$= 6 + 11$
$= 17$
$=$ R.S.

$x - y = -1$

L.S. $= x - y$
$= 1.2 - 2.2$
$= -1$
$=$ R.S.

Since the left side is equal to the right side for each equation, the solution of the linear system is: $x = 1.2$ and $y = 2.2$

c) $x + y = \frac{23}{4}$ ①

$x - y = \frac{3}{4}$ ②

Graph each equation.

For equation ①:

$x + y = \frac{23}{4}$

Since the constant term is a rational number, there are no integer coordinates that satisfy the equation.

Determine the x- and y-intercepts, then graph the equation on a grid with a larger scale.

$x + y = \frac{23}{4}$

When $x = 0$,

$0 + y = \frac{23}{4}$

$y = \frac{23}{4}$, or $5\frac{3}{4}$

When $y = 0$,

$x + 0 = \frac{23}{4}$

$x = \frac{23}{4}$, or $5\frac{3}{4}$

On a grid, use a scale of 4 squares represents 1 unit, mark a point at $5\frac{3}{4}$ on the y-axis and mark a point at $5\frac{3}{4}$ on the x-axis, then draw a line through the points.

For equation ②:

$x - y = \frac{3}{4}$

When $x = 0$,

$0 - y = \frac{3}{4}$

$y = -\frac{3}{4}$

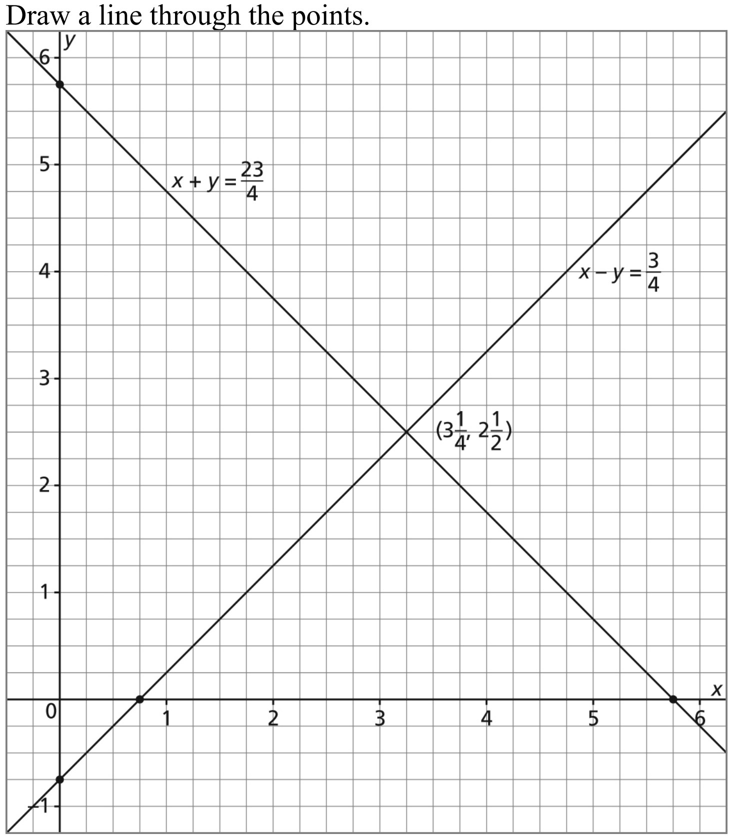
When $y = 0$,

$x - 0 = \frac{3}{4}$

$x = \frac{3}{4}$

On the grid, mark a point at $-\frac{3}{4}$ on the y-axis and mark a point at $\frac{3}{4}$ on the x-axis.

Draw a line through the points.



The solution appears to be: $x = 3\frac{1}{4}$ and $y = 2\frac{1}{2}$

Write these numbers as decimals: $x = 3.25$ and $y = 2.5$

Substitute these values for x and y into each equation to check. Write the rational numbers in the equations as decimals.

$x - y = \frac{3}{4}$

$x - y = 0.75$

L.S. $= x - y$

$= 3.25 - 2.5$

$x + y = \frac{23}{4}$

$x + y = 5.75$

L.S. $= 3.25 + 2.5$

$= 5.75$

$= 0.75$ $\qquad\qquad\qquad\qquad$ $= \text{R.S.}$

$= \text{R.S.}$

Since the left side is equal to the right side for each equation, the solution of the linear system is $x = 3.25$ and $y = 2.5$.

d) $3x + y = 6$ ①

$x + y = -\frac{4}{3}$ ②

Graph each equation.

For equation ①:

$3x + y = 6$

Determine the x- and y-intercepts.

$3x + y = 6$

When $x = 0$,

$3(0) + y = 6$

$y = 6$

When $y = 0$,

$3x + 0 = 6$

$3x = 6$

$x = 2$

On a grid, mark a point at 6 on the y-axis and mark a point at 2 on the x-axis, then draw a line through the points.

For equation ②:

$x + y = -\frac{4}{3}$

When $x = 0$,

$0 + y = -\frac{4}{3}$

$y = -\frac{4}{3}$

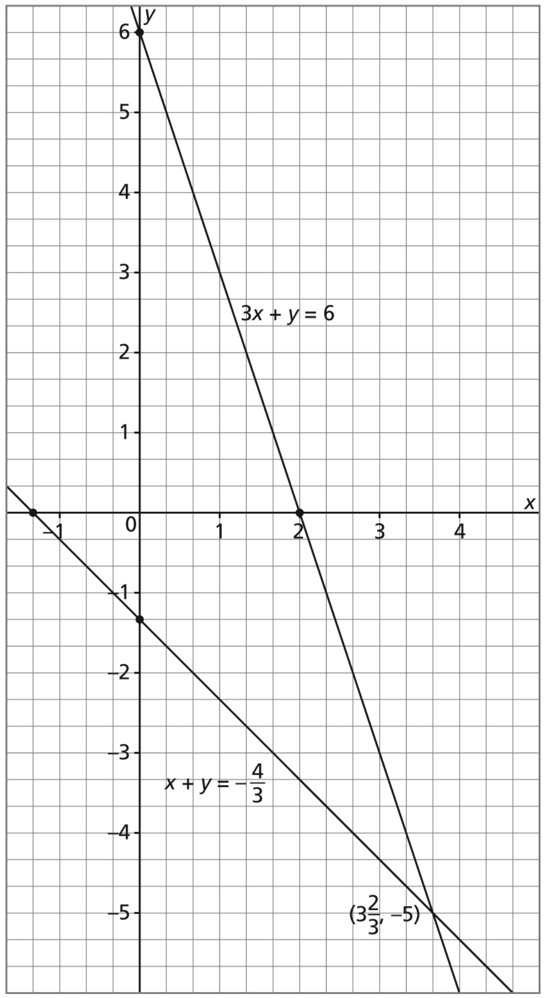
When $y = 0$,

$x - 0 = -\frac{4}{3}$

$x = -\frac{4}{3}$

On the grid, use a scale of 3 squares to 1 unit. Mark a point at $x + y = -\frac{4}{3}$ on the y-axis and mark a point at $-\frac{4}{3}$ on the x-axis.

Draw a line through the points.



The solution appears to be: $x = 3\frac{2}{3}$ and $y = -5$

Substitute these values for x and y into each equation to check.

$x + y = -\frac{4}{3}$

L.S. $= x + y$

$= 3\frac{2}{3} - 5$

$= \frac{11}{3} - \frac{15}{3}$

$= -\frac{4}{3}$

= R.S.

$3x + y = 6$

L.S. $= 3x + y$

$= 3\left(3\frac{2}{3}\right) - 5$

$= (3)\left(\frac{11}{3}\right) - 5$

$= \frac{33}{3} - 5$

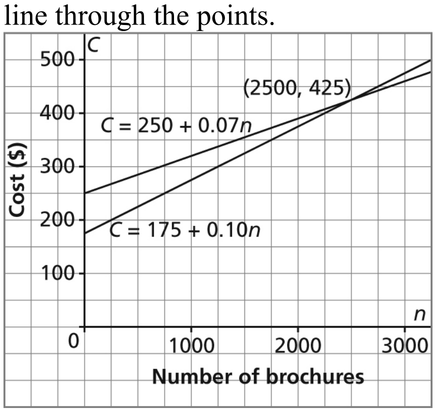
$= 11 - 5$

$= 6$

= R.S.

Since the left side is equal to the right side for each equation, the solution of the linear system is $x = 3\frac{2}{3}$ and $y = -5$.

8. a) $C = 175 + 0.10n$ ①
$C = 250 + 0.07n$ ②
For each equation, determine the C-intercept and the coordinates of another point on the line.
For equation ①:
$C = 175 + 0.10n$
Substitute: $n = 0$
$C = 175$
Substitute: $n = 1000$
$C = 175 + 0.10 \times 1000$
$C = 175 + 100$
$C = 275$
On a grid, use a scale of 2 squares to 100 units on the C-axis, and a scale of 2 squares to 500 units on the n-axis. Mark a point at 175 on the C-axis and mark a point at (1000, 275). Draw a line through the points. The data are discrete, but the scale is so small that if a point were plotted for each brochure, these points would form a line.
For equation ②:
$C = 250 + 0.07n$
Substitute: $n = 0$
$C = 250$
Substitute: $n = 1000$
$C = 250 + 0.07 \times 1000$
$C = 250 + 70$
$C = 320$
On the grid, mark a point at 250 on the C-axis and mark a point at (1000, 320). Draw a line through the points.



b) i) From the graph, the cost will be the same at both companies at the point of intersection of the lines; that is, when 2500 brochures are printed for $425.
Check that this solution satisfies both equations.
Substitute $C = 425$ and $n = 2500$ in each equation.
For equation ①:
$C = 175 + 0.10n$

L.S. $= C$
$= 425$

R.S. $= 175 + 0.10n$
$= 175 + 0.10(2500)$
$= 175 + 250$
$= 425$

For equation ②:
$C = 250 + 0.07n$

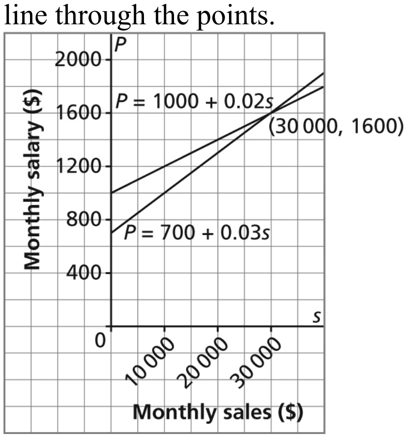
L.S. $= C$
$= 425$

R.S. $= 250 + 0.07n$
$= 250 + 0.07(2500)$
$= 250 + 175$
$= 425$

Since the left side is equal to the right side for each equation, the solution is correct.

ii) The equation that represents Company A is $C = 175 + 0.10n$. It is cheaper to use Company A when its line is beneath the line for Company B; that is, for any number of brochures less that 2500.

9. a) $P = 700 + 0.03s$ ①
$P = 1000 + 0.02s$ ②
For each equation, determine the P-intercept and the coordinates of another point on the line.
For equation ①:
$P = 700 + 0.03s$
Substitute: $s = 0$
$P = 700$
Substitute: $s = 10\,000$
$P = 700 + 0.03 \times 10\,000$
$P = 700 + 300$
$P = 1000$
On a grid, use a scale of 1 square to 200 units on the P-axis, and a scale of 2 squares to 10 000 units on the s-axis. Mark a point at 700 on the P-axis and mark a point at (10 000, 1000). Draw a line through the points.
For equation ②:
$P = 1000 + 0.02s$
Substitute: $s = 0$
$P = 1000$
Substitute: $s = 10\,000$
$P = 1000 + 0.02 \times 10\,000$
$P = 1000 + 200$
$P = 1200$
On the grid, mark a point at 1000 on the P-axis and mark a point at (1000, 1200). Draw a line through the points.



b) **i)** From the graph, a clerk will receive the same salary with both plans at the point of intersection of the lines; that is, when \$30 000 in sales earns a salary of \$1600.
Check that this solution satisfies both equations.
Substitute $P = 1600$ and $s = 30\ 000$ in each equation.
For equation ①:
$P = 700 + 0.03s$

L.S. $= P$	R.S. $= 700 + 0.03s$
$= 1600$	$= 700 + 0.03(30\ 000)$
	$= 700 + 900$
	$= 1600$

For equation ②:
$P = 1000 + 0.02s$

L.S. $= P$	R.S. $= 1000 + 0.02s$
$= 1600$	$= 1000 + 0.02(30\ 000)$
	$= 1000 + 600$
	$= 1600$

Since the left side is equal to the right side for each equation, the solution is correct.

ii) The equation that represents Plan B is $P = 1000 + 0.02s$. A person on Plan B is earning more than he or she would on Plan A when the line for Plan B is above the line for Plan A; that is, for any value of sales up to \$30 000.

10. Let the area of the forested part be represented by f hectares.
Let the unforested area be represented by r hectares.
The area of Stanley park is 391 hectares. This is the sum of f hectares and r hectares.
So, one equation is: $f + r = 391$
The forested area is 141 hectares more than the unforested area.
So, another equation is: $f = r + 141$
A linear system is:
$f + r = 391$ ①
$f = r + 141$ ②
Graph the equations.
Determine the f- and r-intercepts.
For equation ①:
$f + r = 391$

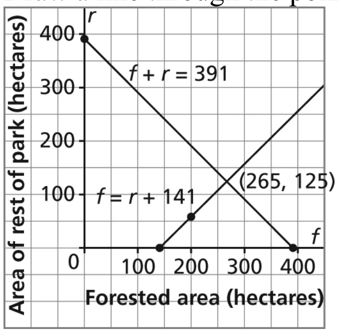
Substitute: $r = 0$	Substitute: $f = 0$
$f + 0 = 391$	$0 + r = 391$
$f = 391$	$r = 391$

On a grid, plot r as a function of f; that is, plot r on the vertical axis and f on the horizontal axis. Use a scale of 1 square to 50 units on each axis. Mark a point at 391 on each axis Draw a line through the points. Since the point at 391 is an approximate position, the graph is approximate.
For equation ②:
$f = r + 141$

Substitute: $r = 0$	Substitute: $f = 0$
$f = 0 + 141$	$0 = r + 141$
$f = 141$	$r = -141$

A negative value of r makes no sense when we are considering areas. This value of r is not in the range. Substitute a different value of f, such as $f = 200$.
$200 = r + 141$
$r = 59$
On the grid, mark a point at 141 on the f-axis and mark a point at (200, 59).

Draw a line through the points.



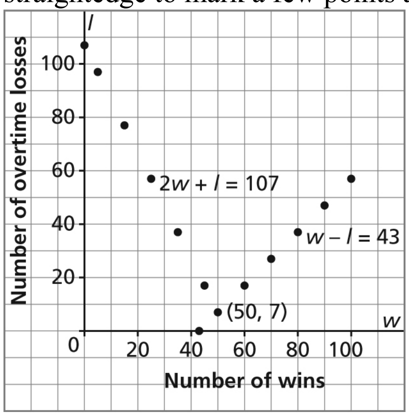
From the graph, the coordinates of the point of intersection indicate the two areas.
The f-coordinate is approximately 265 and the r-coordinate is approximately 125.
So, the forested area is about 265 hectares and the rest of the park is about 125 hectares.
Use the given information to check this solution.
The sum of the areas, in hectares, is: 265 + 125 = 390; this is close to the given sum of 391.
The difference of the areas is: 265 – 125 = 140; this is close to the given difference of 141.
The solution is approximate.

11. Let the number of wins be represented by w.
Let the number of overtime losses be represented by l.
The team had a total of 107 points.
The team gets 2 points for a win and 1 point for an overtime loss.
So, one equation is: $2w + l = 107$
The team had 43 more wins than overtime losses.
So, another equation is: $w - l = 43$
A linear system is:
$2w + l = 107$ ①
$w - l = 43$ ②
Graph the equations.
Determine the w- and l-intercepts.
For equation ①:
$2w + l = 107$ ①

Substitute: $l = 0$	Substitute: $w = 0$
$2w + 0 = 107$	$2(0) + l = 107$
$2w = 107$	$l = 107$
$w = 53.5$	

Since the w-intercept is not a whole number, substitute a different value of l, and determine the corresponding w-coordinate.
Substitute: $l = 17$
$2w + 17 = 107$
$2w = 90$
$w = 45$
On a grid, plot l as a function of w; that is, plot l on the vertical axis and w on the horizontal axis. Use a scale of 1 square to 10 units on each axis. Mark a point at (45, 17) and mark a point at 107 on the l-axis. Since the data are discrete, the graph is a set of points that lie on a line through the plotted points. Use a straightedge to mark a few points along the line at whole number values of w and l.

For equation ②:
$w - l = 43$ ②
Substitute: $l = 0$
$w - 0 = 43$
$w = 43$
Substitute: $w = 0$
$0 - l = 43$
$l = -43$
A negative value of l makes no sense when we are considering numbers of wins and overtime losses. This value of l is not in the range. Substitute a different value of w, such as $w = 80$.
$80 - l = 43$
$l = 37$
On the grid, mark a point at 43 on the w-axis and mark a point at (80, 37). Since the data are discrete, the graph is a set of points that lie on a line through the plotted points. Use a straightedge to mark a few points along the line at whole number values of w and l.



From the graph, the coordinates of the point of intersection are (50, 7).
The w-coordinate is approximately 50 and the l-coordinate is approximately 7.
So, the number of wins is about 50 and the number of overtime losses is about 7.
Use the given information to check this solution.
The number of points is: $2(50) + 7 = 107$; this is equal to the given number of points.
The difference in wins and overtime losses is: $50 - 7 = 43$; this is equal to the given difference.
The solution is correct and the numbers are exact.

12. Let the number of \$5 gift cards sold be represented by f.
Let the number of \$10 gift cards be represented by t.
The class raised \$800.
So, one equation is: $5f + 10t = 800$
The class sold a total of 115 gift cards.
So, another equation is: $f + t = 115$
A linear system is:
$5f + 10t = 800$ ①
$f + t = 115$ ②
Graph the equations.
Determine the f- and t-intercepts.
For equation ①:
$5f + 10t = 800$ ①
Substitute: $t = 0$
$5f + 10(0) = 800$
$5f = 800$
$f = 160$
Substitute: $f = 0$
$5(0) + 10t = 800$
$10t = 800$
$t = 80$
On a grid, plot t as a function of f; that is, plot t on the vertical axis and f on the horizontal

axis. Use a scale of 1 square to 10 units on each axis. Mark a point at 160 on the *f*-axis and mark a point at 80 on the *t*-axis. Since the data are discrete, the graph is a set of points that lie on a line through the plotted points. Use a straightedge to mark a few points along the line at whole number values of *f* and *t*.

For equation ②:

$f + t = 115$ ②

Substitute: $t = 0$

$f + 0 = 115$

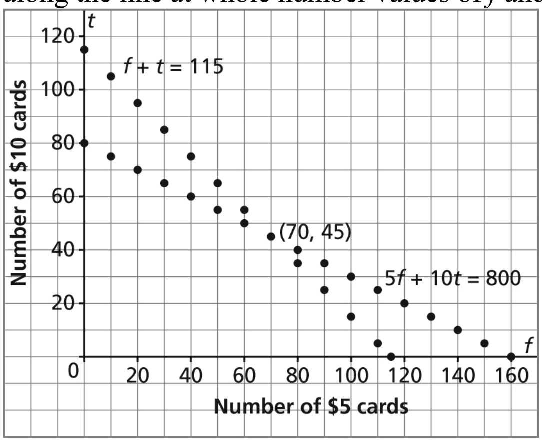
$f = 115$

Substitute: $f = 0$

$0 + t = 115$

$t = 115$

On the grid, mark a point at 115 on each axis. Since the data are discrete, the graph is a set of points that lie on a line through the plotted points. Use a straightedge to mark a few points along the line at whole number values of *f* and *t*.



From the graph, the coordinates of the point of intersection appear to be (70, 45).

The *f*-coordinate is approximately 70 and the *t*-coordinate is approximately 45.

So, the number of $5 gift cards sold is about 70 and the number of $10 gift cards sold is about 45.

Use the given information to check this solution.

The amount of money raised, in dollars, is: 5(70) + 10(45) = 350 + 450, or 800; this is equal to the given amount.

The total number of gift cards sold is: 70 + 45 = 115; this is equal to the total number of cards sold.

The solution is correct and the numbers are exact.

13. Let the number of student tickets sold be represented by *s*.

Let the number of adult tickets sold be represented by *a*.

The total admission fee was $152.

So, one equation is: $4.80s + 8a = 152$

There were 13 more students than adults.

So, another equation is: $s - a = 13$

A linear system is:

$4.80s + 8a = 152$ ①

$s - a = 13$ ②

Graph the equations.

Determine the *s*- and *a*-intercepts.

For equation ①:

$4.80s + 8a = 152$

Substitute: $a = 0$ | Substitute: $s = 0$

$4.80s + 8(0) = 152$ | $4.80(0) + 8a = 152$

$4.80s = 152$ | $8a = 152$

$s = 31.\overline{6}$ | $a = 19$

Since the s-intercept is not a whole number, substitute a different value for a, then solve for s.

Substitute: $a = 10$

$4.80s + 8(10) = 152$

$4.80s + 80 = 152$

$4.80s = 72$

$s = 15$

On a grid, plot a as a function of s; that is, plot a on the vertical axis and s on the horizontal axis. Use a scale of 1 square to 2 units on each axis. Mark a point at 19 on the a-axis and mark a point at (15, 10). Since the data are discrete, the graph is a set of points that lie on a line through the plotted points. Use a straightedge to mark a few points along the line at whole number values of a and s.

For equation ②:

$s - a = 13$ ②

Substitute: $a = 0$ | Substitute: $s = 0$

$s - 0 = 13$ | $0 - a = 13$

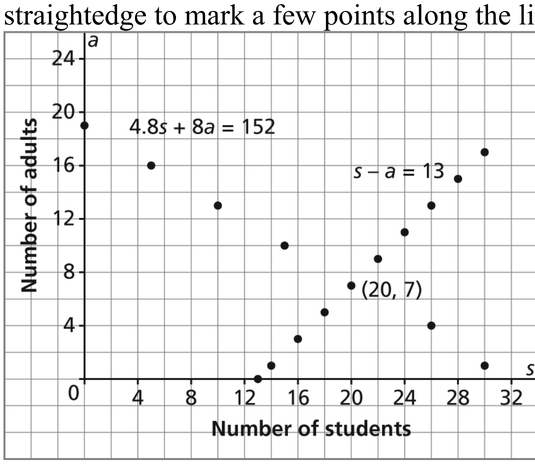
$s = 13$ | $a = -13$

A negative value of a makes no sense when we are considering numbers of adults. This value of a is not in the range. Substitute a different value of s, such as $s = 30$.

$30 - a = 13$

$a = 17$

On the grid, mark a point at 13 on the s-axis and mark a point at (30, 17). Since the data are discrete, the graph is a set of points that lie on a line through the plotted points. Use a straightedge to mark a few points along the line at whole number values of s and a.



From the graph, the coordinates of the point of intersection appear to be (20, 7).

The s-coordinate is approximately 20 and the a-coordinate is approximately 7.

So, the number of student tickets sold is about 20 and the number of adult tickets sold is about 7.

Use the given information to check this solution.

The total admission fee, in dollars, is: $4.8(20) + 7(8) = 96 + 56$, or 152; this is equal to the given admission fee.

The difference in the numbers of students and adults is: $20 - 7 = 13$; this is equal to the given difference.

The solution is correct and the numbers are exact.

14. a) Let the mass of the box be represented by b grams.
Let the mass of a golf ball be represented by g grams.
The box and 36 golf balls have a mass of 1806 g.
So, one equation is: $b + 36g = 1806$
When 12 balls are removed, 24 balls remain, so the mass of the box and 24 balls is 1254 g.
$b + 24g = 1254$
A linear system is:
$b + 36g = 1806$ ①
$b + 24g = 1254$ ②

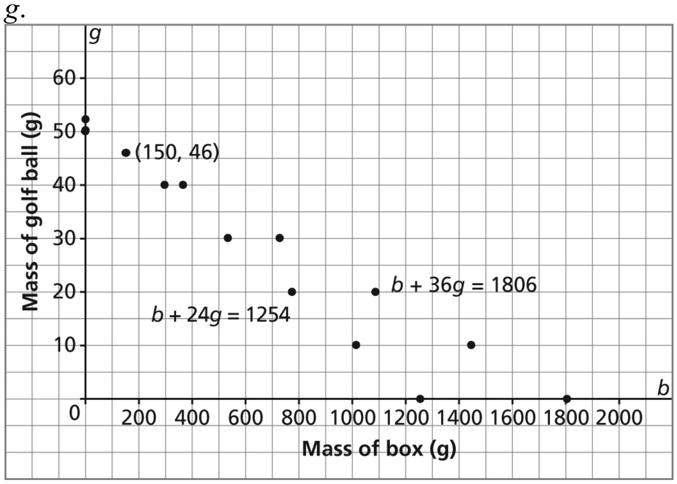
b) To graph the equations, determine the b- and g-intercepts.
For equation ①:
$b + 36g = 1806$

Substitute: $g = 0$	Substitute: $b = 0$
$b + 36(0) = 1806$	$(0) + 36g = 1806$
$b = 1806$	$36g = 1806$
	$g = 50.1\overline{6}$

The g-intercept is not a whole number, and it would be difficult to determine a whole number value, so the graph and solution will be approximate.
On a grid, plot g as a function of b; that is, plot g on the vertical axis and b on the horizontal axis. Use a scale of 1 square to 100 units on the b-axis and 1 square to 5 units on the g-axis. Mark a point at 50 on the g-axis and mark a point at (1806). Since the data are discrete, the graph is a set of points that lie on a line through the plotted points. Use a straightedge to mark a few points along the line at whole number values of g.
For equation ②:
$b + 24g = 1254$ ②

Substitute: $g = 0$	Substitute: $b = 0$
$b + 24(0) = 1254$	$0 + 24g = 1254$
$b = 1254$	$24g = 1254$
	$g = 52.25$

As with equation ①, the g-intercept is not a whole number, so the graph and solution will be approximate.
On the grid, mark a point at 1254 on the b-axis and mark a point at 52 on the g-axis. Since the data are discrete, the graph is a set of points that lie on a line through the plotted points. Use a straightedge to mark a few points along the line at whole number values of g.



From the graph, the coordinates of the point of intersection appear to be (150, 45).
The b-coordinate is approximately 150 and the g-coordinate is approximately 45.
So, the mass of the box is about 150 g and the mass of a golf ball is about 45 g.
Use the given information to check this solution.
The mass of the box and 36 balls, in grams, is: $150 + 36(45) = 1770$; this is close to the given mass of 1806 g.
The mass of the box and 24 balls, in grams, is: $150 + 24(45) = 1230$; this is close to the given mass of 1254 g.
The mass of the box is about 150 g and the mass of one golf ball is about 45 g.

c) It was difficult to determine the exact solution because I could not plot the intercepts accurately using the scale on the graph paper I had. When the intercepts are not accurate, the solution of the linear system will not be accurate.

15. The pentagon has perimeter 58 in.
So, $x + y + y + x + 17 = 58$ Collect like terms.
$2x + 2y = 41$
So, one equation is: $2x + 2y = 41$ ①

y is greater than x, and the difference between y and x is $3\frac{1}{2}$.

So, another equation is: $y - x = 3\frac{1}{2}$

A linear system is:
$2x + 2y = 41$ ①
$y - x = 3\frac{1}{2}$ ②
Graph the equations.
Determine the x- and y-intercepts.
For equation ①:
$2x + 2y = 41$ ①

Substitute: $y = 0$
$2x + 2(0) = 41$
$2x = 41$
$x = 20\frac{1}{2}$

Substitute: $x = 0$
$2(0) + 2y = 41$
$2y = 41$
$y = 20\frac{1}{2}$

On a grid, plot y as a function of x. Mark a point at $20\frac{1}{2}$ on each axis. Draw a line through the points.
For equation ②:
$y - x = 3\frac{1}{2}$ ②

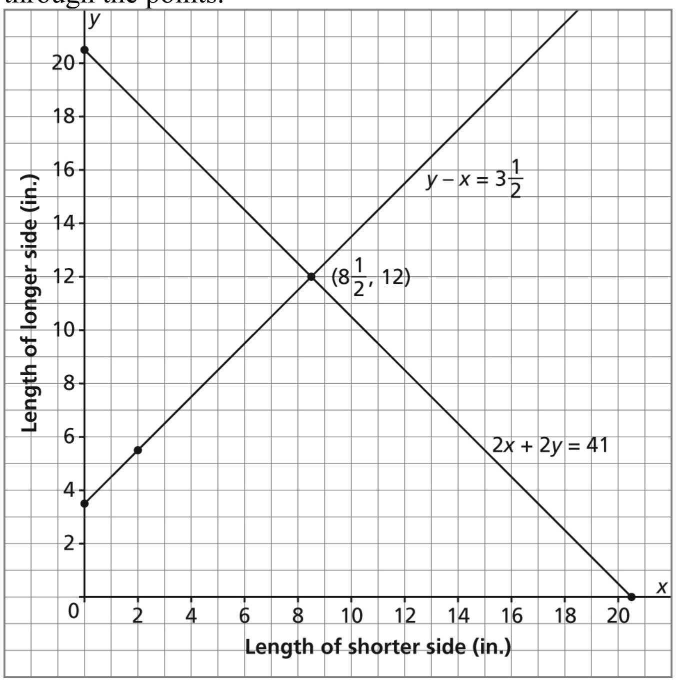
Substitute: $y = 0$
$0 - x = 3\frac{1}{2}$
$x = -3\frac{1}{2}$

Substitute: $x = 0$
$y - 0 = 3\frac{1}{2}$
$y = 3\frac{1}{2}$

A negative value of x makes no sense when we are considering the length of the side of a pentagon. This value of x is not in the domain. Substitute a different value of x, such as $x = 2$.

$$y - 2 = 3\frac{1}{2}$$
$$y = 5\frac{1}{2}$$

On the grid, mark a point at $3\frac{1}{2}$ on the y-axis and mark a point at $(2, 5\frac{1}{2})$. Draw a line through the points.



From the graph, the coordinates of the point of intersection appear to be $(8\frac{1}{2}, 12)$.

The x-coordinate is approximately $8\frac{1}{2}$ and the y-coordinate is 12.

So, the value of x is about $8\frac{1}{2}$ in. and the value of y is about 12 in.

Use the given information to check this solution.

The perimeter, in inches, is: $17 + 2\left(8\frac{1}{2}\right) + 2(12) = 17 + 17 + 24$, or 58 in.; this is equal to the given perimeter.

The difference between y and x is: $12 - 8\frac{1}{2} = 3\frac{1}{2}$; this is equal to the given difference.

The solution is correct and the numbers are exact; that is, x is $8\frac{1}{2}$ in. and y is 12 in.

16. a) $2x + 7y = 3$ ①
$4x + 3y = 7$ ②
To graph this linear system, determine the coordinates of two points on each line.
By looking at the equations, I can tell that the intercepts are fractions, so I choose other points.
For equation ①:

Since it is easier to divide by 2 than by 7, I choose values of y to substitute, then solve for x.

$2x + 7y = 3$

Substitute: $y = 1$	Substitute: $y = -1$
$2x + 7(1) = 3$	$2x + 7(-1) = 3$
$2x = -4$	$2x = 10$
$x = -2$	$x = 5$

Two points on equation ① have coordinates: $(-2, 1)$ and $(5, -1)$

For equation ②:

Since it is easier to divide by 4 than by 3, I choose values of y to substitute, then solve for x.

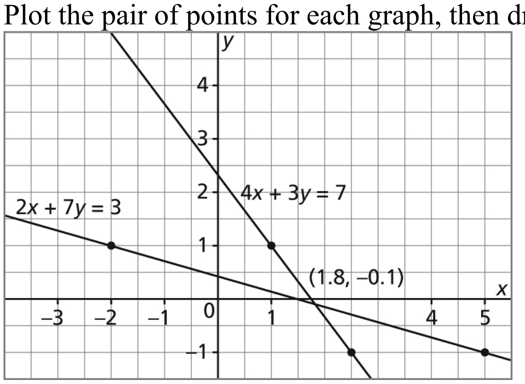
$4x + 3y = 7$

Substitute: $y = 1$	Substitute: $y = -1$
$4x + 3(1) = 7$	$4x + 3(-1) = 7$
$4x = 4$	$4x = 10$
$x = 1$	$x = 2.5$

Two points on equation ② have coordinates: $(1, 1)$ and $(2.5, -1)$

To plot the decimal values of the coordinates, use a scale of 2 squares to 1 unit on each axis.

Plot the pair of points for each graph, then draw a line through the points.



From the graph, the solution appears to be: $x = 1.8$ and $y = -0.1$

Check whether this solution satisfies both equations.

Substitute $x = 1.8$ and $y = -0.1$ in each equation.

For equation ①:

$2x + 7y = 3$

L.S. $= 2x + 7y$ R.S. $= 3$

$= 2(1.8) + 7(-0.1)$

$= 3.6 - 0.7$

$= 2.9$

For equation ②:

$4x + 3y = 7$

L.S. $= 4x + 3y$ R.S. $= 7$

$= 4(1.8) + 3(-0.1)$

$= 7.2 - 0.3$

$= 6.9$

For each equation, the left side is not equal to the right side, but the values are close.

So, the solution $x = 1.8$ and $y = -0.1$ is approximate.

b) The solution is approximate because I cannot read the exact fraction or decimal value from the graph.

C

17. a) Equation ① has x-intercept 5 and y-intercept 5.
So, the graph of the equation passes through the points with coordinates (5, 0) and (0, 5).
Use the formula for the equation of a line when the coordinates of two points on the line are known: $\dfrac{y-y_1}{x-x_1}=\dfrac{y_2-y_1}{x_2-x_1}$

Substitute: $y_1 = 0$, $x_1 = 5$, $y_2 = 5$, and $x_2 = 0$

$$\frac{y-0}{x-5}=\frac{5-0}{0-5}$$

$$\frac{y}{x-5}=-1 \qquad \text{Multiply each side by } (x-5).$$

$$y=-1(x-5)$$

$$y=-x+5 \qquad \text{This is equation ①.}$$

Equation ② has x-intercept 4 and y-intercept 6.
So, the graph of the equation passes through the points with coordinates (4, 0) and (0, 6).

Use this formula: $\dfrac{y-y_1}{x-x_1}=\dfrac{y_2-y_1}{x_2-x_1}$

Substitute: $y_1 = 0$, $x_1 = 4$, $y_2 = 6$, and $x_2 = 0$

$$\frac{y-0}{x-4}=\frac{6-0}{0-4}$$

$$\frac{y}{x-4}=-\frac{3}{2} \qquad \text{Multiply each side by } (x-4).$$

$$y=-\frac{3}{2}(x-4) \qquad \text{Remove brackets.}$$

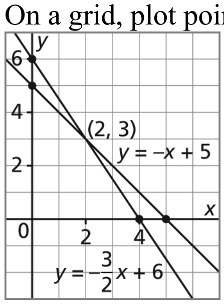
$$y=-\frac{3}{2}x+6 \qquad \text{This is equation ②.}$$

The linear system is:

$y=-x+5$ ①

$y=-\dfrac{3}{2}x+6$ ②

b) On a grid, plot points at the intercepts for each graph, then draw a line through the points.



The solution appears to be: $x = 2$ and $y = 3$
Check that this solution satisfies both equations.
Substitute $x = 2$ and $y = 3$ in each equation.
For equation ①:
$y = -x + 5$

L.S. $= y$
$= 3$

R.S. $= -x + 5$
$= -2 + 5$

$= 3$

For equation ②:

$y = -\frac{3}{2}x + 6$

L.S. $= y$ R.S. $= -\frac{3}{2}x + 6$

$= 3$ $= -\frac{3}{2}(2) + 6$

$= -3 + 6$

$= 3$

Since the left side is equal to the right side for each equation, the solution is correct.

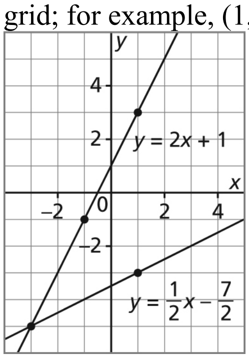
18. Graph the equation $y = 2x + 1$. Substitute two values for x.

When $x = 1$: When $x = -1$:

$y = 2(1) + 1$ $y = 2(-1) + 1$

$y = 3$ $y = -1$

On a grid, plot points at (1, 3) and (–1, –1), then draw a line through them. Extend the line to the third quadrant. Choose a point on the line in the third quadrant; for example, (–3, –5). Let this be the coordinates of the point that is the solution of the linear system. To determine a second equation in the linear system, first choose the coordinates of any other point on the grid; for example, (1, –3).



Use the formula for the equation of a line when the coordinates of two points on the line are known: $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

Substitute: $y_1 = -5$, $x_1 = -3$, $y_2 = -3$, and $x_2 = 1$

$$\frac{y - (-5)}{x - (-3)} = \frac{-3 - (-5)}{1 - (-3)}$$

$$\frac{y + 5}{x + 3} = \frac{2}{4}$$

$$\frac{y + 5}{x + 3} = \frac{1}{2}$$ Multiply each side by $(x + 3)$.

$$y + 5 = \frac{1}{2}(x + 3)$$

$$y + 5 = \frac{1}{2}x + \frac{3}{2}$$

$$y = \frac{1}{2}x - \frac{7}{2}$$

The second equation could be: $y = \frac{1}{2}x - \frac{7}{2}$

19. a) $2x + 3y = -5$ ①

$\frac{x}{2} - \frac{y}{3} = 2$ ②

Write each equation in slope-intercept form.

For equation ①:

$2x + 3y = -5$ Subtract $2x$ from each side.

$3y = -2x - 5$ Divide each side by 3.

$y = -\frac{2}{3}x - \frac{5}{3}$

The slope of the graph of equation ① is $-\frac{2}{3}$.

For equation ②:

$\frac{x}{2} - \frac{y}{3} = 2$ Solve for y. Subtract $\frac{x}{2}$ from each side.

$-\frac{y}{3} = -\frac{x}{2} + 2$ Multiply each side by –3.

$-3\left(-\frac{y}{3}\right) = -3\left(-\frac{x}{2}\right) + (-3)(2)$ Simplify.

$y = \frac{3}{2}x - 6$

The slope of the graph of the equation ② is $\frac{3}{2}$.

The slopes $\frac{3}{2}$ and $-\frac{2}{3}$ are negative reciprocals, so the lines are perpendicular.

b) I chose two numbers that are negative reciprocals; such as $\frac{5}{3}$ and $-\frac{3}{5}$. I used these numbers as the slopes of two lines.

I chose two y-intercepts, such as 4 and –2.

To get two equations of a linear system, I substituted a slope and y-intercept into this form of the equation of a line: $y = mx + b$

A linear system is:

$$y = \frac{5}{3}x + 4$$

$$y = -\frac{3}{5}x - 2$$

Lesson 7.3 **Math Lab: Using Graphing Technology to Solve a System of Linear Equations** **Assess Your Understanding, pages 412-413**

1. a) In the table:
The X-values increase by 1 each time. The values go from X = 0 to X = 6, with corresponding Y-values for each equation.
Gerard should use the table to find the X-value for which $Y_1 = Y_2$.
When X = 4, $Y_1 = 2$ and $Y_2 = 2$, so the solution of the linear system is approximately:
X = 4 and Y = 2

b) To solve this linear system, Gerard could graph each line and determine the coordinates of their point of intersection.

2. a) $3x - 6y = 14$ ①

$x + y = \frac{7}{6}$ ②

To solve the linear system above, I would use a graphing calculator.
I first have to write each equation in the form $y = mx + b$.
For equation ①:

$3x - 6y = 14$ Solve for y. Subtract $3x$ from each side.

$-6y = -3x + 14$ Divide each side by –6.

$y = \frac{-3}{-6}x + \frac{14}{-6}$

$y = \frac{1}{2}x - \frac{7}{3}$

For equation ②:

$x + y = \frac{7}{6}$ Solve for y. Subtract x from each side.

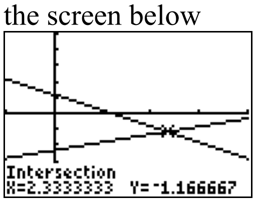
$y = -x + \frac{7}{6}$

The linear system is:

$y = \frac{1}{2}x - \frac{7}{3}$

$y = -x + \frac{7}{6}$

On a TI-83 graphing calculator, I pressed $\boxed{\text{Y=}}$, then next to Y1 = I input the expression: (1/2)X–7/3. I moved the cursor down to Y2= and input the expression –X+7/6.
I pressed $\boxed{\text{GRAPH}}$. To see the point of intersection, I set the $\boxed{\text{WINDOW}}$ to Xmin = –1, Xmax = 4, Ymin = –5, and Ymax = 5. To show the coordinates of the point of intersection, I pressed $\boxed{\text{2nd}}$ $\boxed{\text{TRACE}}$ for CALC, then selected 5:intersect. I pressed $\boxed{\text{ENTER}}$ 3 times to get the screen below



b) From the calculator screen, the solution is: $x = 2.\overline{3}$ and $y = -1.1\overline{6}$

In fraction form, the solution is: $x = \frac{7}{3}$ and $y = -\frac{7}{6}$

Verify the solution.

Substitute $x = \frac{7}{3}$ and $y = -\frac{7}{6}$ into each equation.

$y = \frac{1}{2}x - \frac{7}{3}$

$$\text{L.S.} = y = -\frac{7}{6}$$

$$\begin{aligned}\text{R.S.} &= \frac{1}{2}x - \frac{7}{3}\\ &= \frac{1}{2}\left(\frac{7}{3}\right) - \frac{7}{3}\\ &= \frac{7}{6} - \frac{7}{3}\\ &= \frac{7}{6} - \frac{14}{6}\\ &= -\frac{7}{6}\\ &= \text{L.S.}\end{aligned}$$

$y = -x + \frac{7}{6}$

$$\text{L.S.} = y = -\frac{7}{6}$$

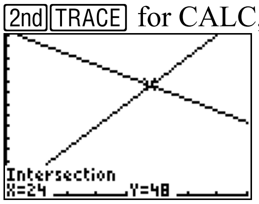
$$\begin{aligned}\text{R.S.} &= -x + \frac{7}{6}\\ &= -\frac{7}{3} + \frac{7}{6}\\ &= -\frac{14}{6} + \frac{7}{6}\\ &= -\frac{7}{6}\\ &= \text{L.S.}\end{aligned}$$

Since the left side is equal to the right side in each equation, then $x = \frac{7}{3}$ and $y = -\frac{7}{6}$ is the solution of the linear system.

3. Let c represent the number of cedar tree seedlings planted.
Let s represent the number of spruce tree seedlings planted.
A total of 72 seedlings was purchased.
So, one equation is: $c + s = 72$
There were twice as many cedar trees as spruce trees.
So, another equation is: $c = 2s$
A linear system is:
$c + s = 72$ ①
$c = 2s$ ②
Write equation ① in $y = mx + b$ form.
$c + s = 72$ Subtract s from each side.
$c = -s + 72$

On a TI-83 graphing calculator, press [Y=], then next to Y1 = input the expression –X+72.
Move the cursor down to Y2= and input the expression 2X.

Press [GRAPH]. To see the point of intersection, set the [WINDOW] to Xmin = 0, Xmax = 40, Ymin = 0, and Ymax = 70. To show the coordinates of the point of intersection, press [2nd][TRACE] for CALC, then selected 5:intersect. Press [ENTER] 3 times to get the screen below:



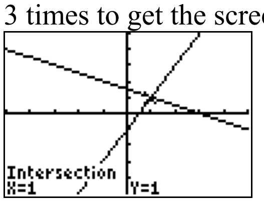
From the calculator screen, the solution is: $x = 24$ and $y = 48$
The value of x is the value of s, so the number of spruce tree seedlings is 24.

The value of y is the value of c, so the number of cedar tree seedlings is 48.

Verify the solution.
The total number of tree seedlings is: 24 + 48 = 72; this is the same as the given information.
The number of cedar tree seedlings is 48, which is twice the number of spruce tree seedlings, 24; this is the same as the given information.
The solution is correct.

4. a) i) $x + 2y = 3$ ①
$2x - y = 1$ ②
Write each equation in $y = mx + b$ form.
For equation ①:
$x + 2y = 3$ Subtract x from each side.
$2y = -x + 3$ Divide each side by 2.
$y = -\frac{1}{2}x + \frac{3}{2}$
For equation ②:
$2x - y = 1$ Subtract $2x$ from each side.
$-y = -2x + 1$ Multiply each side by -1.
$y = 2x - 1$

On a TI-83 graphing calculator, press $\boxed{\text{Y=}}$, then next to Y1 = input the expression (–1/2)X+3/2. Move the cursor down to Y2= and input the expression 2X–1.
Press $\boxed{\text{GRAPH}}$. To see the point of intersection, set the $\boxed{\text{WINDOW}}$ to Xmin = –5, Xmax = 5, Ymin = –5, and Ymax = 5. To show the coordinates of the point of intersection, press $\boxed{\text{2nd}}\boxed{\text{TRACE}}$ for CALC, then selected 5:intersect. Press $\boxed{\text{ENTER}}$ 3 times to get the screen below.



From the calculator screen, the solution is: $x = 1$ and $y = 1$

Verify the solution.
Substitute $x = 1$ and $y = 1$ into each equation.

$x + 2y = 3$		$2x - y = 1$	
L.S. $= x + 2y$	R.S. = 3	L.S. $= 2x - y$	R.S. = 1
$= 1 + 2(1)$		$= 2(1) - 1$	
$= 3$		$= 1$	

Since the left side is equal to the right side in each equation, then $x = 1$ and $y = 1$ is the solution of the linear system.

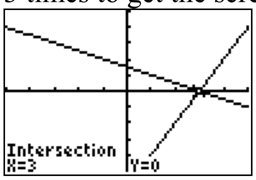
ii) $x + 2y = 3$ ①
$2x - y = 6$ ②
Write each equation in $y = mx + b$ form.
For equation ①:
From part i:
$y = -\frac{1}{2}x + \frac{3}{2}$

For equation ②:

$2x - y = 6$ Subtract $2x$ from each side.
$-y = -2x + 6$ Multiply each side by -1.
$y = 2x - 6$

On a TI-83 graphing calculator, press $\boxed{\text{Y=}}$, then next to Y1 = input the expression (–1/2)X+3/2. Move the cursor down to Y2= and input the expression 2X–6.

Press $\boxed{\text{GRAPH}}$. To see the point of intersection, set the $\boxed{\text{WINDOW}}$ to Xmin = –5, Xmax = 5, Ymin = –5, and Ymax = 5. To show the coordinates of the point of intersection, press $\boxed{\text{2nd}}\boxed{\text{TRACE}}$ for CALC, then selected 5:intersect. Press $\boxed{\text{ENTER}}$ 3 times to get the screen below.



From the calculator screen, the solution is: $x = 3$ and $y = 0$

Verify the solution.
Substitute $x = 3$ and $y = 0$ into each equation.

$x + 2y = 3$
L.S. $= x + 2y$ R.S. $= 3$
$= 3 + 2(0)$
$= 3$

$2x - y = 6$
L.S. $= 2x - y$ R.S. $= 6$
$= 2(3) - 0$
$= 6$

Since the left side is equal to the right side in each equation, then $x = 3$ and $y = 0$ is the solution of the linear system.

iii) $x + 2y = 3$ ①
$2x - y = 11$ ②

Write each equation in $y = mx + b$ form.
For equation ①:
From part i:

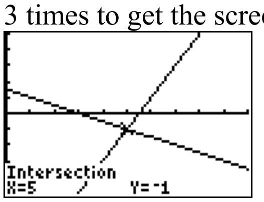
$$y = -\frac{1}{2}x + \frac{3}{2}$$

For equation ②:

$2x - y = 11$ Subtract $2x$ from each side.
$-y = -2x + 11$ Multiply each side by -1.
$y = 2x - 11$

On a TI-83 graphing calculator, press $\boxed{\text{Y=}}$, then next to Y1 = input the expression (–1/2)X+3/2. Move the cursor down to Y2= and input the expression 2X–11.

Press $\boxed{\text{GRAPH}}$. To see the point of intersection, set the $\boxed{\text{WINDOW}}$ to Xmin = 0, Xmax = 10, Ymin = –5, and Ymax = 5. To show the coordinates of the point of intersection, press $\boxed{\text{2nd}}$ $\boxed{\text{TRACE}}$ for CALC, then selected 5:intersect. Press $\boxed{\text{ENTER}}$ 3 times to get the screen below.



From the calculator screen, the solution is: $x = 5$ and $y = -1$

Verify the solution.
Substitute $x = 5$ and $y = -1$ into each equation.

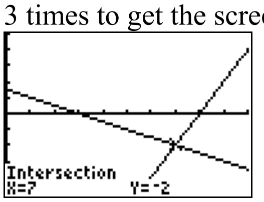
$x + 2y = 3$		$2x - y = 11$	
L.S. $= x + 2y$	R.S. $= 3$	L.S. $= 2x - y$	R.S. $= 11$
$= 5 + 2(-1)$		$= 2(5) - (-1)$	
$= 5 - 2$		$= 10 + 1$	
$= 3$		$= 11$	

Since the left side is equal to the right side in each equation, then $x = 5$ and $y = -1$ is the solution of the linear system.

iv) $x + 2y = 3$ ①
$2x - y = 16$ ②
Write each equation in $y = mx + b$ form.
For equation ①:
From part i:
$$y = -\frac{1}{2}x + \frac{3}{2}$$
For equation ②:

$2x - y = 16$	Subtract $2x$ from each side.
$-y = -2x + 16$	Multiply each side by -1.
$y = 2x - 16$	

On a TI-83 graphing calculator, press $\boxed{\text{Y=}}$, then next to Y1 = input the expression (–1/2)X+3/2. Move the cursor down to Y2= and input the expression 2X–16.
Press $\boxed{\text{GRAPH}}$. To see the point of intersection, set the $\boxed{\text{WINDOW}}$ to Xmin = 0, Xmax = 10, Ymin = –5, and Ymax = 5. To show the coordinates of the point of intersection, press $\boxed{\text{2nd}}\boxed{\text{TRACE}}$ for CALC, then selected 5:intersect. Press $\boxed{\text{ENTER}}$ 3 times to get the screen below.



From the calculator screen, the solution is: $x = 7$ and $y = -2$

Verify the solution.
Substitute $x = 7$ and $y = -2$ into each equation.

$x + 2y = 3$		$2x - y = 16$	
L.S. $= x + 2y$	R.S. $= 3$	L.S. $= 2x - y$	R.S. $= 16$
$= 7 + 2(-2)$		$= 2(7) - (-2)$	
$= 7 - 4$		$= 14 + 2$	
$= 3$		$= 16$	

Since the left side is equal to the right side in each equation, then $x = 7$ and $y = -2$ is the solution of the linear system.

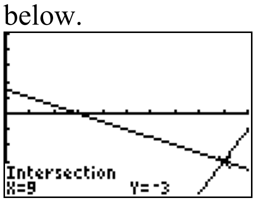
b) The first equation in each linear system is the same.
In the second equations, the x-coefficients are equal, the y-coefficients are equal, and the constant term increases by 5 each time.

So, the next linear system in the pattern will be:
$x + 2y = 3$ ①
$2x - y = 21$ ②
The solutions are:
$x = 1$ and $y = 1$
$x = 3$ and $y = 0$
$x = 5$ and $y = -1$
$x = 7$ and $y = -2$
The x-coordinate increases by 2 each time and the y-coordinate decreases by 1 each time.
So, the solution of the next linear system in the pattern is: $x = 9$ and $y = -3$

c) Write each equation in part b in $y = mx + b$ form.
For equation ①:
From part a) i:
$$y = -\frac{1}{2}x + \frac{3}{2}$$
For equation ②:
$2x - y = 21$ Subtract $2x$ from each side.
$-y = -2x + 21$ Multiply each side by -1.
$y = 2x - 21$

On a TI-83 graphing calculator, press $\boxed{\text{Y=}}$, then next to Y1 = input the expression (–1/2)X+3/2. Move the cursor down to Y2= and input the expression 2X–21.

Press $\boxed{\text{GRAPH}}$. To see the point of intersection, set the $\boxed{\text{WINDOW}}$ to Xmin = 0, Xmax = 10, Ymin = –5, and Ymax = 5. To show the coordinates of the point of intersection, press $\boxed{\text{2nd}}\boxed{\text{TRACE}}$ for CALC, then selected 5:intersect. Press $\boxed{\text{ENTER}}$ 3 times to get the screen below.



From the calculator screen, the solution is: $x = 9$ and $y = -3$

Verify the solution.
Substitute $x = 9$ and $y = -3$ into each equation.

$x + 2y = 3$
L.S. $= x + 2y$ R.S. $= 3$
$= 9 + 2(-3)$
$= 9 - 6$
$= 3$

$2x - y = 21$
L.S. $= 2x - y$ R.S. $= 21$
$= 2(9) - (-3)$
$= 18 + 3$
$= 21$

Since the left side is equal to the right side in each equation, then $x = 9$ and $y = -3$ is the solution of the linear system.

5. For System A:
$\frac{1}{2}x + y = 3$ ①
$x + \frac{1}{2}y = 3$ ②
Write each equation in $y = mx + b$ form.

For equation ①:

$\frac{1}{2}x + y = 3$ Subtract $\frac{1}{2}x$ from each side.

$y = -\frac{1}{2}x + 3$

For equation ②:

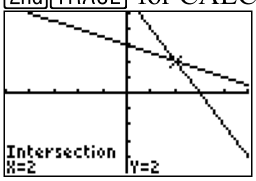
$x + \frac{1}{2}y = 3$ Subtract x from each side.

$\frac{1}{2}y = -x + 3$ Multiply each side by 2.

$y = -2x + 6$

On a TI-83 graphing calculator, press [Y=], then next to Y1 = input the expression (–1/2)X+3. Move the cursor down to Y2= and input the expression –2X+6.

Press [GRAPH]. To see the point of intersection, set the [WINDOW] to Xmin = –5, Xmax = 5, Ymin = –5, and Ymax = 5. To show the coordinates of the point of intersection, press [2nd][TRACE] for CALC, then select 5:intersect. Press [ENTER] 3 times to get the screen below.



From the calculator screen, the solution is: $x = 2$ and $y = 2$

Verify the solution.
Substitute $x = 2$ and $y = 2$ into each equation.

$\frac{1}{2}x + y = 3$

$$\begin{aligned} \text{L.S.} &= \frac{1}{2}x + y \\ &= \frac{1}{2}(2) + 2 \\ &= 1 + 2 \\ &= 3 \\ &= \text{R.S.} \end{aligned}$$

$x + \frac{1}{2}y = 3$

$$\begin{aligned} \text{L.S.} &= x + \frac{1}{2}y \\ &= 2 + \frac{1}{2}(2) \\ &= 2 + 1 \\ &= 3 \\ &= \text{R.S.} \end{aligned}$$

Since the left side is equal to the right side in each equation, then $x = 2$ and $y = 2$ is the solution of the linear system.
So, a linear system with fractional coefficients does not always have an approximate solution.

For System B:

$2x + y = \frac{23}{6}$ ①

$\frac{x}{3} + \frac{y}{2} = \frac{55}{36}$ ②

Write each equation in $y = mx + b$ form.

For equation ①:

$2x + y = \frac{23}{6}$ Subtract $2x$ from each side.

$y = -2x + \frac{23}{6}$

For equation ②:

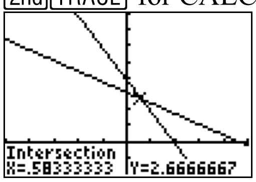
$\frac{x}{3} + \frac{y}{2} = \frac{55}{36}$ Subtract $\frac{x}{3}$ from each side.

$\frac{y}{2} = -\frac{x}{3} + \frac{55}{36}$ Multiply each side by 2.

$y = -\frac{2x}{3} + \frac{55}{18}$

On a TI-83 graphing calculator, press [Y=], then next to Y1 = input the expression –2X+23/6. Move the cursor down to Y2= and input the expression –2X/3+55/18.

Press [GRAPH]. To see the point of intersection, set the [WINDOW] to Xmin = –5, Xmax = 5, Ymin = –2, and Ymax = 8. To show the coordinates of the point of intersection, press [2nd][TRACE] for CALC, then select 5:intersect. Press [ENTER] 3 times to get the screen below.



From the calculator screen, the solution is: $x = 0.58\overline{3}$ and $y = 2.\overline{6}$

Verify the solution.
Substitute $x = 0.58333333$ and $y = 2.6666667$ into each equation.

$2x + y = \frac{23}{6}$

L.S. $= 2x + y$
$= 2(0.58333333) + 2.6666667$
$= 3.833333360$
= R.S.

R.S. $= \frac{23}{6}$
$= 3.8\overline{3}$

$\frac{x}{3} + \frac{y}{2} = \frac{55}{36}$

L.S. $= \frac{x}{3} + \frac{y}{2}$
$= \frac{0.58333333}{3} + \frac{2.6666667}{2}$
$= 1.527777793$
= R.S.

R.S. $= \frac{55}{36}$
$= 1.52\overline{7}$

Since the left side is equal to the right side in each equation, then $x = 0.58\overline{3}$ and $y = 2.\overline{6}$ is the solution of the linear system.

These numbers are probably exact, and can be written as $x = \frac{7}{12}$ and $y = \frac{8}{3}$.

So, a linear system with fractional coefficients does not always have an approximate solution.

Checkpoint 1 **Assess Your Understanding (page 415)**

7.1

1. a) Let the length of the JumboTron be represented by l feet.
Let the width of the JumboTron be represented by w feet.
The perimeter of the JumboTron is 128 ft.
So, $l + l + w + w = 128$
One equation is: $2l + 2w = 128$
The width of the JumboTron is 16 ft. less than its length.
So, another equation is: $w = l – 16$
A linear system is:
$2l + 2w = 128$ ①
$w = l – 16$ ②

b) i) Substitute $l = 40$ and $w = 24$ into each equation of the linear system.

For equation ①:
$2l + 2w = 128$
L.S. $= 2l + 2w$
$= 2(40) + 2(24)$
$= 80 + 48$
$= 128$
= R.S.

For equation ②:
$w = l – 16$
L.S. $= w$
$= 24$
R.S. $= l – 16$
$= 40 – 16$
$= 24$

For each equation, the left side is equal to the right side, so the solution is correct.

ii) Use the fact that the length is 40 ft. and the width is 24 ft.
Then, the perimeter is: 40 ft. + 40 ft. + 24 ft. + 24 ft. = 128 ft.
This is the same as the given perimeter.
The difference between the length and the width is: 40 ft. – 24 ft. = 16 ft.
This is the same as the given information that the width is 16 ft. less than the length.
So, the solution is correct.

2. $10x + 5y = 850$ ①
$x – y = 10$ ②
The coefficients in the first equation are 5 and 10, which are the values of a nickel and a dime, so use these coins and their values to create a situation.
Let x represent the number of dimes and let y represent the number of nickels.
The value of x dimes is $10x$ cents and the value of y nickels is $5y$ cents.
From equation ①, the value of x dimes and y nickels is 850¢.
From equation ②, the difference in the numbers of dimes and nickels is 10.
A related problem is:
A collection of nickels and dimes has a value of $8.50.
There are 10 more dimes than nickels.
How many dimes are there? How many nickels are there?

7.2

3. $2x + y = 1$ ①
$x + 2y = –1$ ②
Determine the coordinates of two points on each line.
For equation ①:

Substitute: $x = 0$
$2x + y = 1$
$2(0) + y = 1$
$y = 1$

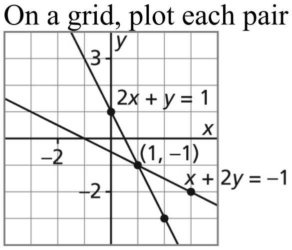
Substitute: $x = 2$
$2x + y = 1$
$2(2) + y = 1$
$4 + y = 1$
$y = –3$

For equation ①, two points have coordinates: (0, 1) and (2, –3)
For equation ②:

Substitute: $x = 1$
$x + 2y = –1$
$1 + 2y = –1$
$2y = –2$
$y = –1$

Substitute: $x = 3$
$x + 2y = –1$
$3 + 2y = –1$
$2y = –4$
$y = –2$

For equation ②, two points have coordinates: (1, –1) and (3, –2)
On a grid, plot each pair of points, then draw a line through them.



From the graph, the solution appears to be: $x = 1$ and $y = –1$

Verify the solution.
The solution satisfies equation ② because the coordinates were determined to graph the line.
Substitute $x = 1$ and $y = –1$ in equation ①.
$2x + y = 1$
L.S. $= 2x + y$ R.S. $= 1$
$= 2(1) + (–1)$
$= 2 – 1$
$= 1$
Since the left side is equal to the right side, the solution is correct.

4. **a)** $F = 75 + 5v$ ①
$F = 10v$ ②
Determine the coordinates of two points on each line.
For equation ①:

Substitute: $v = 0$
$F = 75 + 5v$
$F = 75 + 5(0)$
$F = 75$

Substitute: $v = 10$
$F = 75 + 5v$
$F = 75 + 5(10)$
$F = 75 + 50$
$F = 125$

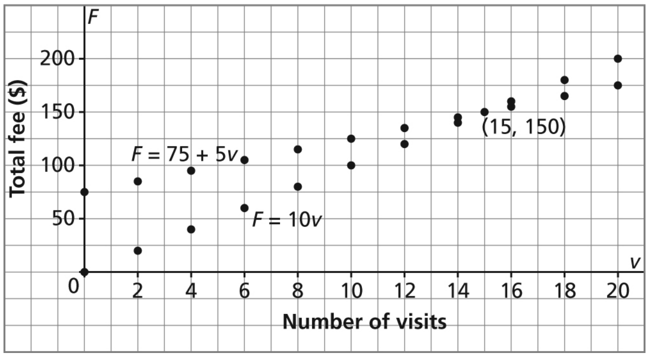
For equation ①, two points have coordinates: (0, 75) and (10, 125)
For equation ②:

Substitute: $v = 0$
$F = 10v$
$F = 10(0)$
$F = 0$

Substitute: $v = 10$
$F = 10v$
$F = 10(10)$
$F = 100$

For equation ②, two points have coordinates: (0, 0) and (10, 100)
The data are discrete. On a grid, plot each pair of points, then use a straightedge to mark more points on each line, with whole number values of v. Use a scale of 1 square to 1 unit

on the v-axis and 1 square to 25 units on the F-axis.



From the graph, the solution appears to be: $v = 15$ and $F = 150$

Verify the solution.
Substitute $v = 15$ and $F = 150$ into each equation.

For $F = 75 + 5v$ ①

L.S. $= F$
$= 150$

R.S. $= 75 + 5v$
$= 75 + 5(15)$
$= 75 + 75$
$= 150$

For $F = 10v$ ②

L.S. $= F$
$= 150$

R.S. $= 10v$
$= 10(15)$
$= 150$

In each equation, the left side is equal to the right side, so the solution is correct.

b) Plan A is represented by the equation $F = 75 + 5v$. Since the vertical axis represents the total fee, Plan A is cheaper when the graph of the line $F = 75 + 5v$ is below the other line on the graph. This occurs for values of v greater than 15.
So, Plan A is cheaper for 16 or more visits.

5. a) Let the number of students who went to the aquarium be represented by s.
Let the number of adults who went to the aquarium be represented by a.
The cost for a student is \$21 and the cost for an adult is \$27, with a total cost of \$396.
So, one equation is: $21s + 27a = 396$
Eighteen people went to the aquarium.
So, another equation is: $s + a = 18$
A linear system is:
$21s + 27a = 396$ ①
$s + a = 18$ ②

b) Determine the coordinates of two points on each line.
For equation ①:
Use guess and test to determine the whole-number coordinates of two points on the line.

Substitute: $s = 6$
$21s + 27a = 396$
$21(6) + 27a = 396$
$126 + 27a = 396$
$27a = 396 - 126$
$27a = 270$
$a = 10$

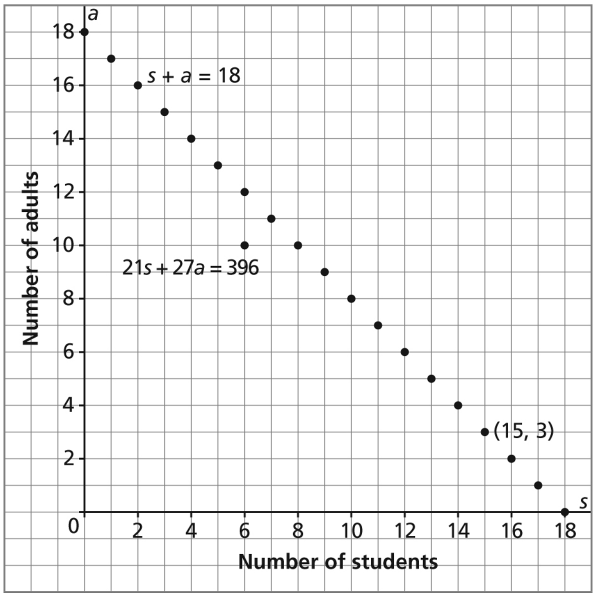
Substitute: $s = 15$
$21s + 27a = 396$
$21(15) + 27a = 396$
$315 + 27a = 396$
$27a = 396 - 315$
$27a = 81$
$a = 3$

For equation ①, two points have coordinates: (6, 10) and (15, 3)
For equation ②, use intercepts.

Substitute: $a = 0$
$s + a = 18$
$s + 0 = 18$
$s = 18$

Substitute: $s = 0$
$s + a = 18$
$0 + a = 18$
$a = 18$

For equation ②, two points have coordinates: (18, 0) and (0, 18)
Graph a as a function of s; that is, plot a on the vertical axis and s on the horizontal axis. The data are discrete. On a grid, plot each pair of points, then use a straightedge to mark more points on each line, with whole number values of v. Use a scale of 1 square to 1 unit on each axis.



From the graph, the solution appears to be: $s = 15$ and $a = 3$; that is, 15 students and 3 adults went to the aquarium.

Verify the solution.
The total cost, in dollars, is: $15(21) + 3(27) = 315 + 81$, or 396; this agrees with the given information.
The total number of people who went to the aquarium is: $15 + 3 = 18$; this agrees with the given information.
So, the solution is correct.

7.3

6. a) Let l represent the number of large trees.
Let s represent the number of small trees.
A large tree removes 1.4 kg of pollution and a small tree removes 0.02 kg of pollution, with a total of 7200 kg of pollution being removed.
So, one equation is: $1.4l + 0.02s = 7200$
There is a total of 15 000 trees.
So, another equation is: $l + s = 15\,000$
A linear system is:
$1.4l + 0.02s = 7200$ ①
$l + s = 15\,000$ ②

b) Write each equation in $y = mx + b$ form.
For equation ①:
$1.4l + 0.02s = 7200$ Subtract $1.4l$ from each side.

$0.02s = -1.4l + 7200$ Divide each side by 0.02.

$s = \frac{-1.4}{0.02}l + \frac{7200}{0.02}$ Simplify.

$s = -70l + 360\,000$

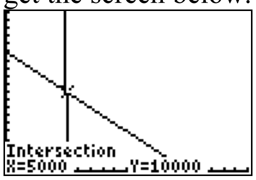
For equation ②:

$l + s = 15\,000$ Subtract l from each side.

$s = -l + 15\,000$

On a TI-83 graphing calculator, press Y=, then next to Y1 = input the expression –70X+360000. Move the cursor down to Y2= and input the expression –X+15000.

Press GRAPH. To see the point of intersection, set the WINDOW to Xmin = 0, Xmax = 20000, Ymin = 0, and Ymax = 20000. To show the coordinates of the point of intersection, press 2nd TRACE for CALC, then selected 5:intersect. Press ENTER 3 times to get the screen below.



From the calculator screen, the solution is: $x = 5000$ and $y = 10\,000$
The value of x is the value of l, so the number of large trees is 5000.
The value of y is the value of s, so the number of small trees is 10 000.

Verify the solution.
The total number of trees is: 5000 + 10 000 = 15 000; this is the same as the given information.
The total mass of pollution removed, in kilograms, is:
1.4(5000) + 0.02(10 000) = 7000 + 200, or 7200; this is the same as the given information.
The solution is correct.

Lesson 7.4 **Using a Substitution Strategy to Solve a System of Linear Equations** **Exercises (pages 425-427)**

A

4. a) $y = 9 - x$ ①
$2x + 3y = 11$ ②
Equation ① is solved for y, so substitute $y = 9 - x$ in equation ②.

$2x + 3y = 11$ ②
$2x + 3(9 - x) = 11$ Remove brackets.
$2x + 27 - 3x = 11$ Collect like terms.
$-x = 11 - 27$
$-x = -16$ Multiply each side by –1.
$x = 16$

Substitute $x = 16$ in equation ①.
$y = 9 - x$ ①
$y = 9 - 16$
$y = -7$

Verify the solution.
In each equation, substitute: $x = 16$ and $y = -7$

$y = 9 - x$ ①
L.S. $= y$
$= -7$
R.S. $= 9 - x$
$= 9 - 16$
$= -7$

$2x + 3y = 11$ ②
L.S. $= 2x + 3y$
$= 2(16) + 3(-7)$
$= 32 - 21$
$= 11$
= R.S.

For each equation, the left side is equal to the right side, so the solution is:
$x = 16$ and $y = -7$

b) $x = y - 1$ ①
$3x - y = 11$ ②
Equation ① is solved for x, so substitute $x = y - 1$ in equation ②.

$3x - y = 11$ ②
$3(y - 1) - y = 11$ Remove brackets.
$3y - 3 - y = 11$ Collect like terms.
$2y = 11 + 3$
$2y = 14$ Divide each side by 2.
$y = 7$

Substitute $y = 7$ in equation ①.
$x = y - 1$ ①
$x = 7 - 1$
$x = 6$

Verify the solution.
In each equation, substitute: $x = 6$ and $y = 7$

$x = y - 1$ ①
L.S. $= x$
$= 6$
R.S. $= y - 1$
$= 7 - 1$
$= 6$

$3x - y = 11$ ②
L.S. $= 3x - y$
$= 3(6) - 7$
$= 18 - 7$
$= 11$
= R.S.

For each equation, the left side is equal to the right side, so the solution is: $x = 6$ and $y = 7$

c) $x = 7 + y$ ①
$2x + y = –10$ ②
Equation ① is solved for x, so substitute $x = 7 + y$ in equation ②.
$2x + y = –10$ ②
$2(7 + y) + y = –10$ Remove brackets.
$14 + 2y + y = –10$ Collect like terms.
$3y = –10 – 14$
$3y = –24$ Divide each side by 3.
$y = –8$
Substitute $y = –8$ in equation ①.
$x = 7 + y$ ①
$x = 7 – 8$
$x = –1$

Verify the solution.
In each equation, substitute: $x = –1$ and $y = –8$

$x = 7 + y$ ①
L.S. $= x$
$= –1$
R.S. $= 7 + y$
$= 7 + (–8)$
$= –1$

$2x + y = –10$ ②
L.S. $= 2x + y$
$= 2(–1) + (–8)$
$= –2 – 8$
$= –10$
$=$ R.S.

For each equation, the left side is equal to the right side, so the solution is:
$x = –1$ and $y = –8$

d) $3x + y = 7$ ①
$y = x + 3$ ②
Equation ② is solved for y, so substitute $y = x + 3$ in equation ①.
$3x + y = 7$ ①
$3x + (x + 3) = 7$ Remove brackets.
$3x + x + 3 = 7$ Collect like terms.
$4x = 7 – 3$
$4x = 4$ Divide each side by 4.
$x = 1$
Substitute $x = 1$ in equation ②.
$y = x + 3$ ②
$y = 1 + 3$
$y = 4$

Verify the solution.
In each equation, substitute: $x = 1$ and $y = 4$

$3x + y = 7$ ①
L.S. $= 3x + y$
$= 3(1) + 4$
$= 7$
R.S. $= 7$

$y = x + 3$ ②
L.S. $= y$
$= 4$
R.S. $= x + 3$
$= 1 + 3$
$= 4$

For each equation, the left side is equal to the right side, so the solution is: $x = 1$ and $y = 4$

5. a) $2x + 3y = 11$ ①
$4x - y = -13$ ②
Solve equation ② for y.
$4x - y = -13$ Subtract $4x$ from each side.
$-y = -4x - 13$ Multiply each side by -1.
$y = 4x + 13$
Substitute $y = 4x + 13$ in equation ①.
$2x + 3y = 11$ ①
$2x + 3(4x + 13) = 11$ Remove brackets.
$2x + 12x + 39 = 11$ Collect like terms.
$14x = 11 - 39$
$14x = -28$ Divide each side by 14.
$x = -2$
Substitute $x = -2$ in equation ②.
$4x - y = -13$ ②
$4(-2) - y = -13$ Solve for y.
$-8 - y = -13$
$-y = -13 + 8$
$-y = -5$
$y = 5$

Verify the solution.
In each equation, substitute: $x = -2$ and $y = 5$

$2x + 3y = 11$ ①
L.S. $= 2x + 3y$
$= 2(-2) + 3(5)$
$= -4 + 15$
$= 11$
= R.S.

$4x - y = -13$ ②
L.S. $= 4x - y$
$= 4(-2) - 5$
$= -8 - 5$
$= -13$
= R.S.

For each equation, the left side is equal to the right side, so the solution is:
$x = -2$ and $y = 5$

b) $4x + y = -5$ ①
$2x + 3y = 5$ ②
Solve equation ① for y.
$4x + y = -5$ Subtract $4x$ from each side.
$y = -4x - 5$
Substitute $y = -4x - 5$ in equation ②.
$2x + 3y = 5$ ②
$2x + 3(-4x - 5) = 5$ Remove brackets.
$2x - 12x - 15 = 5$ Collect like terms.
$-10x = 5 + 15$
$-10x = 20$ Divide each side by -2.
$x = -2$
Substitute $x = -2$ in equation ①.
$4x + y = -5$ ①
$4(-2) + y = -5$ Solve for y.
$-8 + y = -5$
$y = -5 + 8$
$y = 3$

Verify the solution.
In each equation, substitute: $x = -2$ and $y = 3$

$4x + y = -5$ ①
L.S. $= 4x + y$
$= 4(-2) + 3$
$= -8 + 3$
$= -5$
$=$ R.S.

$2x + 3y = 5$ ②
L.S. $= 2x + 3y$
$= 2(-2) + 3(3)$
$= -4 + 9$
$= 5$
$=$ R.S.

For each equation, the left side is equal to the right side, so the solution is:
$x = -2$ and $y = 3$

c) $x + 2y = 13$ ①
$2x - 3y = -9$ ②
Solve equation ① for x.
$x + 2y = 13$ Subtract $2y$ from each side.
$x = -2y + 13$
Substitute $x = -2y + 13$ in equation ②.
$2x - 3y = -9$ ②
$2(-2y + 13) - 3y = -9$ Remove brackets.
$-4y + 26 - 3y = -9$ Collect like terms.
$-7y = -9 - 26$
$-7y = -35$ Divide each side by -7.
$y = 5$
Substitute $y = 5$ in equation ①.
$x + 2y = 13$ ①
$x + 2(5) = 13$
$x + 10 = 13$ Solve for x.
$x = 3$

Verify the solution.
In each equation, substitute: $x = 3$ and $y = 5$

$x + 2y = 13$ ①
L.S. $= x + 2y$
$= 3 + 2(5)$
$= 3 + 10$
$= 13$
$=$ R.S.

$2x - 3y = -9$ ②
L.S. $= 2x - 3y$
$= 2(3) - 3(5)$
$= 6 - 15$
$= -9$
$=$ R.S.

For each equation, the left side is equal to the right side, so the solution is: $x = 3$ and $y = 5$

d) $3x + y = 7$ ①
$5x + 2y = 13$ ②
Solve equation ① for y.
$3x + y = 7$ Subtract $3x$ from each side.
$y = -3x + 7$
Substitute $y = -3x + 7$ in equation ②.
$5x + 2y = 13$ ②
$5x + 2(-3x + 7) = 13$ Remove brackets.
$5x - 6x + 14 = 13$ Collect like terms.
$-x = 13 - 14$
$-x = -1$
$x = 1$

Substitute $x = 1$ in equation ①.

$3x + y = 7$ ①
$3(1) + y = 7$
$3 + y = 7$
$y = 4$

Verify the solution.
In each equation, substitute: $x = 1$ and $y = 4$

$3x + y = 7$ ①
L.S. $= 3x + y$ R.S. $= 7$
$= 3(1) + 4$
$= 7$

$5x + 2y = 13$ ②
L.S. $= 5x + 2y$
$= 5(1) + 2(4)$
$= 5 + 8$
$= 13$

For each equation, the left side is equal to the right side, so the solution is: $x = 1$ and $y = 4$

B

6. a) i) $2x - 3y = 2$ ①
$4x - 4y = 2$ ②
The term $4x$ in equation ② is 2 times the term $2x$ in equation ①.

ii) $40x + 10y = 10$ ①
$3x + 5y = 5$ ②
The term $10y$ in equation ① is 2 times the term $5y$ in equation ②.

iii) $-3x + 6y = 9$ ①
$5x - 2y = -7$ ②
The term $6y$ in equation ① is 3 times the term $2y$ in equation ②.

iv) $-3x + 4y = 6$ ①
$9x + 3y = 27$ ②
The term $9x$ in equation ② is 3 times the term $3x$ in equation ①.

b) i) $2x - 3y = 2$ ①
$4x - 4y = 2$ ②
In equation ②, the term $4x$ can be written as $2(2x)$:
$2(2x) - 4y = 2$ ③
Solve equation ① for $2x$.
$2x - 3y = 2$ ①
$2x = 3y + 2$
Substitute for $2x$ in equation ③.
$2(2x) - 4y = 2$ ③
$2(3y + 2) - 4y = 2$ Simplify, then solve for y.
$6y + 4 - 4y = 2$
$2y = -2$
$y = -1$
Substitute $y = -1$ in equation ①.
$2x - 3y = 2$ ①
$2x - 3(-1) = 2$ Simplify, then solve for x.
$2x + 3 = 2$
$2x = -1$

$x = -\frac{1}{2}$

Verify the solution.

In each equation, substitute: $x = -\frac{1}{2}$ and $y = -1$

$2x - 3y = 2$ ①

L.S. $= 2x - 3y$
$= 2\left(-\frac{1}{2}\right) - 3(-1)$
$= -1 + 3$
$= 2$
$=$ R.S.

$4x - 4y = 2$ ②

L.S. $= 4x - 4y$
$= 4\left(-\frac{1}{2}\right) - 4(-1)$
$= -2 + 4$
$= 2$
$=$ R.S.

For each equation, the left side is equal to the right side, so the solution is: $x = -\frac{1}{2}$ and $y = -1$

ii) $40x + 10y = 10$ ①
$3x + 5y = 5$ ②
In equation ①, the term $10y$ can be written as $2(5y)$:
$40x + 2(5y) = 10$ ③
Solve equation ② for $5y$.
$3x + 5y = 5$ ②
$5y = -3x + 5$
Substitute for $5y$ in equation ③.
$40x + 2(5y) = 10$ ③
$40x + 2(-3x + 5) = 10$ Simplify, then solve for x.
$40x - 6x + 10 = 10$
$34x = 0$
$x = 0$
Substitute $x = 0$ in equation ①.
$40x + 10y = 10$ ①
$40(0) + 10y = 10$ Simplify, then solve for y.
$10y = 10$
$y = 1$

Verify the solution.
In each equation, substitute: $x = 0$ and $y = 1$

$40x + 10y = 10$ ①
L.S. $= 40x + 10y$
$= 40(0) + 10(1)$
$= 10$
$=$ R.S.

$3x + 5y = 5$ ②
L.S. $= 3x + 5y$
$= 3(0) + 5(1)$
$= 5$
$=$ R.S.

For each equation, the left side is equal to the right side, so the solution is:
$x = 0$ and $y = 1$

iii) $-3x + 6y = 9$ ①
$5x - 2y = -7$ ②
In equation ①, the term $6y$ can be written as $3(2y)$:

$-3x + 3(2y) = 9$ ③
Solve equation ② for $2y$.
$5x - 2y = -7$ ②
$-2y = -5x - 7$
$2y = 5x + 7$
Substitute for $2y$ in equation ③.
$-3x + 3(2y) = 9$ ③
$-3x + 3(5x + 7) = 9$ Simplify, then solve for x.
$-3x + 15x + 21 = 9$
$12x = -12$
$x = -1$
Substitute $x = -1$ in equation ①.
$-3x + 6y = 9$ ①
$-3(-1) + 6y = 9$
$3 + 6y = 9$ Simplify, then solve for y.
$6y = 6$
$y = 1$

Verify the solution.
In each equation, substitute: $x = -1$ and $y = 1$

$-3x + 6y = 9$ ①
L.S. $= -3x + 6y$
$= -3(-1) + 6(1)$
$= 3 + 6$
$= 9$
= R.S.

$5x - 2y = -7$ ②
L.S. $= 5x - 2y$
$= 5(-1) - 2(1)$
$= -5 - 2$
$= -7$
= R.S.

For each equation, the left side is equal to the right side, so the solution is:
$x = -1$ and $y = 1$

iv) $-3x + 4y = 6$ ①
$9x + 3y = 27$ ②
In equation ②, the term $9x$ can be written as $3(3x)$:
$3(3x) + 3y = 27$ ③
Solve equation ① for $3x$.
$-3x + 4y = 6$ ①
$-3x = -4y + 6$
$3x = 4y - 6$
Substitute for $3x$ in equation ③.
$3(3x) + 3y = 27$ ③
$3(4y - 6) + 3y = 27$ Simplify, then solve for y.
$12y - 18 + 3y = 27$
$15y = 45$
$y = 3$
Substitute $y = 3$ in equation ①.
$-3x + 4y = 6$ ①
$-3x + 4(3) = 6$ Simplify, then solve for x.
$-3x + 12 = 6$
$-3x = -6$
$x = 2$

Verify the solution.
In each equation, substitute: $x = 2$ and $y = 3$

$-3x + 4y = 6$ ①
L.S. $= -3x + 4y$
$= -3(2) + 4(3)$
$= -6 + 12$
$= 6$
= R.S.

$9x + 3y = 27$ ②
L.S. $= 9x + 3y$
$= 9(2) + 3(3)$
$= 18 + 9$
$= 27$
= R.S.

For each equation, the left side is equal to the right side, so the solution is:
$x = 2$ and $y = 3$

7. a) I would choose the linear system in part i because one of the equations is already solved for one of the variables.

b) i) $x - y = -5$ ①
$x = -1$ ②
Equation ② is solved for x, so substitute $x = -1$ in equation ①.
$x - y = -5$ ①
$-1 - y = -5$
$-y = -4$
$y = 4$

Verify the solution.
In each equation, substitute: $x = -1$ and $y = 4$

$x - y = -5$ ①
L.S. $= x - y$
$= -1 - 4$
$= -5$
= R.S.

$x = -1$ ②
L.S. $= x$
$= -1$
= R.S.

For each equation, the left side is equal to the right side, so the solution is:
$x = -1$ and $y = 4$

ii) $x - y = -5$ ①
$-x - y = 3$ ②
Solve equation ① for x.
$x - y = -5$ ①
$x = y - 5$
Substitute $x = y - 5$ in equation ②.
$-x - y = 3$ ②
$-(y - 5) - y = 3$ Remove brackets.
$-y + 5 - y = 3$ Collect like terms.
$-2y = -2$ Divide each side by –2.
$y = 1$
Substitute $y = 1$ in equation ①.
$x - y = -5$ ①
$x - 1 = -5$ Solve for x.
$x = -4$

Verify the solution.

In each equation, substitute: $x = -4$ and $y = 1$

$x - y = -5$ ①
L.S. $= x - y$
$= -4 - 1$
$= -5$
$=$ R.S.

$-x - y = 3$ ②
L.S. $= -x - y$
$= -(-4) - 1$
$= 4 - 1$
$= 3$
$=$ R.S.

For each equation, the left side is equal to the right side, so the solution is:
$x = -4$ and $y = 1$

iii) $2x - 3y = 7$ ①
$x - 2y = 3$ ②
Solve equation ② for x.
$x - 2y = 3$ ②
$x = 2y + 3$
Substitute $x = 2y + 3$ in equation ①.
$2x - 3y = 7$ ①
$2(2y + 3) - 3y = 7$ Remove brackets.
$4y + 6 - 3y = 7$ Collect like terms.
$y = 1$
Substitute $y = 1$ in equation ②.
$x - 2y = 3$ ②
$x - 2(1) = 3$ Solve for x.
$x - 2 = 3$
$x = 5$

Verify the solution.
In each equation, substitute: $x = 5$ and $y = 1$

$2x - 3y = 7$ ①
L.S. $= 2x - 3y$
$= 2(5) - 3(1)$
$= 10 - 3$
$= 7$
$=$ R.S.

$x - 2y = 3$ ②
L.S. $= x - 2y$
$= 5 - 2(1)$
$= 5 - 2$
$= 3$
$=$ R.S.

For each equation, the left side is equal to the right side, so the solution is:
$x = 5$ and $y = 1$

8. a) $\frac{x}{3} - \frac{y}{2} = 2$ ①

$\frac{5x}{6} + \frac{3y}{4} = 1$ ②

I multiply each term in equation ① by 6 because that is the lowest common denominator of the fractions; that is, 6 is the lowest common multiple of 2 and 3.

$6\left(\frac{x}{3}\right) - 6\left(\frac{y}{2}\right) = 6(2)$ Simplify.

$2x - 3y = 12$

I multiply each term in equation ② by 12 because that is the lowest common denominator of the fractions; that is, 12 is the lowest common multiple of 6 and 4.

$12\left(\frac{5x}{6}\right) + 12\left(\frac{3y}{4}\right) = 12$ Simplify.

$10x + 9y = 12$

An equivalent linear system is:

$2x - 3y = 12$ ③

$10x + 9y = 12$ ④

b) Solve this linear system:

$2x - 3y = 12$ ③

$10x + 9y = 12$ ④

In equation ④, the term $10x$ can be written as $5(2x)$:

$5(2x) + 9y = 12$ ⑤

Solve equation ③ for $2x$.

$2x - 3y = 12$ ③

$2x = 3y + 12$

Substitute for $2x$ in equation ⑤.

$5(2x) + 9y = 12$ ⑤

$5(3y + 12) + 9y = 12$ Simplify, then solve for y.

$15y + 60 + 9y = 12$

$24y = -48$

$y = -2$

Substitute $y = -2$ in equation ③.

$2x - 3y = 12$ ③

$2x - 3(-2) = 12$ Simplify, then solve for x.

$2x + 6 = 12$

$2x = 6$

$x = 3$

Verify the solution.

In each equation, substitute: $x = 3$ and $y = -2$

$2x - 3y = 12$ ③

L.S. $= 2x - 3y$

$= 2(3) - 3(-2)$

$= 6 + 6$

$= 12$

= R.S.

$10x + 9y = 12$ ④

L.S. $= 10x + 9y$

$= 10(3) + 9(-2)$

$= 30 - 18$

$= 12$

= R.S.

For each equation, the left side is equal to the right side, so the solution is:

$x = 3$ and $y = -2$

Verify the solution for the linear system in part a.

$\frac{x}{3} - \frac{y}{2} = 2$ ①

$\frac{5x}{6} + \frac{3y}{4} = 1$ ②

In each equation, substitute: $x = 3$ and $y = –2$

$\frac{x}{3} - \frac{y}{2} = 2$ ①

L.S. $= \frac{x}{3} - \frac{y}{2}$

$= \frac{3}{3} - \frac{-2}{2}$

$= 1 - (-1)$

$= 2$

= R.S.

$\frac{5x}{6} + \frac{3y}{4} = 1$ ②

L.S. $= \frac{5(3)}{6} + \frac{3(-2)}{4}$

$= \frac{15}{6} + \frac{-6}{4}$

$= \frac{5}{2} - \frac{3}{2}$

$= \frac{2}{2}$

$= 1$

= R.S.

For each equation, the left side is equal to the right side, so the solution is:
$x = 3$ and $y = –2$

9. a) $2x + 2y = –4$ ①
$–12x + 4y = –24$ ②
I divide each term in equation ① by 2 because 2 is the greatest common factor of the coefficients and constant term.
$2x + 2y = –4$ Divide each term by 2.
$x + y = –2$
I divide each term in equation ② by 4 because 4 is the greatest common factor of the coefficients and constant term.
$–12x + 4y = –24$ Divide each term by 4.
$–3x + y = –6$
An equivalent linear system is:
$x + y = –2$ ③
$–3x + y = –6$ ④

b) Solve this linear system:
$x + y = –2$ ③
$–3x + y = –6$ ④
Solve equation ③ for y.
$x + y = –2$
$y = –x – 2$ ⑤
Substitute $y = –x – 2$ in equation ④.
$–3x + y = –6$ ④
$–3x + (–x – 2) = –6$ Simplify, then solve for x.
$–3x – x – 2 = –6$
$–4x = –4$
$x = 1$
Substitute $x = 1$ in equation ③.
$x + y = –2$ ③
$1 + y = –2$ Simplify, then solve for y.
$y = –3$

Verify the solution.
In each equation, substitute: $x = 1$ and $y = -3$

$x + y = -2$ ③
L.S. $= x + y$
$= 1 + (-3)$
$= -2$
= R.S.

$-3x + y = -6$ ④
L.S. $= -3x + y$
$= -3(1) + (-3)$
$= -3 - 3$
$= -6$
= R.S.

For each equation, the left side is equal to the right side, so the solution is:
$x = 1$ and $y = -3$

Verify the solution for the linear system in part a.
$2x + 2y = -4$ ①
$-12x + 4y = -24$ ②
In each equation, substitute: $x = 1$ and $y = -3$

$2x + 2y = -4$ ①
L.S. $= 2x + 2y$
$= 2(1) + 2(-3)$
$= 2 - 6$
$= -4$
= R.S.

$-12x + 4y = -24$ ②
L.S. $= -12x + 4y$
$= -12(1) + 4(-3)$
$= -12 - 12$
$= -24$
= R.S.

For each equation, the left side is equal to the right side, so the solution is:
$x = 1$ and $y = -3$

10. Let r represent the number of bears that did appear to respond.
Let n represent the number of bears that did not appear to respond.
A total of 186 bears were investigated.
So, one equation is: $r + n = 186$
There were 94 more bears that did not respond than did respond. So, the difference in bears that did not respond and bears that did respond is 94.
Another equation is: $n - r = 94$
A linear system is:
$r + n = 186$ ①
$n - r = 94$ ②
Solve equation ① for r.
$r + n = 186$ ①
$r = 186 - n$
Substitute $r = 186 - n$ in equation ②.
$n - r = 94$ ②
$n - (186 - n) = 94$ Remove brackets.
$n - 186 + n = 94$ Collect like terms.
$2n = 94 + 186$
$2n = 280$ Divide each side by 2.
$n = 140$
Substitute $n = 140$ in equation ①.
$r + n = 186$ ①
$r + 140 = 186$ Solve for r.
$r = 186 - 140$
$r = 46$

Verify the solution.
The total number of bears is: 140 + 46 = 186; this is the same as the given information.
The difference between the numbers of bears that did not respond and that did respond is: 140 – 46 = 94; this is the same as the given information.
The solution is correct; that is, 46 bears responded and 140 bears did not appear to respond.

11. Let l centimetres represent the length of the flag.
Let w centimetres represent the width of the flag.
The length is 90 cm longer than the width.
So, one equation is: $l = 90 + w$
The perimeter of the flag is 540 cm.
So, $l + l + w + w = 540$ Simplify.
Another equation is: $2l + 2w = 540$
A linear system is:
$l = 90 + w$ ①
$2l + 2w = 540$ ②
To solve this system:
From equation ①, substitute for $l = 90 + w$ in equation ②.
$2l + 2w = 540$ ②
$2(90 + w) + 2w = 540$ Remove brackets.
$180 + 2w + 2w = 540$ Simplify, then solve for w.
$4w = 360$
$w = 90$
Substitute $w = 90$ in equation ①.
$l = 90 + w$ ①
$l = 90 + 90$
$l = 180$
The length of the flag is 180 cm and the width is 90 cm.

Verify the solution.
The difference between the length and width is: 180 cm – 90 cm = 90 cm; this is the same as the given information.
The perimeter of the flag is: 180 cm + 180 cm + 90 cm + 90 cm = 540 cm; this is the same as the given information.
So, the solution is correct.

12. Let s represent the number of students in the study.
Let a represent the number of adults in the study.
A total of 45 people were in the study.
So, one equation is: $s + a = 45$
80% of the students is: $0.8s$
60% of the adults is: $0.6a$
80% of the students and 60% of the adults is a total of 31 people.
So, another equation is: $0.8s + 0.6a = 31$
A linear system is:
$s + a = 45$ ①
$0.8s + 0.6a = 31$ ②
To solve this system:
Solve equation ① for s.
$s + a = 45$ ①
$s = 45 - a$

Substitute $s = 45 – a$ in equation ②.

$0.8s + 0.6a = 31$ ②
$0.8(45 – a) + 0.6a = 31$ Simplify, then solve for a.
$36 – 0.8a + 0.6a = 31$
$–0.2a = –5$ Divide each side by –0.2.
$a = 25$

Substitute $a = 25$ in equation ①.

$s + a = 45$ ①
$s + 25 = 45$
$s = 20$

There were 20 students and 25 adults in the study.

Verify the solution.
The total number of people in the study is: $20 + 25 = 45$; this is the same as the given information.
80% of the students is: $0.8 \times 20 = 16$
60% of the adults is: $0.6 \times 25 = 15$
The number of people who reported a heavy use of the internet is: $16 + 15 = 31$; this is the same as the given information.
So, the solution is correct.

13. Let the number of groups of 4 students be represented by x.
Let the number of groups of 5 students be represented by y.
A total of 47 students were in the groups.
So, one equation is: $4x + 5y = 47$
There were 11 groups of students.
So, another equation is: $x + y = 11$
A linear system is:

$4x + 5y = 47$ ①
$x + y = 11$ ②

To solve this system:
Solve equation ② for x.

$x + y = 11$ ②
$x = 11 – y$

Substitute $x = 11 – y$ in equation ①.

$4x + 5y = 47$ ①
$4(11 – y) + 5y = 47$ Simplify, then solve for y.
$44 – 4y + 5y = 47$
$y = 3$

Substitute $y = 3$ in equation ②.

$x + y = 11$ ②
$x + 3 = 11$
$x = 8$

There were 8 groups of 4 students and 3 groups of 5 students.

Verify the solution.
8 groups of 4 is 32 students.
3 groups of 5 is 15 students.
So, the total number of students is: $32 + 15 = 47$; this is the same as the given information.
The total number of groups is: $8 + 3 = 11$; this is the same as the given information.
So, the solution is correct.

14. Let p represent the number of people masks.
Let a represent the number of animal masks.
The total number of masks is 85.
So, one equation is: $p + a = 85$
60% of the people masks is: $0.6p$
40% of the animal masks is: $0.4a$
60% of the people masks and 40% of the animal masks is a total of 38 masks made from yellow cedar.
So, another equation is: $0.6p + 0.4a = 38$
A linear system is:
$p + a = 85$ ①
$0.6p + 0.4a = 38$ ②
To solve this system:
Solve equation ① for p.
$p + a = 85$ ①
$p = 85 - a$
Substitute $p = 85 - a$ in equation ②.
$0.6p + 0.4a = 38$ ②
$0.6(85 - a) + 0.4a = 38$ Simplify, then solve for a.
$51 - 0.6a + 0.4a = 38$
$-0.2a = -13$ Divide each side by -0.2.
$a = 65$
Substitute $a = 65$ in equation ①.
$p + a = 85$ ①
$p + 65 = 85$
$p = 20$
There were 20 people masks and 65 animal masks.

Verify the solution.
The total number of masks is: $20 + 65 = 85$; this is the same as the given information.
60% of 20 is: $0.6 \times 20 = 12$
40% of 65 is: $0.4 \times 65 = 26$
The total number of yellow cedar masks: $12 + 26 = 38$; this is the same as the given information.
So, the solution is correct.

15. Let A represent the number of marks for part A of the test.
Let B represent the number of marks for part B of the test.
The total possible mark is 75.
So, one equation is: $A + B = 75$
80% of the marks for part A is: $0.8A$
92% of the marks for part B is: $0.92B$
80% of the marks for part A and 92% of the marks for part B is 63.
So, another equation is: $0.8A + 0.92B = 63$
A linear system is:
$A + B = 75$ ①
$0.8A + 0.92B = 63$ ②

To solve this system:
Solve equation ① for A.
$A + B = 75$ ①
$A = 75 - B$
Substitute $A = 75 - B$ in equation ②.
$0.8A + 0.92B = 63$ ②
$0.8(75 - B) + 0.92B = 63$
$60 - 0.8B + 0.92B = 63$ Simplify, then solve for B.
$0.12B = 3$ Divide each side by 0.12.
$B = 25$
Substitute $B = 25$ in equation ①.
$A + B = 75$ ①
$A + 25 = 75$
$A = 50$
There 50 marks for part A and 25 marks for part B of the test.

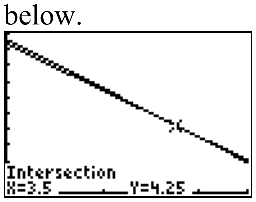
Verify the solution.
The total possible mark is: 50 + 25 = 75; this is the same as the given information.
80% of 50 is: $0.8 \times 50 = 40$
92% of 25 is: $0.92 \times 25 = 23$
The total number of marks that Sam scored is: 40 + 23 = 63; this is the same as the given information.
So, the solution is correct.

16. Let x dollars represent the amount invested at 2.5%.
Let y dollars represent the amount invested at 3.75%.
The total amount invested is \$5000.
So, one equation is: $x + y = 5000$
x dollars invested at 2.5% earns $0.025x$ dollars interest.
y dollars invested at 3.75% earns $0.0375y$ dollars.
The total interest earned is \$162.50.
So, another equation is: $0.025x + 0.0375y = 162.5$
A linear system is:
$x + y = 5000$ ①
$0.025x + 0.0375y = 162.5$ ②
To solve this system:
Solve equation ① for x.
$x + y = 5000$ ①
$x = 5000 - y$
Substitute $x = 5000 - y$ in equation ②.
$0.025x + 0.0375y = 162.5$ ②
$0.025(5000 - y) + 0.0375y = 162.5$ Simplify, then solve for y.
$125 - 0.025y + 0.0375y = 162.5$
$0.0125y = 162.5 - 125$
$0.0125y = 37.5$ Divide each side by 0.0125.
$y = 3000$
Substitute $y = 3000$ in equation ①.
$x + 3000 = 5000$ ①
$x = 2000$
\$2000 was invested at 2.5% and \$3000 was invested at 3.75%.

Verify the solution.
The total amount invested is: \$2000 + \$3000 = \$5000; this is the same as the given information.
\$2000 invested at 2.5% earns: \$2000 × 0.025 = \$50 interest
\$3000 invested at 3.75% earns: \$3000 × 0.0375 = \$112.50 interest
The total interest is: \$50 + \$112.50 = \$162.50; this is the same as the given information.
So, the solution is correct.

17. Let s dollars represent the cost of a single-scoop cone.
Let d dollars represent the cost of a double-scoop cone.
76 single-scoop cones and 49 double-scoop cones cost \$474.25.
So, one equation is: $76s + 49d = 474.25$
54 single-scoop cones and 37 double-scoop cones cost \$346.25.
So, another equation is: $54s + 37d = 346.25$
A linear system is:
$76s + 49d = 474.25$ ①
$54s + 37d = 346.25$ ②
To solve this system, use a graphing calculator.
Write each equation in the form $y = mx + b$.
For equation ①:
$76s + 49d = 474.25$ Solve for d. Subtract $76s$ from each side.
$49d = -76s + 474.25$ Divide each side by 49.
$$d = \frac{-76}{49}s + \frac{474.25}{49}$$
For equation ②:
$54s + 37d = 346.25$ Solve for d. Subtract $54s$ from each side.
$37d = -54s + 346.25$ Divide each side by 37.
$$d = \frac{-54}{37}s + \frac{346.25}{37}$$
An equivalent linear system is:
$$d = \frac{-76}{49}s + \frac{474.25}{49}$$
$$d = \frac{-54}{37}s + \frac{346.25}{37}$$
On a TI-83 graphing calculator, press Y=, then next to Y1 = I input the expression (–76/49)X+474.25/49. Move the cursor down to Y2= and input the expression (–54/37)X+346.25/37.

Press GRAPH. To see the point of intersection, I set the WINDOW to Xmin = 0, Xmax = 5, Ymin = 0, and Ymax = 10. To show the coordinates of the point of intersection, I pressed 2nd TRACE for CALC, then selected 5:intersect. I pressed ENTER 3 times to get the screen below.



From the calculator screen, the solution is: $s = 3.5$ and $d = 4.25$
A single-scoop cone costs \$3.50 and a double-scoop cone costs \$4.25.

Verify the solution.
76 single-scoop cones and 49 double-scoop cones cost: 76($3.50) + 49($4.25) = $474.25;
this is the same as the given information.
54 single-scoop cones and 37 double-scoop cones cost: 54($3.50) + 37($4.25) = $346.25;
this is the same as the given information.
So, the solution is correct.

18. Let p dollars represent the cost for each person to work w weekends.
Joel is paid $40 per weekend.
So, for w weekends, Joel's earnings in dollars are: $40w$
One equation is: $p = 40w$
Sue is paid $150 plus $30 each weekend.
So, for w weekends, Sue's earnings in dollars are: $150 + 30w$
Another equation is: $p = 150 + 30w$
A linear system is:
$p = 40w$ ①
$p = 150 + 30w$ ②
To solve this system:
Both equations are solved for p.
From equation ①, substitute $p = 40w$ in equation ②.
$p = 150 + 30w$ ②
$40w = 150 + 30w$ Solve for w.
$10w = 150$
$w = 15$
Joel has to work 15 weekends before he earns the same as Sue.
Check the solution.
When Joel works for 15 weekends, he earns: 15($40) = $600
When Sue works for 15 weekends, she earns: $150 + 15($30) = $600
Since both people earn $600 after working 15 weekends, the solution is correct.

19. a) $\frac{1}{2}x + \frac{2}{3}y = 1$ ①

$\frac{1}{4}x - \frac{1}{3}y = \frac{5}{2}$ ②

Write an equivalent system with integer coefficients.
For equation ①, the common denominator is the lowest common multiple of 2 and 3, which is 6:

$\frac{1}{2}x + \frac{2}{3}y = 1$ Multiply each term by 6.

$6\left(\frac{1}{2}x\right) + 6\left(\frac{2}{3}y\right) = 6(1)$ Simplify.

$3x + 4y = 6$ ③

For equation ②, the common denominator is the lowest common multiple of 4 and 3, which is 12:

$\frac{1}{4}x - \frac{1}{3}y = \frac{5}{2}$ Multiply each term by 12.

$12\left(\frac{1}{4}x\right) - 12\left(\frac{1}{3}y\right) = 12\left(\frac{5}{2}\right)$ Simplify.

$3x - 4y = 30$ ④

Solve equation ④ for $3x$.

$3x - 4y = 30$ ④

$3x = 4y + 30$

Substitute for $3x$ in equation ③.

$3x + 4y = 6$ ③

$(4y + 30) + 4y = 6$ Simplify, then solve for y.

$8y = -24$

$y = -3$

Substitute $y = -3$ into equation ④.

$3x - 4y = 30$ ④

$3x - 4(-3) = 30$ Simplify, then solve for x.

$3x + 12 = 30$

$3x = 18$

$x = 6$

Verify the solution.

In each original equation, substitute: $x = 6$ and $y = -3$

$\frac{1}{2}x + \frac{2}{3}y = 1$ ①

$\text{L.S.} = \frac{1}{2}x + \frac{2}{3}y$

$= \frac{1}{2}(6) + \frac{2}{3}(-3)$

$= 3 - 2$

$= 1$

$= \text{R.S.}$

$\frac{1}{4}x - \frac{1}{3}y = \frac{5}{2}$ ②

$\text{L.S.} = \frac{1}{4}x - \frac{1}{3}y$

$= \frac{1}{4}(6) - \frac{1}{3}(-3)$

$= \frac{3}{2} + 1$

$= \frac{5}{2}$

$= \text{R.S.}$

For each equation, the left side is equal to the right side, so the solution is:
$x = 6$ and $y = -3$

b) $\frac{3}{4}x + \frac{1}{2}y = -\frac{7}{12}$ ①

$x - y = -\frac{4}{3}$ ②

Write an equivalent system with integer coefficients.

For equation ①, the common denominator is the lowest common multiple of 4, 2, and 12, which is 12:

$\frac{3}{4}x + \frac{1}{2}y = -\frac{7}{12}$ Multiply each term by 12.

$12\left(\frac{3}{4}x\right) + 12\left(\frac{1}{2}y\right) = 12\left(-\frac{7}{12}\right)$ Simplify.

$9x + 6y = -7$ ③

For equation ②, the common denominator is 3:

$x - y = -\frac{4}{3}$ Multiply each term by 3.

$3x - 3y = 3\left(-\frac{4}{3}\right)$ Simplify.

$3x - 3y = -4$ ④

Solve equation ④ for $3x$.

$3x - 3y = -4$ ④

$3x = 3y - 4$

Substitute for $3x$ in equation ③.

$9x + 6y = -7$ ③

$3(3y - 4) + 6y = -7$ Simplify, then solve for y.

$9y - 12 + 6y = -7$

$15y = 5$

$y = \frac{5}{15}$, or $\frac{1}{3}$

Substitute $y = \frac{1}{3}$ into equation ③.

$9x + 6y = -7$ ③

$9x + 6\left(\frac{1}{3}\right) = -7$ Simplify, then solve for x.

$9x + 2 = -7$

$9x = -9$

$x = -1$

Verify the solution.

In each original equation, substitute: $x = -1$ and $y = \frac{1}{3}$

$\frac{3}{4}x + \frac{1}{2}y = -\frac{7}{12}$ ①

$$\begin{aligned}
\text{L.S.} &= \frac{3}{4}x + \frac{1}{2}y \\
&= \frac{3}{4}(-1) + \frac{1}{2}\left(\frac{1}{3}\right) \\
&= -\frac{3}{4} + \frac{1}{6} \\
&= -\frac{9}{12} + \frac{2}{12} \\
&= -\frac{7}{12} \\
&= \text{R.S.}
\end{aligned}$$

$x - y = -\frac{4}{3}$ ②

$$\begin{aligned}
\text{L.S.} &= x - y \\
&= -1 - \frac{1}{3} \\
&= -\frac{3}{3} - \frac{1}{3} \\
&= -\frac{4}{3} \\
&= \text{R.S.}
\end{aligned}$$

For each equation, the left side is equal to the right side, so the solution is:

$x = -1$ and $y = \frac{1}{3}$

c) $\frac{1}{3}x - \frac{3}{8}y = 1$ ①

$-\frac{1}{4}x - \frac{1}{8}y = \frac{3}{2}$ ②

Write an equivalent system with integer coefficients.
For equation ①, the common denominator is the lowest common multiple of 3 and 8, which is 24:

$\frac{1}{3}x - \frac{3}{8}y = 1$ Multiply each term by 24.

$24\left(\frac{1}{3}x\right) - 24\left(\frac{3}{8}y\right) = 24(1)$ Simplify.

$8x - 9y = 24$ ③

For equation ②, the common denominator is the lowest common multiple of 4, 8, and 2, which is 8:

$-\frac{1}{4}x - \frac{1}{8}y = \frac{3}{2}$ Multiply each term by 8.

$8\left(-\frac{1}{4}x\right) - 8\left(\frac{1}{8}y\right) = 8\left(\frac{3}{2}\right)$ Simplify.

$-2x - y = 12$ ④

Solve equation ④ for y.

$-2x - y = 12$ ④
$-y = 2x + 12$
$y = -2x - 12$

Substitute for y in equation ③.

$8x - 9y = 24$ ③
$8x - 9(-2x - 12) = 24$ Simplify, then solve for x.
$8x + 18x + 108 = 24$
$26x = 24 - 108$
$26x = -84$
$x = \frac{-84}{26}$, or $-\frac{42}{13}$

To determine the value of y, solve equation ④ for $2x$.

$-2x - y = 12$ ④
$-2x = y + 12$
$2x = -y - 12$

Substitute for $2x$ in equation ③.

$8x - 9y = 24$ ③
$4(-y - 12) - 9y = 24$ Simplify, then solve for y.
$-4y - 48 - 9y = 24$
$-13y = 24 + 48$
$-13y = 72$
$y = \frac{72}{-13}$, or $-\frac{72}{13}$

Verify the solution.

In each original equation, substitute: $x = -\frac{42}{13}$ and $y = -\frac{72}{13}$

$$\frac{1}{3}x - \frac{3}{8}y = 1 \quad ①$$

$$\begin{aligned}\text{L.S.} &= \frac{1}{3}x - \frac{3}{8}y \\ &= \frac{1}{3}\left(-\frac{42}{13}\right) - \frac{3}{8}\left(-\frac{72}{13}\right) \\ &= -\frac{14}{13} + \frac{27}{13} \\ &= \frac{13}{13} \\ &= 1 \\ &= \text{R.S.}\end{aligned}$$

$$-\frac{1}{4}x - \frac{1}{8}y = \frac{3}{2} \quad ②$$

$$\begin{aligned}\text{L.S.} &= -\frac{1}{4}x - \frac{1}{8}y \\ &= -\frac{1}{4}\left(-\frac{42}{13}\right) - \frac{1}{8}\left(-\frac{72}{13}\right) \\ &= \frac{21}{26} + \frac{18}{26} \\ &= \frac{39}{26} \\ &= \frac{3}{2} \\ &= \text{R.S.}\end{aligned}$$

For each equation, the left side is equal to the right side, so the solution is:

$x = -\frac{42}{13}$ and $y = -\frac{72}{13}$

d) $\frac{7}{4}x + \frac{4}{3}y = 3 \quad ①$

$\frac{1}{2}x - \frac{5}{6}y = 2 \quad ②$

Write an equivalent system with integer coefficients.

For equation ①, the common denominator is the lowest common multiple of 4 and 3, which is 12:

$$\frac{7}{4}x + \frac{4}{3}y = 3 \qquad \text{Multiply each term by 12.}$$

$$12\left(\frac{7}{4}x\right) + 12\left(\frac{4}{3}y\right) = 12(3) \qquad \text{Simplify.}$$

$$21x + 16y = 36 \quad ③$$

For equation ②, the common denominator is the lowest common multiple of 2 and 6, which is 6:

$$\frac{1}{2}x - \frac{5}{6}y = 2 \qquad \text{Multiply each term by 6.}$$

$$6\left(\frac{1}{2}x\right) - 6\left(\frac{5}{6}y\right) = 6(2) \qquad \text{Simplify.}$$

$$3x - 5y = 12 \quad ④$$

Solve equation ④ for $3x$.

$$\begin{aligned}3x - 5y &= 12 \quad ④ \\ 3x &= 5y + 12\end{aligned}$$

Substitute for $3x$ in equation ③.

$$\begin{aligned}21x + 16y &= 36 \quad ③ \\ 7(5y + 12) + 16y &= 36 \qquad \text{Simplify, then solve for } y. \\ 35y + 84 + 16y &= 36 \\ 51y &= 36 - 84 \\ 51y &= -48\end{aligned}$$

$$y = \frac{-48}{51}\text{, or } -\frac{16}{17}$$

Substitute $y = -\frac{16}{17}$ into equation ④.

$$3x - 5y = 12 \qquad ④$$

$$3x - 5\left(-\frac{16}{17}\right) = 12 \qquad \text{Simplify, then solve for } x.$$

$$3x + \frac{80}{17} = 12$$

$$3x = 12 - \frac{80}{17}$$

$$3x = \frac{204}{17} - \frac{80}{17}$$

$$3x = \frac{124}{17}$$

$$x = \frac{124}{51}$$

Verify the solution.

In each original equation, substitute: $x = \frac{124}{51}$ and $y = -\frac{16}{17}$

$$\frac{7}{4}x + \frac{4}{3}y = 3 \qquad ①$$

$$\text{L.S.} = \frac{7}{4}x + \frac{4}{3}y$$
$$= \frac{7}{4}\left(\frac{124}{51}\right) + \frac{4}{3}\left(-\frac{16}{17}\right)$$
$$= \frac{217}{51} - \frac{64}{51}$$
$$= \frac{153}{51}$$
$$= 3$$
$$= \text{R.S.}$$

$$\frac{1}{2}x - \frac{5}{6}y = 2 \qquad ②$$

$$\text{L.S.} = \frac{1}{2}x - \frac{5}{6}y$$
$$= \frac{1}{2}\left(\frac{124}{51}\right) - \frac{5}{6}\left(-\frac{16}{17}\right)$$
$$= \frac{62}{51} + \frac{40}{51}$$
$$= \frac{102}{51}$$
$$= 2$$
$$= \text{R.S.}$$

For each equation, the left side is equal to the right side, so the solution is:

$x = \frac{124}{51}$ and $y = -\frac{16}{17}$

20. a) $7.50r + 45c = 375$ ①

$r - c = 15$ ②

An ink cartridge is more expensive than a ream of paper, so assume an ink cartridge costs \$45.00 and a ream of paper costs \$7.50.
Let r represent the number of reams of paper bought.
Let c represent the number of ink cartridges bought.
Then equation ① represents the sum of the cost of r reams of paper at \$7.50/ream and the cost of c ink cartridges at \$45.00/cartridge. This sum is \$375.00.
Equation ② represents the difference in numbers of reams of paper and ink cartridges

bought. This difference is 15.
A related problem is:
An ink cartridge costs $45.00.
A ream of paper costs $7.50.
The cost of buying some ink cartridges and some reams of paper was $375.00.
Fifteen more reams of paper were bought than ink cartridges.
How many reams of paper and how many ink cartridges were bought?

b) $7.50r + 45c = 375$ ①
$r - c = 15$ ②
Solve equation ② for r.
$r - c = 15$ ②
$r = c + 15$
Substitute $r = c + 15$ in equation ①.
$7.50r + 45c = 375$ ①
$7.50(c + 15) + 45c = 375$ Simplify, then solve for c.
$7.50c + 112.5 + 45c = 375$
$52.50c = 375 - 112.5$
$52.50c = 262.5$ Divide each side by 52.50.
$c = 5$
Substitute $c = 5$ into equation ②.
$r - c = 15$ ②
$r - 5 = 15$ Solve for r.
$r = 20$
20 reams of paper and 5 ink cartridges were bought.

Verify the solution.
In each equation, substitute: $r = 20$ and $c = 5$

$7.50r + 45c = 375$ ①
L.S. $= 7.50r + 45c$
$= 7.50(20) + 45(5)$
$= 150 + 225$
$= 375$
$=$ R.S.

$r - c = 15$ ②
L.S. $= r - c$
$= 20 - 5$
$= 15$
$=$ R.S.

For each equation, the left side is equal to the right side, so the solution is correct.

21. $2x + 4y = 98$ ①
$x + y = 27$ ②
Each variable term in equation ① is the product of a constant that is not 1 and a variable.
This suggests a problem about two groups of people who pay different amounts.
Equation ② is the sum of two variable terms with coefficient 1, which suggests that the numbers of people in equation ① are added.
A situation that could be modelled by this linear system is students and adults attending a fall fair.
A related problem is:
The cost for a student to attend a local fall fair is $2.
The cost for an adult to attend the fair is $4.
In the first 5 min after the fair opened, 27 people paid a total of $98 to go to the fair.
How many students and how many adults went to the fair in the first 5 min?

In this problem, x represents the number of students and y represents the number of adults.
$2x + 4y = 98$ ①
$x + y = 27$ ②
To solve the system:
Solve equation ② for x.
$x + y = 27$ ②
$x = 27 - y$
Substitute $x = 27 - y$ in equation ①.
$2x + 4y = 98$ ①
$2(27 - y) + 4y = 98$ Simplify, then solve for y.
$54 - 2y + 4y = 98$
$2y = 98 - 54$
$2y = 44$
$y = 22$
Substitute $y = 22$ into equation ②.
$x + y = 27$ ②
$x + 22 = 27$
$x = 5$
5 students and 22 adults went to the fair in the first 5 min.

Verify the solution.
Substitute $x = 5$ and $y = 22$ into each equation.
$2x + 4y = 98$ ①
L.S. $= 2x + 4y$
$= 2(5) + 4(22)$
$= 10 + 88$
$= 98$
= R.S.

$x + y = 27$ ②
L.S. $= x + y$
$= 5 + 22$
$= 27$
= R.S.

For each equation, the left side is equal to the right side, so the solution is correct.

22. a) $2x - y = -4$ ①
$3x + 2y = 1$ ②
To write an equivalent system, multiply the terms in each equation by the same constant.
For equation ①, multiply each term by 2:
$2(2x) - 2(y) = 2(-4)$
$4x - 2y = -8$ ③
For equation ②, multiply each term by 3:
$3(3x) + 3(2y) = 3(1)$
$9x + 6y = 3$ ④
An equivalent system is:
$4x - 2y = -8$ ③
$9x + 6y = 3$ ④

b) $2x - y = -4$ ①
$3x + 2y = 1$ ②
To solve this system:
Solve equation ① for y.
$2x - y = -4$ ①
$-y = -4 - 2x$
$y = 4 + 2x$
Substitute $y = 4 + 2x$ in equation ②.

$3x + 2y = 1$ ②
$3x + 2(4 + 2x) = 1$ Simplify, then solve for x.
$3x + 8 + 4x = 1$
$7x = -7$
$x = -1$
Substitute $x = -1$ into equation ①.
$2x - y = -4$ ①
$2(-1) - y = -4$
$-2 - y = -4$
$-y = -2$
$y = 2$

Verify the solution.
Substitute $x = -1$ and $y = 2$ into each equation.

$2x - y = -4$ ①	$3x + 2y = 1$ ②
L.S. $= 2x - y$	L.S. $= 3x + 2y$
$= 2(-1) - 2$	$= 3(-1) + 2(2)$
$= -2 - 2$	$= -3 + 4$
$= -4$	$= 1$
= R.S.	= R.S.

For each equation, the left side is equal to the right side, so the solution is correct.

$4x - 2y = -8$ ③
$9x + 6y = 3$ ④
To solve this system:
Solve equation ③ for $2y$.
$4x - 2y = -8$ ③
$-2y = -8 - 4x$
$2y = 8 + 4x$
Substitute $2y = 8 + 4x$ in equation ④.
$9x + 6y = 3$ ④
$9x + 3(8 + 4x) = 3$ Simplify, then solve for x.
$9x + 24 + 12x = 3$
$21x = -21$
$x = -1$
Substitute $x = -1$ into equation ③.
$4x - 2y = -8$ ③
$4(-1) - 2y = -8$
$-4 - 2y = -8$
$-2y = -4$
$-y = -2$
$y = 2$

Verify the solution.
Substitute $x = -1$ and $y = 2$ into each equation.

$4x - 2y = -8$ ③	$9x + 6y = 3$ ④
L.S. $= 4x - 2y$	L.S. $= 9x + 6y$
$= 4(-1) - 2(2)$	$= 9(-1) + 6(2)$
$= -4 - 4$	$= -9 + 12$
$= -8$	$= 3$
= R.S.	= R.S.

For each equation, the left side is equal to the right side, so the solution is correct.
Equivalent linear systems have the same solution; so, since the solutions are the same, the systems are equivalent.

C

23. Write a linear system to represent the situation.
Let s kilometres per hour represent the usual average speed.
Let d kilometres represent the distance from Penticton to Chute Lake.
The cyclists' usual average speed was reduced by 6 km/h, so the new average speed is:
$(s - 6)$ km/h
The cyclists took 4 h to travel d kilometres at $(s - 6)$ km/h.
Use the formula:
distance travelled = average speed × time
$d = (s - 6) \times 4$ Simplify.
$d = 4s - 24$
On the return trip, the usual average speed was increased by 4 km/h, so the new average speed is:
$(s + 4)$ km/h
The cyclists took 2 h to travel d kilometres at $(s + 4)$ km/h.
So, $d = (s + 4) \times 2$ Simplify.
$d = 2s + 8$
A linear system that represents this situation is:
$d = 4s - 24$ ①
$d = 2s + 8$ ②

a) Solve the linear system.
From equation ①, substitute $d = 4s - 24$ into equation ②.
$d = 2s + 8$ ②
$4s - 24 = 2s + 8$ Simplify, then solve for s.
$2s = 32$
$s = 16$
The usual average speed is 16 km/h.

b) Substitute $s = 16$ into equation ②.
$d = 2s + 8$ ②
$d = 2(16) + 8$
$d = 32 + 8$
$d = 40$
The distance from Penticton to Chute Lake is 40 km.

Check the answers.
Uphill: the cyclists travel at: $(16 - 6)$ km/h = 10 km/h
They travel for 4 h.
So, the distance travelled is: 4 h × 10 km/h = 40 km; this agrees with the given information.
Downhill: the cyclists travel at: $(16 + 4)$ km/h = 20 km/h
They travel for 2 h.
So, the distance travelled is: 2 h × 20 km/h = 40 km; this agrees with the given information.
So, the answers are correct.

24. Write a linear system to represent the situation.
Let the mean mass of the female rattlesnakes be represented by f grams.
Let the mean mass of the male rattlesnakes be represented by m grams.
The mean mass of 45 female rattlesnakes and 100 male rattlesnakes is 194 g.

So, one equation is: $\frac{45f + 100m}{145} = 194$

The mean mass of the male rattlesnakes is 37.7 g greater than the mean mass of the female rattlesnakes.
So, another equation is: $m = 37.7 + f$
A linear system is:

$$\frac{45f + 100m}{145} = 194 \quad ①$$
$$m = 37.7 + f \quad ②$$

To solve the linear system:
From equation ②, substitute $m = 37.7 + f$ into equation ①.

$$\frac{45f + 100m}{145} = 194 \quad ①$$
$$\frac{45f + 100(37.7 + f)}{145} = 194 \quad \text{Multiply each side by 145.}$$
$$45f + 3770 + 100f = 28\,130 \quad \text{Simplify, then solve for } f.$$
$$45f + 100f = 28\,130 - 3770$$
$$145f = 24\,360 \quad \text{Divide each side by 145.}$$
$$f = 168$$

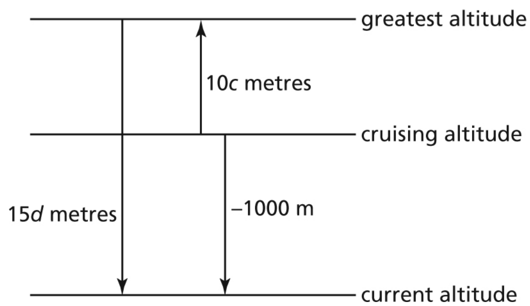
Substitute $f = 168$ into equation ②.

$$m = 37.7 + f \quad ②$$
$$m = 37.7 + 168 \quad \text{Solve for } m.$$
$$m = 205.7$$

The mean mass of the male rattlesnakes is 205.7 g and the mean mass of the female rattlesnakes is 168 g.

Check the answers.
The mean mass, in grams, of all the rattlesnakes is:
$\frac{45(168) + 100(205.7)}{145} = \frac{28\,130}{145}$, or 194; this agrees with the given information.
The difference between the mean mass of the male rattlesnakes and the mean mass of female rattlesnakes is: 205.7 g – 168 g = 37.7 g; this agrees with the given information.
So, the answers are correct.

25. Write a linear system to represent the situation.
Let the rate of climb be represented by c metres per minute.
Let the rate of descent be represented by d metres per minute.
The difference between the rate of climb and rate of descent is 400 m/min.
So, one equation is: $c - d = 400$
The distance travelled in 10 min at a rate of c metres per minute is: $10c$ metres
The distance travelled in 15 min at a rate of d metres per minute is: $15d$ metres
Sketch a diagram:



Another equation is: $10c + 15d = -1000$
A linear system is:
$c - d = 400$ ①
$10c + 15d = -1000$ ②
To solve the system:
Solve equation ① for c.
$c - d = 400$ ①
$c = d + 400$
Substitute $c = d + 400$ into equation ②.
$10c + 15d = -1000$ ②
$10(d + 400) + 15d = -1000$ Simplify, then solve for d.
$10d + 4000 + 15d = -1000$
$25d = -5000$
$d = -200$
Substitute $d = -200$ into equation ①.
$c - d = 400$ ①
$c - (-200) = 400$
$c + 200 = 400$
$c = 200$
The rate of climb is 200 m/min and the rate of descent is –200 m/min.
Check these answers.
The airplane climbed for 10 min at a rate of 200 m/min, so it travelled: 10×200 m = 2000 m
The airplane descended for 15 min at a rate of –200 m/min, so it travelled:
$15 \times (-200$ m$) = -3000$ m
The sum of these the two distances is: 2000 m + (–3000 m) = –1000 m; this agrees with the given information.
The difference between the rate of climb and rate of descent was:
200 m/min – (–200 m/min) = 400 m/min; this agrees with the given information.
The solution is correct.

26. $Ax + By = C$ ①
$Bx + Ay = C$ ②
To solve the linear system:
Solve equation ① for x.
$Ax + By = C$ Subtract By from each side.
$Ax = -By + C$ Divide each side by A.
$$x = \frac{-B}{A}y + \frac{C}{A}$$
Substitute $x = \dfrac{-B}{A}y + \dfrac{C}{A}$ into equation ②.

$$Bx + Ay = C \quad ②$$

$$B\left(-\frac{B}{A}y + \frac{C}{A}\right) + Ay = C$$ Simplify.

$$-\frac{B^2}{A}y + \frac{BC}{A} + Ay = C$$ Multiply each side by A.

$$-B^2 + BC + A^2y = CA$$ Solve for y.

$$y(A^2 - B^2) = CA - BC$$ Divide each side by $(A^2 - B^2)$.

$$y = \frac{CA - BC}{A^2 - B^2}$$ Factor the numerator and denominator.

$$y = \frac{C(A - B)}{(A - B)(A + B)}$$ Since $A \neq B$, divide the numerator and denominator by $(A - B)$.

$$y = \frac{C}{A + B}$$

Substitute for y in equation ①.

$$Ax + By = C \quad ①$$

$$Ax + B\left(\frac{C}{A + B}\right) = C$$

$$Ax + \frac{BC}{A + B} = C$$ Multiply each side by $(A + B)$.

$$Ax(A + B) + \frac{BC}{A + B}(A + B) = C(A + B)$$

$$Ax(A + B) + BC = CA + CB$$

$$Ax(A + B) = CA$$ Solve for x. Divide each side by $A(A + B)$.

Assume $A \neq -B$.

$$x = \frac{CA}{A(A + B)}$$

$$x = \frac{C}{A + B}$$

So, the values of x and y are equal for all given values of A, B, and C.
Check the solution.

In each equation, substitute: $x = \frac{C}{A + B}$ and $y = \frac{C}{A + B}$

$Ax + By = C$ ①
L.S. $= Ax + By$

$$= A\left(\frac{C}{A + B}\right) + B\left(\frac{C}{A + B}\right)$$

$$= \frac{AC + BC}{A + B}$$

$$= \frac{C(A + B)}{A + B}$$

$$= C$$

$Bx + Ay = C$ ②
L.S. $= Bx + Ay$

$$= B\left(\frac{C}{A + B}\right) + A\left(\frac{C}{A + B}\right)$$

$$= \frac{BC + AC}{A + B}$$

$$= \frac{C(B + A)}{A + B}$$

$$= C$$

= R.S. = R.S.

For each equation, the left side is equal to the right side, so the solution is correct.

27. $Ax + By = -17$ ①
$Bx + Ay = 18$ ②
Substitute $x = -2$ and $y = 3$ into each equation.
In equation ①:
$Ax + By = -17$
$A(-2) + B(3) = -17$
$-2A + 3B = -17$ ③
In equation ②:
$Bx + Ay = 18$
$B(-2) + A(3) = 18$
$-2B + 3A = 18$ ④
Equations ③ and ④ form a linear system.
$-2A + 3B = -17$ ③
$-2B + 3A = 18$ ④
To solve the system:
Solve equation ③ for A.
$-2A + 3B = -17$ ③
$-2A + 3B = -17$ Subtract $3B$ from each side.
$-2A = -3B - 17$ Divide each side by –2.
$A = \frac{3}{2}B + \frac{17}{2}$
Substitute $A = \frac{3}{2}B + \frac{17}{2}$ in equation ④.
$-2B + 3A = 18$ ④
$-2B + 3\left(\frac{3}{2}B + \frac{17}{2}\right) = 18$ Simplify.
$-2B + \frac{9}{2}B + \frac{51}{2} = 18$
$-\frac{4}{2}B + \frac{9}{2}B = -\frac{51}{2} + \frac{36}{2}$
$\frac{5}{2}B = -\frac{15}{2}$
$\frac{5}{2}B = -\frac{15}{2}$ Multiply each side by $\frac{2}{5}$.
$B = -3$
Substitute $B = -3$ in equation ④.
$-2B + 3A = 18$ ④
$-2(-3) + 3A = 18$
$6 + 3A = 18$
$3A = 12$
$A = 4$
Check the solution.

In each equation, substitute: $A = 4$ and $B = –3$

$-2A + 3B = -17$ ③	$-2B + 3A = 18$ ④
L.S. $= -2A + 3B$	L.S. $= -2B + 3A$
$= -2(4) + 3(-3)$	$= -2(-3) + 3(4)$
$= -8 - 9$	$= 6 + 12$
$= -17$	$= 18$
= R.S.	= R.S.

For each equation, the left side is equal to the right side, so the solution is correct.

Lesson 7.5 **Using an Elimination Strategy to Solve a System of Linear Equations** **Exercises (pages 437-439)**

A

3. **a)** $x - 4y = 1$ ①
$x - 2y = -1$ ②
Since the x-terms are equal, subtract the equations.

$$\begin{aligned} x - 4y &= 1 \quad ① \\ \underline{-(x - 2y = -1)} &\quad ② \\ -4y + 2y &= 1 + 1 \\ -2y &= 2 \\ y &= -1 \end{aligned}$$

Substitute $y = -1$ into equation ①.

$$\begin{aligned} x - 4y &= 1 \quad ① \\ x - 4(-1) &= 1 \\ x + 4 &= 1 \\ x &= -3 \end{aligned}$$

Verify the solution.
In each equation, substitute: $x = -3$ and $y = -1$

$x - 4y = 1$ ①
$$\begin{aligned} \text{L.S.} &= x - 4y \\ &= -3 - 4(-1) \\ &= -3 + 4 \\ &= 1 \\ &= \text{R.S.} \end{aligned}$$

$x - 2y = -1$ ②
$$\begin{aligned} \text{L.S.} &= x - 2y \\ &= -3 - 2(-1) \\ &= -3 + 2 \\ &= -1 \\ &= \text{R.S.} \end{aligned}$$

Since the left side is equal to the right side for each equation, the solution is correct: $x = -3$ and $y = -1$

b) $3a + b = 5$ ①
$9a - b = 15$ ②
Since the b-terms are opposites, add the equations.

$$\begin{aligned} 3a + b &= 5 \quad ① \\ \underline{+(9a - b = 15)} &\quad ② \\ 12a &= 20 \\ a &= \frac{20}{12}, \text{ or } \frac{5}{3} \end{aligned}$$

Substitute $a = \frac{5}{3}$ into equation ①.

$$\begin{aligned} 3a + b &= 5 \quad ① \\ 3\left(\frac{5}{3}\right) + b &= 5 \\ 5 + b &= 5 \\ b &= 0 \end{aligned}$$

Verify the solution.

In each equation, substitute: $a = \frac{5}{3}$ and $b = 0$

$3a + b = 5$ ①
L.S. $= 3a + b$
$= 3\left(\frac{5}{3}\right) + 0$
$= 5$
$=$ R.S.

$9a - b = 15$ ②
L.S. $= 9a - b$
$= 9\left(\frac{5}{3}\right) - 0$
$= 15$
$=$ R.S.

Since the left side is equal to the right side for each equation, the solution is correct:
$a = \frac{5}{3}$ and $b = 0$

c) $3x - 4y = 1$ ①
$3x - 2y = -1$ ②
Since the x-terms are equal, subtract the equations.

$$\begin{aligned} 3x - 4y &= 1 \quad ① \\ \underline{-(3x - 2y} &\underline{= -1)} \quad ② \\ -4y + 2y &= 1 + 1 \\ -2y &= 2 \\ y &= -1 \end{aligned}$$

Substitute $y = -1$ into equation ①.

$$\begin{aligned} 3x - 4y &= 1 \quad ① \\ 3x - 4(-1) &= 1 \\ 3x + 4 &= 1 \\ 3x &= -3 \\ x &= -1 \end{aligned}$$

Verify the solution.
In each equation, substitute: $x = -1$ and $y = -1$

$3x - 4y = 1$ ①
L.S. $= 3x - 4y$
$= 3(-1) - 4(-1)$
$= -3 + 4$
$= 1$
$=$ R.S.

$3x - 2y = -1$ ②
L.S. $= 3x - 2y$
$= 3(-1) - 2(-1)$
$= -3 + 2$
$= -1$
$=$ R.S.

Since the left side is equal to the right side for each equation, the solution is correct:
$x = -1$ and $y = -1$

d) $3x - 4y = 0$ ①
$5x - 4y = 8$ ②
Since the y-terms are equal, subtract the equations.

$$\begin{aligned} 3x - 4y &= 0 \quad ① \\ \underline{-(5x - 4y} &\underline{= 8)} \quad ② \\ 3x - 5x &= 0 - 8 \\ -2x &= -8 \\ x &= 4 \end{aligned}$$

Substitute $x = 4$ into equation ①.

$$\begin{aligned} 3x - 4y &= 0 \quad ① \\ 3(4) - 4y &= 0 \\ 12 - 4y &= 0 \\ -4y &= -12 \\ y &= 3 \end{aligned}$$

Verify the solution.
In each equation, substitute: $x = 4$ and $y = 3$

$3x - 4y = 0$ ①
L.S. $= 3x - 4y$
$= 3(4) - 4(3)$
$= 12 - 12$
$= 0$
$=$ R.S.

$5x - 4y = 8$ ②
L.S. $= 5x - 4y$
$= 5(4) - 4(3)$
$= 20 - 12$
$= 8$
$=$ R.S.

Since the left side is equal to the right side for each equation, the solution is correct:
$x = 4$ and $y = 3$

4. a) $x - 2y = -6$ ①
$3x - y = 2$ ②

i) The coefficients of x are 1 and 3.
Multiply equation ① by 3.
$3(x - 2y = -6)$
$3x - 6y = -18$ ③
An equivalent system is:
$3x - y = 2$ ②
$3x - 6y = -18$ ③

ii) The coefficients of y are –2 and –1.
Multiply equation ② by 2.
$2(3x - y = 2)$
$6x - 2y = 4$ ④
An equivalent system is:
$x - 2y = -6$ ①
$6x - 2y = 4$ ④

b) $15x - 2y = 9$ ①
$5x + 4y = 17$ ②

i) The coefficients of x are 15 and 5.
Multiply equation ② by 3.
$3(5x + 4y = 17)$ ②
$15x + 12y = 51$ ③
An equivalent system is:
$15x - 2y = 9$ ①
$15x + 12y = 51$ ③

ii) The coefficients of y are –2 and 4.
Multiply equation ① by –2.
$-2(15x - 2y = 9)$
$-30x + 4y = -18$ ④
An equivalent system is:
$5x + 4y = 17$ ②
$-30x + 4y = -18$ ④

c) $7x + 3y = 9$ ①
$5x + 2y = 7$ ②

i) The coefficients of x are 7 and 5.
Multiply equation ① by 5.
$5(7x + 3y = 9)$
$35x + 15y = 45$ ③
Multiply equation ② by 7.
$7(5x + 2y = 7)$
$35x + 14y = 49$ ④
An equivalent system is:
$35x + 15y = 45$ ③
$35x + 14y = 49$ ④

ii) The coefficients of y are 3 and 2.
Multiply equation ① by 2.
$2(7x + 3y = 9)$
$14x + 6y = 18$ ⑤
Multiply equation ② by 3.
$3(5x + 2y = 7)$
$15x + 6y = 21$ ⑥
An equivalent system is:
$14x + 6y = 18$ ⑤
$15x + 6y = 21$ ⑥

d) $14x + 15y = 16$ ①
$21x + 10y = -1$ ②

i) The coefficients of x are 14 and 21; their least common multiple is 42.
Multiply equation ① by 3.
$3(14x + 15y = 16)$
$42x + 45y = 48$ ③
Multiply equation ② by 2.
$2(21x + 10y = -1)$
$42x + 20y = -2$ ④
An equivalent system is:
$42x + 45y = 48$ ③
$42x + 20y = -2$ ④

ii) The coefficients of y are 15 and 10; their least common multiple is 30.
Multiply equation ① by 2.
$2(14x + 15y = 16)$
$28x + 30y = 32$ ⑤
Multiply equation ② by 3.
$3(21x + 10y = -1)$
$63x + 30y = -3$ ⑥
An equivalent system is:
$28x + 30y = 32$ ⑤
$63x + 30y = -3$ ⑥

5. a) $x - 2y = -6$ ①
$3x - y = 2$ ②
Use the first equivalent system to eliminate x.
$3x - y = 2$ ②
$3x - 6y = -18$ ③
Subtract equation ③ from equation ②.
$3x - y = 2$ ②
$\underline{-(3x - 6y = -18)}$ ③
$-y + 6y = 2 + 18$
$5y = 20$
$y = 4$
Use the second equivalent system to eliminate y.
$x - 2y = -6$ ①
$6x - 2y = 4$ ④
Subtract equation ④ from equation ①.
$x - 2y = -6$ ①
$\underline{-(6x - 2y = 4)}$ ④
$x - 6x = -6 - 4$
$-5x = -10$
$x = 2$

Verify the solution.
In each original equation, substitute: $x = 2$ and $y = 4$

$x - 2y = -6$ ①
L.S. $= x - 2y$
$= 2 - 2(4)$
$= 2 - 8$
$= -6$
= R.S.

$3x - y = 2$ ②
L.S. $= 3x - y$
$= 3(2) - 4$
$= 6 - 4$
$= 2$
= R.S.

Since the left side is equal to the right side for each equation, the solution is correct:
$x = 2$ and $y = 4$

b) $15x - 2y = 9$ ①
$5x + 4y = 17$ ②
Use the first equivalent system to eliminate x.
$15x - 2y = 9$ ①
$15x + 12y = 51$ ③
Subtract equation ③ from equation ①.
$15x - 2y = 9$ ①
$\underline{-(15x + 12y = 51)}$ ③
$-2y - 12y = 9 - 51$
$-14y = -42$
$y = 3$
Use the second equivalent system to eliminate y.
$5x + 4y = 17$ ②
$-30x + 4y = -18$ ④
Subtract equation ④ from equation ②.
$5x + 4y = 17$ ②
$\underline{-(-30x + 4y = -18)}$ ④
$5x + 30x = 17 + 18$
$35x = 35$

$x = 1$

Verify the solution.
In each original equation, substitute: $x = 1$ and $y = 3$

$15x - 2y = 9$ ①
L.S. $= 15x - 2y$
$= 15(1) - 2(3)$
$= 15 - 6$
$= 9$
$=$ R.S.

$5x + 4y = 17$ ②
L.S. $= 5x + 4y$
$= 5(1) + 4(3)$
$= 5 + 12$
$= 17$
$=$ R.S.

Since the left side is equal to the right side for each equation, the solution is correct:
$x = 1$ and $y = 3$

c) $7x + 3y = 9$ ①
$5x + 2y = 7$ ②
Use the first equivalent system to eliminate x.
$35x + 15y = 45$ ③
$35x + 14y = 49$ ④
Subtract equation ④ from equation ③.
$35x + 15y = 45$ ③
$\underline{-(35x + 14y = 49)}$ ④
$15y - 14y = 45 - 49$
$y = -4$
Use the second equivalent system to eliminate y.
$14x + 6y = 18$ ⑤
$15x + 6y = 21$ ⑥
Subtract equation ⑥ from equation ⑤.
$14x + 6y = 18$ ⑤
$\underline{-(15x + 6y = 21)}$ ⑥
$14x - 15x = 18 - 21$
$-x = -3$
$x = 3$

Verify the solution.
In each original equation, substitute: $x = 3$ and $y = -4$

$7x + 3y = 9$ ①
L.S. $= 7x + 3y$
$= 7(3) + 3(-4)$
$= 21 - 12$
$= 9$
$=$ R.S.

$5x + 2y = 7$ ②
L.S. $= 5x + 2y$
$= 5(3) + 2(-4)$
$= 15 - 8$
$= 7$
$=$ R.S.

Since the left side is equal to the right side for each equation, the solution is correct:
$x = 3$ and $y = -4$

d) $14x + 15y = 16$ ①
$21x + 10y = -1$ ②
Use the first equivalent system to eliminate x.
$42x + 45y = 48$ ③
$42x + 20y = -2$ ④

Subtract equation ④ from equation ③.

$$\begin{aligned} 42x + 45y &= 48 && ③ \\ \underline{-(42x + 20y} &\underline{= -2)} && ④ \\ 45y - 20y &= 48 + 2 \\ 25y &= 50 \\ y &= 2 \end{aligned}$$

Use the second equivalent system to eliminate y.

$28x + 30y = 32$ ⑤

$63x + 30y = -3$ ⑥

Subtract equation ⑥ from equation ⑤.

$$\begin{aligned} 28x + 30y &= 32 && ⑤ \\ \underline{-(63x + 30y} &\underline{= -3)} && ⑥ \\ 28x - 63x &= 32 + 3 \\ -35x &= 35 \\ x &= -1 \end{aligned}$$

Verify the solution.

In each original equation, substitute: $x = -1$ and $y = 2$

$14x + 15y = 16$ ①

$$\begin{aligned} \text{L.S.} &= 14x + 15y \\ &= 14(-1) + 15(2) \\ &= -14 + 30 \\ &= 16 \\ &= \text{R.S.} \end{aligned}$$

$21x + 10y = -1$ ②

$$\begin{aligned} \text{L.S.} &= 21x + 10y \\ &= 21(-1) + 10(2) \\ &= -21 + 20 \\ &= -1 \\ &= \text{R.S.} \end{aligned}$$

Since the left side is equal to the right side for each equation, the solution is correct: $x = -1$ and $y = 2$

B

6. a) $2x + y = -5$ ①

$3x + 5y = 3$ ②

Multiply equation ① by 5 so the y-terms are equal.

$5(2x + y = -5)$

$10x + 5y = -25$ ③

Subtract equation ② from equation ③ to eliminate y.

$$\begin{aligned} 10x + 5y &= -25 && ③ \\ \underline{-(3x + 5y} &\underline{= 3)} && ② \\ 10x - 3x &= -25 - 3 \\ 7x &= -28 \\ x &= -4 \end{aligned}$$

Substitute $x = -4$ in equation ①.

$$\begin{aligned} 2x + y &= -5 && ① \\ 2(-4) + y &= -5 \\ -8 + y &= -5 \\ y &= 3 \end{aligned}$$

Verify the solution.

In each equation, substitute: $x = -4$ and $y = 3$

$2x + y = -5$ ①
L.S. $= 2x + y$
$= 2(-4) + 3$
$= -8 + 3$
$= -5$
$=$ R.S.

$3x + 5y = 3$ ②
L.S. $= 3x + 5y$
$= 3(-4) + 5(3)$
$= -12 + 15$
$= 3$
$=$ R.S.

Since the left side is equal to the right side for each equation, the solution is correct: $x = -4$ and $y = 3$

b) $3m - 6n = 0$ ①
$9m + 3n = -7$ ②
Multiply equation ② by 2 so the n-terms are opposites.
$2(9m + 3n = -7)$
$18m + 6n = -14$ ③
Add equations ① and ③ to eliminate n.
$3m - 6n = 0$ ①
$\underline{+(18m + 6n = -14)}$ ③
$3m + 18m = -14$
$21m = -14$ Divide each side by 21.
$m = \dfrac{-14}{21}$, or $-\dfrac{2}{3}$
Multiply equation ① by 3 so the m-terms are equal.
$3(3m - 6n = 0)$
$9m - 18n = 0$ ④
Subtract equation ④ from equation ② to eliminate m.
$9m + 3n = -7$ ②
$\underline{-(9m - 18n = 0)}$ ④
$3n + 18n = -7$
$21n = -7$ Divide each side by 21.
$n = -\dfrac{1}{3}$

Verify the solution.
In each equation, substitute: $m = -\dfrac{2}{3}$ and $n = -\dfrac{1}{3}$

$3m - 6n = 0$ ①
L.S. $= 3m - 6n$
$= 3\left(-\dfrac{2}{3}\right) - 6\left(-\dfrac{1}{3}\right)$
$= -2 + 2$
$= 0$
$=$ R.S.

$9m + 3n = -7$ ②
L.S. $= 9m + 3n$
$= 9\left(-\dfrac{2}{3}\right) + 3\left(-\dfrac{1}{3}\right)$
$= -6 - 1$
$= -7$
$=$ R.S.

Since the left side is equal to the right side for each equation, the solution is correct: $m = -\dfrac{2}{3}$ and $n = -\dfrac{1}{3}$

c) $2s + 3t = 6$ ①
$5s + 10t = 20$ ②
Multiply equation ① by 5 and equation ② by 2 so the s-terms are equal.
$5(2s + 3t = 6)$ $2(5s + 10t = 20)$
$10s + 15t = 30$ ③ $10s + 20t = 40$ ④
Subtract equation ③ from equation ④ to eliminate s.
$10s + 20t = 40$ ④
$\underline{-(10s + 15t = 30)}$ ③
$20t - 15t = 40 - 30$
$5t = 10$
$t = 2$
Substitute $t = 2$ into equation ①.
$2s + 3t = 6$ ①
$2s + 3(2) = 6$
$2s + 6 = 6$
$2s = 0$
$s = 0$

Verify the solution.
In each equation, substitute: $s = 0$ and $t = 2$

$2s + 3t = 6$ ①	$5s + 10t = 20$ ②
L.S. $= 2s + 3t$	L.S. $= 5s + 10t$
$= 2(0) + 3(2)$	$= 5(0) + 10(2)$
$= 6$	$= 20$
= R.S.	= R.S.

Since the left side is equal to the right side for each equation, the solution is correct:
$s = 0$ and $t = 2$

d) $3a + 2b = 5$ ①
$2a + 3b = 0$ ②
Multiply equation ① by 2 and equation ② by 3 so the a-terms are equal.
$2(3a + 2b = 5)$ $3(2a + 3b = 0)$
$6a + 4b = 10$ ③ $6a + 9b = 0$ ④
Subtract equation ④ from equation ③ to eliminate s.
$6a + 4b = 10$ ③
$\underline{-(6a + 9b = 0)}$ ④
$4b - 9b = 10 - 0$
$-5b = 10$
$b = -2$
Substitute $b = -2$ into equation ②.
$2a + 3b = 0$ ②
$2a + 3(-2) = 0$
$2a = 6$
$a = 3$

Verify the solution.

In each equation, substitute: $a = 3$ and $b = -2$

$3a + 2b = 5$ ①
L.S. $= 3a + 2b$
$= 3(3) + 2(-2)$
$= 9 - 4$
$= 5$
= R.S.

$2a + 3b = 0$ ②
L.S. $= 2a + 3b$
$= 2(3) + 3(-2)$
$= 6 - 6$
$= 0$
= R.S.

Since the left side is equal to the right side for each equation, the solution is correct: $a = 3$ and $b = -2$

7. a) $8x - 3y = 38$ ①
$3x - 2y = -1$ ②

Multiply equation ① by 2 and equation ② by 3 so the y-terms are equal.

$2(8x - 3y = 38)$
$16x - 6y = 76$ ③

$3(3x - 2y = -1)$
$9x - 6y = -3$ ④

Subtract equation ④ from equation ③ to eliminate y.

$16x - 6y = 76$ ③
$\underline{-(9x - 6y = -3)}$ ④
$16x - 9x = 76 - (-3)$
$7x = 79$
$x = \dfrac{79}{7}$

Multiply equation ① by 3 and equation ② by 8 so the x-terms are equal.

$3(8x - 3y = 38)$
$24x - 9y = 114$ ⑤

$8(3x - 2y = -1)$
$24x - 16y = -8$ ⑥

Subtract equation ⑥ from equation ⑤ to eliminate y.

$24x - 9y = 114$ ⑤
$\underline{-(24x - 16y = -8)}$ ⑥
$-9y - (-16y) = 114 - (-8)$
$-9y + 16y = 122$
$7y = 122$
$y = \dfrac{122}{7}$

Verify the solution.

In each original equation, substitute: $x = \dfrac{79}{7}$ and $y = \dfrac{122}{7}$

$8x - 3y = 38$ ①
L.S. $= 8x - 3y$
$= 8\left(\dfrac{79}{7}\right) - 3\left(\dfrac{122}{7}\right)$
$= \dfrac{632}{7} - \dfrac{366}{7}$
$= \dfrac{266}{7}$
$= 38$
= R.S.

$3x - 2y = -1$ ②
L.S. $= 3x - 2y$
$= 3\left(\dfrac{79}{7}\right) - 2\left(\dfrac{122}{7}\right)$
$= \dfrac{237}{7} - \dfrac{244}{7}$
$= -\dfrac{7}{7}$
$= -1$
= R.S.

Since the left side is equal to the right side for each equation, the solution is correct:
$x = \frac{79}{7}$ and $y = \frac{122}{7}$

b) $2a - 5b = 29$ ①
$7a - 3b = 0$ ②
Multiply equation ① by 7 and equation ② by 2 so the a-terms are equal.
$7(2a - 5b = 29)$ $2(7a - 3b = 0)$
$14a - 35b = 203$ ③ $14a - 6b = 0$ ④
Subtract equation ④ from equation ③ to eliminate s.

$$\begin{aligned} 14a - 35b &= 203 \quad ③ \\ \underline{-(14a - 6b} &\underline{= 0)} \quad ④ \\ -35b - (-6b) &= 203 - 0 \\ -35b + 6b &= 203 \\ -29b &= 203 \\ b &= -7 \end{aligned}$$

Substitute $b = -7$ into equation ②.

$$\begin{aligned} 7a - 3b &= 0 \quad ② \\ 7a - 3(-7) &= 0 \\ 7a + 21 &= 0 \\ 7a &= -21 \\ a &= -3 \end{aligned}$$

Verify the solution.
In each equation, substitute: $a = -3$ and $b = -7$

$2a - 5b = 29$ ①

$$\begin{aligned} \text{L.S.} &= 2a - 5b \\ &= 2(-3) - 5(-7) \\ &= -6 + 35 \\ &= 29 \\ &= \text{R.S.} \end{aligned}$$

$7a - 3b = 0$ ②

$$\begin{aligned} \text{L.S.} &= 7a - 3b \\ &= 7(-3) - 3(-7) \\ &= -21 + 21 \\ &= 0 \\ &= \text{R.S.} \end{aligned}$$

Since the left side is equal to the right side for each equation, the solution is correct:
$a = -3$ and $b = -7$

c) $18a - 15b = 4$ ①
$10a + 3b = 6$ ②
Multiply equation ② by 5 so the b-terms are opposites.
$5(10a + 3b = 6)$
$50a + 15b = 30$ ③
Add equations ① and ③ to eliminate b.

$$\begin{aligned} 18a - 15b &= 4 \quad ① \\ \underline{+(50a + 15b} &\underline{= 30)} \quad ③ \\ 18a + 50a &= 4 + 30 \\ 68a &= 34 \\ a &= \frac{34}{68}, \text{ or } \frac{1}{2} \end{aligned}$$

Substitute $a = \frac{1}{2}$ into equation ②.

$$10a + 3b = 6 \quad ②$$
$$10\left(\frac{1}{2}\right) + 3b = 6$$
$$5 + 3b = 6$$
$$3b = 1$$
$$b = \frac{1}{3}$$

Verify the solution.

In each equation, substitute: $a = \frac{1}{2}$ and $b = \frac{1}{3}$

$18a – 15b = 4 \quad ①$

L.S. $= 18a – 15b$
$= 18\left(\frac{1}{2}\right) – 15\left(\frac{1}{3}\right)$
$= 9 – 5$
$= 4$
$=$ R.S.

$10a + 3b = 6 \quad ②$

L.S. $= 10a + 3b$
$= 10\left(\frac{1}{2}\right) + 3\left(\frac{1}{3}\right)$
$= 5 + 1$
$= 6$
$=$ R.S.

Since the left side is equal to the right side for each equation, the solution is correct:
$a = \frac{1}{2}$ and $b = \frac{1}{3}$

d) $6x – 2y = 21 \quad ①$
$4x + 3y = 1 \quad ②$

Multiply equation ① by 3 and equation ② by 2 so the y-terms are opposites.

$3(6x – 2y = 21)$
$18x – 6y = 63 \quad ③$

$2(4x + 3y = 1)$
$8x + 6y = 2 \quad ④$

Add equations ③ and ④ to eliminate y.

$$18x – 6y = 63 \quad ③$$
$$\underline{+(8x + 6y = 2)} \quad ②$$
$$18x + 8x = 63 + 2$$
$$26x = 65$$
$$x = \frac{65}{26}, \text{ or } \frac{5}{2}$$

Substitute $x = \frac{5}{2}$ in equation ②.

$$4x + 3y = 1 \quad ②$$
$$4\left(\frac{5}{2}\right) + 3y = 1$$
$$3y = –9$$
$$y = –3$$

Verify the solution.

In each equation, substitute: $x = \frac{5}{2}$ and $y = -3$

$6x - 2y = 21$ ①
L.S. $= 6x - 2y$
$= 6\left(\frac{5}{2}\right) - 2(-3)$
$= 15 + 6$
$= 21$
$=$ R.S.

$4x + 3y = 1$ ②
L.S. $= 4x + 3y$
$= 4\left(\frac{5}{2}\right) + 3(-3)$
$= 10 - 9$
$= 1$
$=$ R.S.

Since the left side is equal to the right side for each equation, the solution is correct:
$x = \frac{5}{2}$ and $y = -3$

8. Let x represent the attendance in 2006.
Let y represent the attendance in 2008.
The mean attendance was 45 265.
So, one equation is:

$\frac{x+y}{2} = 45\,265$ Multiply each side by 2.

$x + y = 90\,530$

The attendance in 2008 was 120 more than the attendance in 2006.
So, another equation is: $y - x = 120$
A linear system is:
$x + y = 90\,530$ ①
$y - x = 120$ ②
Add equations ① and ② to eliminate the x-terms.

$x + y = 90\,530$ ①
$\underline{+\ (y - x = 120)}$ ②
$y + y = 90\,530 + 120$
$2y = 90\,650$
$y = 45\,325$

Substitute $y = 45\,325$ in equation ①.

$x + y = 90\,530$ ①
$x + 45\,325 = 90\,530$
$x = 45\,205$

The attendance in 2006 was 45 205. The attendance in 2008 was 45 325.

Verify the solution.
The mean attendance is: $\frac{45\,205 + 45\,325}{2} = \frac{90\,530}{2}$, or 45 265; this agrees with the given information.
The difference between the attendance in 2008 and 2006 was: $45\,325 - 45\,205 = 120$; this agrees with the given information.
So, the solution is correct.

9. Let t represent the number of cones on Talise's dress.
Let s represent the number of cones on her sister's dress.
The total number of cones is 545.
So, one equation is: $t + s = 545$

Talise's dress had 185 more cones than her sister's dress.
So, another equation is: $t - s = 185$
A linear system is:
$t + s = 545$ ①
$t - s = 185$ ②
Add equations ① and ② to eliminate s.

$$\begin{aligned} t + s &= 545 && ①\\ \underline{+(t - s) = 185} & && ②\\ t + t &= 545 + 185 \\ 2t &= 730 \\ t &= 365 \end{aligned}$$

Substitute $t = 365$ in equation ①.

$$\begin{aligned} t + s &= 545 && ①\\ 365 + s &= 545 \\ s &= 545 - 365 \\ s &= 180 \end{aligned}$$

There were 365 cones on Talise's dress and 180 cones on her sister's dress.

Verify the solution.
The total number of cones is: $365 + 180 = 545$; this agrees with the given information.
The difference in the number of cones is: $365 - 180 = 185$; this agrees with the given information.
So, the solution is correct.

10. Let k represent the cost of a knife in beaver pelts.
Let b represent the cost of a blanket in beaver pelts.
10 knives + 20 blankets = 200 beaver pelts
So, one equation is: $10k + 20b = 200$
Divide both sides by 10 to simplify:
$10k + 20b = 200$
$k + 2b = 20$ ①

15 knives + 25 blankets = 270 beaver pelts
So, another equation is: $15k + 25b = 270$
Divide both sides by 5 to simplify:
$15k + 25b = 270$
$3k + 5b = 54$ ②

So, a linear system is:
$k + 2b = 20$ ①
$3k + 5b = 54$ ②

Multiply equation ① by 3.
$3(k + 2b = 20)$
$3k + 6b = 60$ ③

Subtract equation ② from equation ③ to eliminate k.

$$\begin{aligned} 3k + 6b &= 60 && ③\\ \underline{-(3k + 5b = 54)} & && ②\\ 6b - 5b &= 60 - 54 \\ b &= 6 \end{aligned}$$

Substitute $b = 6$ in equation ①.

$k + 2b = 20$ ①
$k + 2(6) = 20$
$k + 12 = 20$
$k = 8$

One knife cost 8 beaver pelts. One blanket cost 6 beaver pelts.

Verify the solution.
In beaver pelts, 10 knives + 20 blankets cost: $10(8) + 20(6) = 80 + 120$, or 200; this agrees with the given information.
In beaver pelts, 15 knives + 25 blankets cost: $15(8) + 25(6) = 120 + 150$, or 270; this agrees with the given information.
So, the solution is correct.

11. Let m beats per minute represent the moderate tempo.
Let f beats per minute represent the fast tempo.
A moderate tempo for 4.5 min and a fast tempo for 30 s, or 0.5 min produced 620 beats.
So, one equation is: $4.5m + 0.5f = 620$
The moderate tempo was 40 beats/min less than the fast tempo.
So, another equation is: $f - m = 40$
A linear system is:

$4.5m + 0.5f = 620$ ①
$f - m = 40$ ②

Multiply equation ① by 2.

$2(4.5m + 0.5f = 620)$
$9m + f = 1240$ ③

Subtract equation ② from equation ③.

$9m + f = 1240$ ③
$\underline{-(f - m = 40)}$ ②
$9m - (-m) = 1240 - 40$
$10m = 1200$
$m = 120$

Substitute $m = 120$ in equation ②.

$f - m = 40$ ②
$f - 120 = 40$
$f = 160$

The moderate tempo is 120 beats/min and the fast tempo is 160 beats/min.

Verify the solution.
The number of beats in a moderate tempo for 4.5 min and a fast tempo for 0.5 min is:
$4.5(120) + 0.5(160) = 540 + 80$, or 620; this agrees with the given information.
The difference in rates is: 160 beats/min – 120 beats/min = 40 beats/min; this agrees with the given information.
So, the solution is correct.

12. a) $\frac{a}{2} + \frac{b}{3} = 1$ ①

$\frac{a}{4} - \frac{2b}{3} = -1$ ②

The lowest common denominator of the fractions in equation ① is 6, so multiply equation ① by 6.

$6\left(\frac{a}{2} + \frac{b}{3} = 1\right)$

$3a + 2b = 6$ ③

The lowest common denominator of the fractions in equation ② is 12, so multiply equation ② by 12.

$12\left(\frac{a}{4} - \frac{2b}{3} = -1\right)$

$3a - 8b = -12$ ④

An equivalent linear system is:

$3a + 2b = 6$ ③

$3a - 8b = -12$ ④

Subtract equation ④ from equation ③ to eliminate a.

$3a + 2b = 6$ ③

$\underline{-(3a - 8b = -12)}$ ④

$2b + 8b = 6 + 12$

$10b = 18$

$b = \frac{18}{10}$, or $\frac{9}{5}$

Substitute $b = \frac{9}{5}$ in equation ③.

$3a + 2b = 6$ ③

$3a + 2\left(\frac{9}{5}\right) = 6$

$3a + \left(\frac{18}{5}\right) = 6$

$3a = 6 - \frac{18}{5}$

$3a = \frac{30}{5} - \frac{18}{5}$

$3a = \frac{12}{5}$

$a = \frac{4}{5}$

Verify the solution.

In each original equation, substitute: $a = \frac{4}{5}$ and $b = \frac{9}{5}$

$$\frac{a}{2} + \frac{b}{3} = 1 \quad ①$$

$$\begin{aligned}\text{L.S.} &= \frac{1}{2}a + \frac{1}{3}b \\ &= \frac{1}{2}\left(\frac{4}{5}\right) + \frac{1}{3}\left(\frac{9}{5}\right) \\ &= \frac{2}{5} + \frac{3}{5} \\ &= \frac{5}{5} \\ &= 1 \\ &= \text{R.S.}\end{aligned}$$

$$\frac{a}{4} - \frac{2b}{3} = -1 \quad ②$$

$$\begin{aligned}\text{L.S.} &= \frac{1}{4}a - \frac{2}{3}b \\ &= \frac{1}{4}\left(\frac{4}{5}\right) - \frac{2}{3}\left(\frac{9}{5}\right) \\ &= \frac{1}{5} - \frac{6}{5} \\ &= -\frac{5}{5} \\ &= -1 \\ &= \text{R.S.}\end{aligned}$$

Since the left side is equal to the right side for each equation, the solution is correct:
$a = \frac{4}{5}$ and $b = \frac{9}{5}$

b) $\frac{x}{2} + \frac{y}{2} = 7$ ①

$3x + 2y = 48$ ②

The lowest common denominator of the fractions in equation ① is 2, so multiply equation ① by 2.

$$2\left(\frac{x}{2} + \frac{y}{2} = 7\right)$$

$$x + y = 14 \quad ③$$

Multiply equation ③ by 3.

$$3(x + y = 14)$$

$$3x + 3y = 42 \quad ④$$

Subtract equation ② from equation ④ to eliminate x.

$$\begin{aligned} 3x + 3y &= 42 && ④ \\ \underline{-(3x + 2y = 48)} & && ② \\ 3y - 2y &= 42 - 48 \\ y &= -6 \end{aligned}$$

Substitute $y = -6$ in equation ③.

$$\begin{aligned} x + y &= 14 && ③ \\ x + (-6) &= 14 \\ x &= 14 + 6 \\ x &= 20 \end{aligned}$$

Verify the solution.

In each original equation, substitute: $x = 20$ and $y = –6$

$\frac{x}{2} + \frac{y}{2} = 7$ ①

L.S. $= \frac{1}{2}x + \frac{1}{2}y$
$= \frac{1}{2}(20) + \frac{1}{2}(-6)$
$= 10 - 3$
$= 7$
= R.S.

$3x + 2y = 48$ ②

L.S. $= 3x + 2y$
$= 3(20) + 2(-6)$
$= 60 - 12$
$= 48$
= R.S.

Since the left side is equal to the right side for each equation, the solution is correct: $x = 20$ and $y = –6$

c) $0.03x + 0.15y = 0.027$ ①
$-0.5x - 0.5y = 0.05$ ②
Multiply equation ② by 0.3.
$0.3(-0.5x - 0.5y = 0.05)$
$-0.15x - 0.15y = 0.015$ ③
Add equations ① and ③ to eliminate y.
$0.03x + 0.15y = 0.027$ ①
$\underline{+(-0.15x - 0.15y = 0.015)}$ ③
$0.03x - 0.15x = 0.027 + 0.015$
$-0.12x = 0.042$
$x = \frac{0.042}{-0.12}$
$x = -0.35$
Substitute $x = -0.35$ in equation ②.
$-0.5x - 0.5y = 0.05$ ②
$-0.5(-0.35) - 0.5y = 0.05$
$0.175 - 0.5y = 0.05$
$-0.5y = 0.05 - 0.175$
$-0.5y = -0.125$
$y = \frac{-0.125}{-0.5}$
$y = 0.25$

Verify the solution.
In each original equation, substitute: $x = -0.35$ and $y = 0.25$

$0.03x + 0.15y = 0.027$ ①
L.S. $= 0.03x + 0.15y$
$= 0.03(-0.35) + 0.15(0.25)$
$= -0.0105 + 0.0375$
$= 0.027$
= R.S.

$-0.5x - 0.5y = 0.05$ ②
L.S. $= -0.5x - 0.5y$
$= -0.5(-0.35) - 0.5(0.25)$
$= 0.175 - 0.125$
$= 0.05$
= R.S.

Since the left side is equal to the right side for each equation, the solution is correct: $x = -0.35$ and $y = 0.25$

d) $-1.5x + 2.5y = 0.5$ ①
$2x + y = 1.5$ ②
Multiply equation ② by 2.5.
$2.5(2x + y = 1.5)$
$5x + 2.5y = 3.75$ ③
Subtract equation ① from equation ③ to eliminate y.
$5x + 2.5y = 3.75$ ③
$\underline{-(-1.5x + 2.5y = 0.5)}$ ①
$5x + 1.5x = 3.75 - 0.5$
$6.5x = 3.25$
$x = \dfrac{3.25}{6.5}$
$x = 0.5$
Substitute $x = 0.5$ in equation ②.
$2x + y = 1.5$ ②
$2(0.5) + y = 1.5$
$1 + y = 1.5$
$y = 0.5$

Verify the solution.
In each original equation, substitute: $x = 0.5$ and $y = 0.5$

$-1.5x + 2.5y = 0.5$ ①
L.S. $= -1.5x + 2.5y$
$= -1.5(0.5) + 2.5(0.5)$
$= -0.75 + 1.25$
$= 0.5$
= R.S.

$2x + y = 1.5$ ②
L.S. $= 2(0.5) + (0.5)$
$= 1 + 0.5$
$= 1.5$
= R.S.

Since the left side is equal to the right side for each equation, the solution is correct:
$x = 0.5$ and $y = 0.5$

13. Write a linear system to represent the situation.
Let c represent the number of Canadian players.
Let f represent the number of foreign players.
There are 25 players.
So, one equation is: $c + f = 25$
Seven-ninths of the Canadian players and three-sevenths of the foreign players are over 6 ft. tall. There are 17 players who are over 6 ft. tall.
So, another equation is: $\frac{7}{9}c + \frac{3}{7}f = 17$
A linear system is:
$c + f = 25$ ①
$\frac{7}{9}c + \frac{3}{7}f = 17$ ②
Solve the linear system.
Multiply equation ② by the least common denominator of the fractions, which is 63.
$63\left(\frac{7}{9}c + \frac{3}{7}f = 17\right)$
$49c + 27f = 1071$ ③

Multiply equation ① by 27.

$27(c + f = 25)$

$27c + 27f = 675$ ④

Subtract equation ④ from equation ③.

$$\begin{array}{rl} 49c + 27f = 1071 & ③ \\ \underline{-(27c + 27f = 675)} & ④ \\ 49c - 27c = 1071 - 675 & \\ 22c = 396 & \\ c = 18 & \end{array}$$

Substitute $c = 18$ in equation ①.

$$\begin{array}{rl} c + f = 25 & ① \\ 18 + f = 25 & \\ f = 7 & \end{array}$$

There are 18 Canadian players and 7 foreign players.

Verify the solution.

The total number of players is: 18 + 7 = 25; this agrees with the given information.

Seven-ninths of the Canadian players and three-sevenths of the foreign players is:

$\frac{7}{9}(18) + \frac{3}{7}(7) = 14 + 3$, or 17; this agrees with the given information.

So, the solution is correct.

14. Write a linear system to represent the situation.

Let g represent the number of girls surveyed.

Let b represent the number of boys surveyed.

76 students were surveyed.

So, one equation is: $g + b = 76$

One-quarter of the girls and three-quarters of the boys played online games; this is 39 students.

So, another equation is: $\frac{1}{4}g + \frac{3}{4}b = 39$

A linear system is:

$$\begin{array}{ll} g + b = 76 & ① \\ \frac{1}{4}g + \frac{3}{4}b = 39 & ② \end{array}$$

Solve the linear system.

Multiply equation ② by the least common denominator of the fractions, which is 4.

$$4\left(\frac{1}{4}g + \frac{3}{4}b = 39\right)$$

$$g + 3b = 156 \quad ③$$

Subtract equation ① from equation ③.

$$\begin{array}{rl} g + 3b = 156 & ③ \\ \underline{-(g + b = 76)} & ① \\ 3b - b = 156 - 76 & \\ 2b = 80 & \\ b = 40 & \end{array}$$

Substitute $b = 40$ in equation ①.

$g + b = 76$ ①
$g + 40 = 76$
$g = 36$
36 girls and 40 boys were surveyed.

Verify the solution.
The total number of students is: 36 + 40 = 76; this agrees with the given information.
One-quarter of the girls and three-quarters of the boys is: $\frac{1}{4}(36) + \frac{3}{4}(40) = 9 + 30$, or 39; this agrees with the given information.
So, the solution is correct.

15. a) An equation that Balance scales 1 models is:
$x + x + x + y = 10 + 5 + 1 + 1$ Simplify.
$3x + y = 17$
An equation that Balance scales 2 models is:
$x + y = 5 + 1 + 1$ Simplify.
$x + y = 7$
A linear system is:
$3x + y = 17$ ①
$x + y = 7$ ②

b) From Balance scales 2, the sum of mass x and mass y is 7 kg.
So, for Balance scales 1, if masses x and y are removed from the left side and 7 kg is removed from the right side, then the same mass has been removed from both sides, and the scales remain balanced.

c) When masses x and y are removed from the left side of Balance scales 1, a mass of $2x$ remains.
When a mass of 7 kg is removed, a mass of 10 kg remains.
So, $2x = 20$ kg, and $x = 5$ kg
Since masses x and y equal 7 kg, and mass x is 5 kg, then mass y is 2 kg.

d) Removing masses x, y, and 7 kg from Balance scales 1 is the same as subtracting equation ② from equation ①, to eliminate y.

16. Let a dollars represent the cost of an adult's ticket.
Let c dollars represent the cost of a child's ticket.
Use the given information to write a linear system:
$a + 3c = 27.75$ ①
$2a + 2c = 27.50$ ②
Solve the linear system to determine the cost of each ticket.
Multiply equation ① by 2.
$2(a + 3c = 27.75)$
$2a + 6c = 55.50$ ③
Subtract equation ② from equation ③ to eliminate a.
$2a + 6c = 55.50$ ③
$\underline{-(2a + 2c = 27.50)}$ ②
$6c - 2c = 55.50 - 27.50$
$4c = 28$

$c = 7$

Substitute $c = 7$ in equation ①.

$a + 3c = 27.75$ ①

$a + 3(7) = 27.75$

$a + 21 = 27.75$

$a = 6.75$

A child pays \$7.00 and an adult pays \$6.75, so a child's ticket is more expensive.

Verify the solution.

One adult and 3 children would pay: \$6.75 + 3(\$7.00) = \$6.75 + \$21.00, or \$27.75; this agrees with the given information.

Two adults and 2 children would pay: 2(\$6.75) + 2(\$7.00) = \$13.50 + \$14.00, or \$27.50; this agrees with the given information.

So, the solution is correct.

17. Write a linear system to model the situation.

Let p kilograms represent the mass of peas.

Let l kilograms represent the mass of lentils.

The total mass of peas and lentils is 25 kg.

So, one equation is: $p + l = 25$

A mass of p kilograms of peas costs $5p$ dollars.

A mass of l kilograms of lentils costs $6.50l$ dollars.

The total cost is \$140.00.

So, another equation is: $5p + 6.50l = 140$

A linear system is:

$p + l = 25$ ①

$5p + 6.50l = 140$ ②

Multiply equation ① by 5.

$5(p + l = 25)$

$5p + 5l = 125$ ③

Subtract equation ③ from equation ② to eliminate p.

$5p + 6.50l = 140$ ②

$\underline{-(5p + 5l = 125)}$ ③

$6.50l - 5l = 140 - 125$ ③

$1.50l = 15$

$l = 10$

Substitute $l = 10$ in equation ①.

$p + l = 25$ ①

$p + 10 = 25$

$p = 15$

The mass of peas is 15 kg and the mass of lentils is 10 kg.

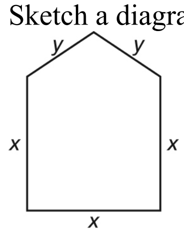
Verify the solution.

The total mass of peas and lentils is: 15 kg + 10 kg = 25 kg; this agrees with the given information.

The total cost of peas and lentils is: 15(\$5) + 10(\$6.50) = \$75 + \$65 , or \$140; this agrees with the given information.

So, the solution is correct.

18. $3x + 2y = 21$ ①
$x - y = 2$ ②
A pentagon has 5 sides.
From equation ①, the coefficients of x and y have a sum of 5. This suggests that x and y could be the lengths of the sides of the pentagon and the equation models the perimeter of the pentagon.
From equation ②, a side with length x units is 2 units longer than a side with length y units.
Sketch a diagram.



A problem could be:
A pentagon has 3 sides with one length and 2 sides with another length.
The perimeter of the pentagon is 21 cm.
A longer side is 2 cm longer than a shorter side.
What are the lengths of the sides of the pentagon?
To solve the problem, solve the linear system.
Multiply equation ② by 3.
$3(x - y = 2)$
$3x - 3y = 6$ ③
Subtract equation ③ from equation ①.

$$\begin{aligned} 3x + 2y &= 21 & ① \\ \underline{-(3x - 3y} &\underline{= 6)} & ③ \\ 2y + 3y &= 21 - 6 \\ 5y &= 15 \\ y &= 3 \end{aligned}$$

Substitute $y = 3$ in equation ②.

$$\begin{aligned} x - y &= 2 & ② \\ x - 3 &= 2 \\ x &= 5 \end{aligned}$$

The polygon has 3 sides with length 5 cm and 2 sides with length 3 cm.

Verify the solution.
The perimeter of the polygon is: 5 cm + 5 cm + 5 cm + 3 cm + 3 cm = 21 cm; this agrees with the given information.
The difference between a longer side and a shorter side is: 5 cm – 3 cm = 2 cm; this agrees with the given information.
So, the solution is correct.

19. a) $3x + y = 17$ ①
$x + y = 7$ ②
Equation ① has coefficients 3 and 1, which could be the numbers of items bought.
Equation ② has coefficients 1, so x and y could be the costs, in dollars, of the items bought.
A possible problem is:
Three binders and 1 pen cost $17.00.
One binder and 1 pen cost $7.00.
What is the cost of 1 binder and 1 pen?

b) x dollars represents the cost of 1 binder.
y dollars represents the cost of 1 pen.
To solve the linear system:
Subtract equation ② from equation ① to eliminate y.

$$\begin{array}{rll} 3x + y = 17 & & ① \\ \underline{-(x + y = 7)} & & ② \\ 3x - x = 17 - 7 & & \\ 2x = 10 & & \\ x = 5 & & \end{array}$$

Substitute $x = 5$ in equation ②.

$$\begin{array}{rl} x + y = 7 & ② \\ 5 + y = 7 & \\ y = 2 & \end{array}$$

One binder costs \$5.00 and 1 pen costs \$2.00.

Verify the solution.
Three binders and one pen cost: 3(\$5.00) + \$2.00 = \$17.00; this agrees with the given information.
One binder and one pen cost: \$5.00 + \$2.00 = \$7.00; this agrees with the given information.
So, the solution is correct.

20. a)

$$\begin{array}{rl} 3x + 4y = 29 & ① \\ 2x - 5y = -19 & ② \end{array}$$

One way to solve the system is to multiply equation ① by 2 and multiply equation ② by 3, then subtract the equations to eliminate x.
Another way to solve the system is to multiply equation ① by 5 and multiply equation ② by 4, then add the equations to eliminate y.

b) Solution 1:
Multiply equation ① by 2 and multiply equation ② by 3.

$$\begin{array}{rlrl} 2(3x + 4y = 29) & & 3(2x - 5y = -19) & \\ 6x + 8y = 58 & ③ & 6x - 15y = -57 & ④ \end{array}$$

Subtract equation ④ from equation ③.

$$\begin{array}{rl} 6x + 8y = 58 & ③ \\ \underline{-(6x - 15y = -57)} & ④ \\ 8y + 15y = 58 + 57 & \\ 23y = 115 & \\ y = 5 & \end{array}$$

Substitute $y = 5$ in equation ①.

$$\begin{array}{rl} 3x + 4y = 29 & ① \\ 3x + 4(5) = 29 & \\ 3x + 20 = 29 & \\ 3x = 9 & \\ x = 3 & \end{array}$$

Solution 2:
Multiply equation ① by 5 and multiply equation ② by 4.

$$\begin{array}{rlrl} 5(3x + 4y = 29) & & 4(2x - 5y = -19) & \\ 15x + 20y = 145 & ③ & 8x - 20y = -76 & ④ \end{array}$$

Add equations ③ and ④.

$15x + 20y = 145$ ③
$+(8x - 20y = -76)$ ④
$15x + 8x = 145 - 76$
$23x = 69$
$x = 3$
Substitute $x = 3$ in equation ①.
$3x + 4y = 29$ ①
$3(3) + 4y = 29$
$9 + 4y = 29$
$4y = 20$
$y = 5$

Verify the solution.
In each equation, substitute: $x = 3$ and $y = 5$

$3x + 4y = 29$ ①
L.S. $= 3x + 4y$
$= 3(3) + 4(5)$
$= 9 + 20$
$= 29$
= R.S.

$2x - 5y = -19$ ②
L.S. $= 2x - 5y$
$= 2(3) - 5(5)$
$= 6 - 25$
$= -19$
= R.S.

Since the left side is equal to the right side for each equation, the solution is correct:
$x = 3$ and $y = 5$

21. a) A possible situation is:
Two hundred students were checked for colour blindness and 14 students were found to be colour blind. Two percent of the females and 12% of the males were colour blind.
Let f represent the number of females in the study.
Let m represent the number of males in the study.
The total number of students in the study is 200.
So, one equation is: $f + m = 200$
2% of females is: $0.02f$
12% of males is: $0.12m$
14 students were colour blind.
So, another equation is: $0.02f + 0.12m = 14$
A linear system is:
$f + m = 200$ ①
$0.02f + 0.12m = 14$ ②

b) A related problem is: How many females and how many males were in the study?
To solve the linear system:
Multiply equation ① by 0.02.
$0.02(f + m = 200)$
$0.02f + 0.02m = 4$ ③
Subtract equation ③ from equation ② to eliminate f.
$0.02f + 0.12m = 14$ ②
$-(0.02f + 0.02m = 4)$ ③
$0.12m - 0.02m = 14 - 4$
$0.10m = 10$
$m = 100$

Substitute $m = 100$ in equation ①.

$f + m = 200$ ①
$f + 100 = 200$
$f = 100$

There were 100 females and 100 males in the study.

Verify the solution.
The total number of people in the study is: 100 + 100 = 200; this agrees with the given information.
2% of 100 + 12% of 100 = 2 + 12, or 14; this agrees with the given information.
So, the solution is correct.

C

22. Write then solve a linear system.
Let s dollars represent the money invested in the stock.
Let b dollars represent the money invested in the bond.
The stock lost 10.5%, so the interest in dollars at the end of the year was: $-0.105s$
The bond gained 3.5%, so the interest in dollars at the end of the year was: $0.035b$
The total loss was $84.00.
So, one equation is: $-0.105s + 0.035b = -84$
If s dollars had been invested in the bond, then the interest in dollars would have been: $0.035s$
If b dollars had been invested in the stock, then the interest in dollars would have been: $-0.105b$
The total loss would have been $14.00.
So, another equation is:
$0.035s - 0.105b = -14$
A linear system is:
$-0.105s + 0.035b = -84$ ①
$0.035s - 0.105b = -14$ ②
Multiply equation ① by 3.
$3(-0.105s + 0.035b = -84)$ ①
$-0.315s + 0.105b = -252$ ③
Add equations ② and ③ to eliminate b.
$0.035s - 0.105b = -14$ ②
$\underline{+(-0.315s + 0.105b = -252)}$ ③
$0.035s - 0.315s = -14 - 252$
$-0.28s = -266$
$s = 950$
Substitute $s = 950$ in equation ①.
$-0.105s + 0.035b = -84$ ①
$-0.105(950) + 0.035b = -84$
$-99.75 + 0.035b = -84$
$0.035b = -84 + 99.75$
$0.035b = 15.75$
$b = 450$
Cam invested \$950 in the stock and \$450 in the bond.

Verify the solution.
The stock lost 10.5% and the bond gained 3.5%, so the interest was:
$-(0.105 \times \$950) + (0.035 \times \$450) = -\$99.75 + \$15.75$
$= -\$84.00$; this agrees with the given information

If the amounts had been interchanged, the interest would have been:
$-(0.105 \times \$450) + (0.035 \times \$950) = -\$47.25 + \$33.25$
$= -\$14$; this agrees with the given information
So, the solution is correct.

23. a) $2x + 5y = 8$
Another equation whose coefficients and constant term differ by 3 is:
$4x + 7y = 10$
Solve this linear system:
$2x + 5y = 8$ ①
$4x + 7y = 10$ ②
Multiply equation ① by 2.
$2(2x + 5y = 8)$
$4x + 10y = 16$ ③
Subtract equation ② from equation ③ to eliminate x.

$$\begin{aligned} 4x + 10y &= 16 \quad ③ \\ \underline{-(4x + 7y = 10)} & \quad ② \\ 10y - 7y &= 16 - 10 \\ 3y &= 6 \\ y &= 2 \end{aligned}$$

Substitute $y = 2$ in equation ①.

$$\begin{aligned} 2x + 5y &= 8 \quad ① \\ 2x + 5(2) &= 8 \\ 2x + 10 &= 8 \\ 2x &= -2 \\ x &= -1 \end{aligned}$$

Verify the solution.
In each original equation, substitute: $x = -1$ and $y = 2$

$2x + 5y = 8$ ①	$4x + 7y = 10$ ②
L.S. $= 2x + 5y$	L.S. $= 4x + 7y$
$= 2(-1) + 5(2)$	$= 4(-1) + 7(2)$
$= -2 + 10$	$= -4 + 14$
$= 8$	$= 10$
= R.S.	= R.S.

Since the left side is equal to the right side for each equation, the solution is correct:
$x = -1$ and $y = 2$

b) Another linear system is:
$5x + 8y = 11$ ①
$14x + 17y = 20$ ②
To solve this linear system:
Multiply equation ① by 14 and equation ② by 5.

$14(5x + 8y = 11)$	$5(14x + 17y = 20)$
$70x + 112y = 154$ ③	$70x + 85y = 100$ ④

Subtract equation ④ from equation ③ to eliminate x.

$$\begin{aligned} 70x + 112y &= 154 \quad ③ \\ \underline{-(70x + 85y = 100)} & \quad ④ \\ 112y - 85y &= 154 - 100 \\ 27y &= 54 \\ y &= 2 \end{aligned}$$

Substitute $y = 2$ in equation ①.

$5x + 8y = 11$ ①
$5x + 8(2) = 11$
$5x + 16 = 11$
$5x = -5$
$x = -1$

Verify the solution.
In each original equation, substitute: $x = -1$ and $y = 2$

$5x + 8y = 11$ ①
L.S. $= 5x + 8y$
$= 5(-1) + 8(2)$
$= -5 + 16$
$= 11$
$=$ R.S.

$14x + 17y = 20$ ②
L.S. $= 14x + 17y$
$= 14(-1) + 17(2)$
$= -14 + 34$
$= 20$
$=$ R.S.

Since the left side is equal to the right side for each equation, the solution is correct:
$x = -1$ and $y = 2$

Another linear system is:
$-2x + y = 4$ ①
$-7x - 4y = -1$ ②
To solve this linear system:
Multiply equation ① by 4.
$4(-2x + y = 4)$
$-8x + 4y = 16$ ③
Add equations ② and ③ to eliminate y.

$-7x - 4y = -1$ ②
$\underline{+(-8x + 4y = 16)}$ ③
$-7x - 8x = -1 + 16$
$-15x = 15$
$x = -1$

Substitute $x = -1$ in equation ①.

$-2x + y = 4$ ①
$-2(-1) + y = 4$
$2 + y = 4$
$y = 2$

Verify the solution.
In each original equation, substitute: $x = -1$ and $y = 2$

$-2x + y = 4$ ①
L.S. $= -2x + y$
$= -2(-1) + 2$
$= 2 + 2$
$= 4$
$=$ R.S.

$-7x - 4y = -1$ ②
L.S. $= -7x - 4y$
$= -7(-1) - 4(2)$
$= 7 - 8$
$= -1$
$=$ R.S.

Since the left side is equal to the right side for each equation, the solution is correct:
$x = -1$ and $y = 2$

c) For each linear system, the solution is: $x = -1$ and $y = 2$

d) Suppose the coefficients differ by a constant d, no coefficient equals 0, and $a \neq b$. Then the linear system can be expressed as:

$ax + (a + d)y = a + 2d$ ①

$bx + (b + d)y = b + 2d$ ②

Multiply equation ① by b and equation ② by a.

$b(ax + (a + d)y = a + 2d)$ $\qquad$ $a(bx + (b + d)y = b + 2d)$

$bax + b(a + d)y = ba + 2bd$ ③ $\qquad$ $abx + a(b + d)y = ab + 2ad$ ④

Subtract equation ④ from equation ③ to eliminate x.

$$\begin{aligned}
bax + b(a + d)y &= ba + 2bd && ③\\
\underline{-[abx + a(b + d)y = ab + 2ad]} & && ④\\
b(a + d)y - a(b + d)y &= ba + 2bd - (ab + 2ad) && \text{Expand and simplify.}\\
aby + bdy - aby - ady &= ba + 2bd - ab - 2ad\\
bdy - ady &= 2bd - 2ad && \text{Factor each side.}\\
dy(b - a) &= 2d(b - a) && \text{Divide each side by } d(b - a).\\
y &= 2
\end{aligned}$$

Substitute $y = 2$ in equation ①.

$$\begin{aligned}
ax + (a + d)y &= a + 2d && ①\\
ax + (a + d)2 &= a + 2d && \text{Simplify.}\\
ax + 2a + 2d &= a + 2d\\
ax &= -a\\
x &= -1
\end{aligned}$$

Verify the solution.

In each original equation, substitute: $x = -1$ and $y = 2$

$ax + (a + d)y = a + 2d$ ①

$$\begin{aligned}
\text{L.S.} &= ax + (a + d)y\\
&= a(-1) + (a + d)2\\
&= -a + 2a + 2d\\
&= a + 2d\\
&= \text{R.S.}
\end{aligned}$$

$bx + (b + d)y = b + 2d$ ②

$$\begin{aligned}
\text{L.S.} &= bx + (b + d)y\\
&= b(-1) + (b + d)2\\
&= -b + 2b + 2d\\
&= b + 2d\\
&= \text{R.S.}
\end{aligned}$$

Since the left side is equal to the right side for each equation, the solution is correct: $x = -1$ and $y = 2$

24. a) Write a linear system to model the situation.

Let the yield of wheat be represented by w bushels.
Let the yield of barley be represented by b bushels.
The total yield was 99 840 bushels.
So, one equation is: $w + b = 99\,840$
The sale of w bushels of wheat at \$6.35/bushel earns: $6.35w$ dollars
The sale of b bushels of barley at \$2.70/bushel earns: $2.70b$ dollars
The total money earned was \$363 008.
So, another equation is: $6.35w + 2.70b = 363\,008$
A linear system is:

$$\begin{aligned}
w + b &= 99\,840 && ①\\
6.35w + 2.70b &= 363\,008 && ②
\end{aligned}$$

To solve this linear system:
Multiply equation ① by 6.35.

$$\begin{aligned}
6.35(w + b &= 99\,840)\\
6.35w + 6.35b &= 633\,984 && ③
\end{aligned}$$

Subtract equation ② from equation ③ to eliminate w.

$$
\begin{aligned}
6.35w + 6.35b &= 633\,984 \quad ③\\
\underline{-(6.35w + 2.70b = 363\,008)} & \quad ②\\
6.35b - 2.70b &= 633\,984 - 363\,008\\
3.65b &= 270\,976\\
b &= 74\,240
\end{aligned}
$$

Substitute $b = 74\,240$ in equation ①.

$$
\begin{aligned}
w + b &= 99\,840 \quad ①\\
w + 74\,240 &= 99\,840\\
w &= 25\,600
\end{aligned}
$$

Verify the solution.
The total yield of grain, in bushels, is: 74 240 + 25 600 = 99 840; this agrees with the given information.
The total income from the sale of grain is: 25 600 × \$6.35 + 74 240 × \$2.70 = \$363 008; this agrees with the given information.
So, the solution is correct.
640 acres of wheat were harvested.

So, the yield in bushels per acre is: $\dfrac{25\,600}{640} = 40$

640 × 2, or 1280 acres of barley were harvested.

So, the yield in bushels per acre is: $\dfrac{74\,240}{1280} = 58$

There were 40 bushels/acre for wheat and 58 bushels/acre for barley.

b) No, I would not have to write then solve a different linear system.
1 acre is 0.4047 ha.
40 bushels per acre is the same as 40 bushels per 0.4047 ha.

In 1 ha, the yield is: $\dfrac{40}{0.4047} \doteq 98.8$

40 bushels/acre is approximately 98.8 bushels/ha.
Similarly, 58 bushels/acre is the same as 58 bushels per 0.4047 ha.

In 1 ha, the yield is: $\dfrac{58}{0.4047} \doteq 143.3$

58 bushels/acre is approximately 143.3 bushels/ha.

Checkpoint 2 **Assess Your Understanding (page 441)**

7.4

1. a) $5x + y = 4$ ①
$x + y = 2$ ②
Solve equation ② for y.
$x + y = 2$ ②
$y = 2 - x$
Substitute $y = 2 - x$ in equation ①.

$$\begin{aligned} 5x + y &= 4 \quad ① \\ 5x + (2 - x) &= 4 \\ 5x + 2 - x &= 4 \\ 4x &= 2 \\ x &= \frac{2}{4}, \text{ or } \frac{1}{2} \end{aligned}$$

Substitute $x = \frac{1}{2}$ in equation ②.

$$\begin{aligned} x + y &= 2 \quad ② \\ \frac{1}{2} + y &= 2 \\ y &= 2 - \frac{1}{2} \\ y &= \frac{3}{2} \end{aligned}$$

Verify the solution.
In each equation, substitute: $x = \frac{1}{2}$ and $y = \frac{3}{2}$

$5x + y = 4$ ①

$$\begin{aligned} \text{L.S.} &= 5x + y \\ &= 5\left(\frac{1}{2}\right) + \frac{3}{2} \\ &= \frac{5}{2} + \frac{3}{2} \\ &= \frac{8}{2} \\ &= 4 \\ &= \text{R.S.} \end{aligned}$$

$x + y = 2$ ②

$$\begin{aligned} \text{L.S.} &= x + y \\ &= \frac{1}{2} + \frac{3}{2} \\ &= \frac{4}{2} \\ &= 2 \\ &= \text{R.S.} \end{aligned}$$

For each equation, the left side is equal to the right side, so the solution is:
$x = \frac{1}{2}$ and $y = \frac{3}{2}$

b) $3x - y = 1$ ①
$2x + y = -1$ ②
Solve equation ② for y.
$2x + y = -1$ ②
$y = -1 - 2x$
Substitute $y = -1 - 2x$ in equation ①.

$$\begin{aligned} 3x - y &= 1 \quad ① \\ 3x - (-1 - 2x) &= 1 \\ 3x + 1 + 2x &= 1 \\ 5x &= 0 \\ x &= 0 \end{aligned}$$

Substitute $x = 0$ in equation ②.

$$\begin{aligned} 2x + y &= -1 \quad ② \\ 2(0) + y &= -1 \\ y &= -1 \end{aligned}$$

Verify the solution.
In each equation, substitute: $x = 0$ and $y = -1$

$3x - y = 1$ ①

$$\begin{aligned} \text{L.S.} &= 3x - y \\ &= 3(0) - (-1) \\ &= 1 \\ &= \text{R.S.} \end{aligned}$$

$2x + y = -1$ ②

$$\begin{aligned} \text{L.S.} &= 2x + y \\ &= 2(0) + (-1) \\ &= -1 \\ &= \text{R.S.} \end{aligned}$$

For each equation, the left side is equal to the right side, so the solution is:
$x = 0$ and $y = -1$

c) $\frac{x}{3} + \frac{y}{4} = -\frac{9}{4}$ ①

$\frac{5x}{6} - \frac{3y}{4} = -\frac{17}{4}$ ②

Write an equivalent system with integer coefficients.
For equation ①, the common denominator is the lowest common multiple of 3 and 4, which is 12:

$\frac{x}{3} + \frac{y}{4} = -\frac{9}{4}$ Multiply each term by 12.

$12\left(\frac{1}{3}x\right) + 12\left(\frac{1}{4}y\right) = 12\left(-\frac{9}{4}\right)$ Simplify.

$4x + 3y = -27$ ③

For equation ②, the common denominator is the lowest common multiple of 6 and 4, which is 12:

$\frac{5x}{6} - \frac{3y}{4} = -\frac{17}{4}$ Multiply each term by 12.

$12\left(\frac{5}{6}x\right) - 12\left(\frac{3}{4}y\right) = 12\left(-\frac{17}{4}\right)$ Simplify.

$10x - 9y = -51$ ④

Solve equation ③ for $3y$.

$$\begin{aligned} 4x + 3y &= -27 \quad ③ \\ 3y &= -27 - 4x \end{aligned}$$

Substitute for $3y$ in equation ④.

$10x - 9y = -51$ ④

$10x - 3(-27 - 4x) = -51$ Simplify, then solve for x.

$$\begin{aligned} 10x + 81 + 12x &= -51 \\ 22x &= -51 - 81 \\ 22x &= -132 \\ x &= -6 \end{aligned}$$

Substitute $x = -6$ into equation ③.

$$
\begin{aligned}
4x + 3y &= -27 && ③\\
4(-6) + 3y &= -27 && \text{Simplify, then solve for } y.\\
-24 + 3y &= -27\\
3y &= -27 + 24\\
3y &= -3\\
y &= -1
\end{aligned}
$$

Verify the solution.
In each original equation, substitute: $x = -6$ and $y = -1$

$$\frac{x}{3} + \frac{y}{4} = -\frac{9}{4} \quad ①$$

$$
\begin{aligned}
\text{L.S.} &= \frac{x}{3} + \frac{y}{4}\\
&= \frac{-6}{3} + \frac{-1}{4}\\
&= -2 - \frac{1}{4}\\
&= -\frac{9}{4}\\
&= \text{R.S.}
\end{aligned}
$$

$$\frac{5x}{6} - \frac{3y}{4} = -\frac{17}{4} \quad ②$$

$$
\begin{aligned}
\text{L.S.} &= \frac{5x}{6} - \frac{3y}{4}\\
&= \frac{5(-6)}{6} - \frac{3(-1)}{4}\\
&= -5 + \frac{3}{4}\\
&= -\frac{20}{4} + \frac{3}{4}\\
&= -\frac{17}{4}\\
&= \text{R.S.}
\end{aligned}
$$

For each equation, the left side is equal to the right side, so the solution is:
$x = -6$ and $y = -1$

2. a) Let a represent the number of Inukshuit with 6 stones.
Let b represent the number of Inukshuit with 7 stones.
The total number of stones in all Inukshuit sold was 494.
So, one equation is: $6a + 7b = 494$
13 more Inukshuit with 6 stones were sold than with 7 stones.
So, another equation is: $a = b + 13$
A linear system is:
$6a + 7b = 494$ ①
$a = b + 13$ ②

b) Solve this linear system:
$6a + 7b = 494$ ①
$a = b + 13$ ②
From equation ②, substitute $a = b + 13$ in equation ①.

$$
\begin{aligned}
6a + 7b &= 494 && ①\\
6(b + 13) + 7b &= 494\\
6b + 78 + 7b &= 494\\
13b &= 494 - 78\\
13b &= 416\\
b &= 32
\end{aligned}
$$

Substitute $b = 32$ into equation ②.
$a = b + 13$ ②
$a = 32 + 13$
$a = 45$
45 Inukshuit with 6 stones were sold and 32 Inukshuit with 7 stones were sold.

Verify the solution.
The total number of stones is: $45(6) + 32(7) = 494$; this agrees with the given information.
The difference in number of types sold is: $45 - 32 = 13$; this agrees with the given information.
So, the solution is correct.

3. Write a linear system to model the situation.
Let a dollars represent the amount invested in a bond that earns 5.5% interest.
Let b dollars represent the amount invested in a bond that earns 4.5% interest.
The total amount invested was \$1000.
So, one equation is: $a + b = 1000$
The interest in dollars earned at 5.5% was: $0.055a$
The interest in dollars earned at 4.5% was: $0.045b$
The total interest earned was \$50.
So, another equation is: $0.055a + 0.045b = 50$
A linear system is:
$a + b = 1000$ ①
$0.055a + 0.045b = 50$ ②
Solve equation ① for a.
$a + b = 1000$ ①
$a = 1000 - b$
Substitute $a = 1000 - b$ in equation ②.
$0.055a + 0.045b = 50$ ②
$0.055(1000 - b) + 0.045b = 50$
$55 - 0.055b + 0.045b = 50$
$-0.01b = -5$
$b = 500$
Substitute $b = 500$ in equation ①.
$a + b = 1000$ ①
$a + 500 = 1000$
$a = 500$
\$500 was invested in each bond.

Verify the solution.
The total amount invested was: \$500 + \$500 = \$1000; this agrees with the given information.
The total interest was: 0.055(\$500) + 0.045(\$500) = \$27.50 + \$22.50, or \$50; this agrees with the given information.
So, the solution is correct.

7.5

4. a) $3x - y = -11$ ①
$-x + y = -1$ ②
Since the y-terms are opposites, add the equations to eliminate y.

$$\begin{aligned} 3x - y &= -11 \quad ① \\ \underline{+(-x + y = -1)} & \quad ② \\ 3x - x &= -11 - 1 \\ 2x &= -12 \\ x &= -6 \end{aligned}$$

Substitute $x = -6$ into equation ②.

$$\begin{aligned} -x + y &= -1 \quad ② \\ -(-6) + y &= -1 \\ 6 + y &= -1 \\ y &= -7 \end{aligned}$$

Verify the solution.
In each equation, substitute: $x = -6$ and $y = -7$

$3x - y = -11$ ①

$$\begin{aligned} \text{L.S.} &= 3x - y \\ &= 3(-6) - (-7) \\ &= -18 + 7 \\ &= -11 \\ &= \text{R.S.} \end{aligned}$$

$-x + y = -1$ ②

$$\begin{aligned} \text{L.S.} &= -x + y \\ &= -(-6) + (-7) \\ &= 6 - 7 \\ &= -1 \\ &= \text{R.S.} \end{aligned}$$

For each equation, the left side is equal to the right side; so the solution is:
$x = -6$ and $y = -7$

b) $\frac{1}{3}x + \frac{5}{6}y = \frac{8}{3}$ ①

$\frac{1}{4}x - \frac{3}{4}y = -\frac{17}{8}$ ②

The lowest common denominator of the fractions in equation ① is 6, so multiply equation ① by 6.

$$6\left(\frac{1}{3}x + \frac{5}{6}y = \frac{8}{3}\right)$$

$$2x + 5y = 16 \quad ③$$

The lowest common denominator of the fractions in equation ② is 8, so multiply equation ② by 8.

$$8\left(\frac{1}{4}x - \frac{3}{4}y = -\frac{17}{8}\right)$$

$$2x - 6y = -17 \quad ④$$

In equation ③ and ④, the coefficients of x are equal.
Subtract equation ④ from equation ③ to eliminate x.

$$\begin{aligned} 2x + 5y &= 16 \quad ③ \\ \underline{-(2x - 6y = -17)} & \quad ④ \\ 5y + 6y &= 16 + 17 \\ 11y &= 33 \\ y &= 3 \end{aligned}$$

Substitute $y = 3$ in equation ③.

$$\begin{aligned} 2x + 5y &= 16 \quad ③ \\ 2x + 5(3) &= 16 \\ 2x + 15 &= 16 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

Verify the solution.

In each original equation, substitute: $x = \frac{1}{2}$ and $y = 3$

$$\frac{1}{3}x + \frac{5}{6}y = \frac{8}{3} \quad ①$$

$$\begin{aligned} \text{L.S.} &= \frac{1}{3}x + \frac{5}{6}y \\ &= \frac{1}{3}\left(\frac{1}{2}\right) + \frac{5}{6}(3) \\ &= \frac{1}{6} + \frac{15}{6} \\ &= \frac{16}{6} \\ &= \frac{8}{3} \\ &= \text{R.S.} \end{aligned}$$

$$\frac{1}{4}x - \frac{3}{4}y = -\frac{17}{8} \quad ②$$

$$\begin{aligned} \text{L.S.} &= \frac{1}{4}x - \frac{3}{4}y \\ &= \frac{1}{4}\left(\frac{1}{2}\right) - \frac{3}{4}(3) \\ &= \frac{1}{8} - \frac{9}{4} \\ &= \frac{1}{8} - \frac{18}{8} \\ &= -\frac{17}{8} \\ &= \text{R.S.} \end{aligned}$$

For each equation, the left side is equal to the right side; so the solution is:

$x = \frac{1}{2}$ and $y = 3$

c) $0.5x - 0.3y = 0.15$ ①

$-0.3x + 0.5y = -0.65$ ②

Multiply equation ① by 0.3 and equation ② by 0.5.

$0.3(0.5x - 0.3y = 0.15)$

$0.15x - 0.09y = 0.045$ ③

$0.5(-0.3x + 0.5y = -0.65)$

$-0.15x + 0.25y = -0.325$ ④

Add equations ③ and ④ to eliminate x.

$$\begin{aligned} 0.15x - 0.09y &= 0.045 \quad ③ \\ +(-0.15x + 0.25y &= -0.325) \quad ④ \\ \hline -0.09y + 0.25y &= 0.045 - 0.325 \\ 0.16y &= -0.28 \\ y &= \frac{-0.28}{0.16} \\ y &= -1.75 \end{aligned}$$

Substitute $y = -1.75$ in equation ①.

$$\begin{aligned} 0.5x - 0.3y &= 0.15 \quad ① \\ 0.5x - 0.3(-1.75) &= 0.15 \\ 0.5x + 0.525 &= 0.15 \\ 0.5x &= 0.15 - 0.525 \\ 0.5x &= -0.375 \end{aligned}$$

$x = \dfrac{-0.375}{0.5}$

$x = -0.75$

Verify the solution.
In each original equation, substitute: $x = -0.75$ and $y = -1.75$

$0.5x - 0.3y = 0.15$ ①

L.S. $= 0.5x - 0.3y$
$= 0.5(-0.75) - 0.3(-1.75)$
$= -0.375 + 0.525$
$= 0.15$
= R.S.

$-0.3x + 0.5y = -0.65$ ②

L.S. $= -0.3x + 0.5y$
$= -0.3(-0.75) + 0.5(-1.75)$
$= 0.225 - 0.875$
$= -0.65$
= R.S.

For each equation, the left side is equal to the right side; so the solution is:
$x = -0.75$ and $y = -1.75$

d) $x + 2y = -2$ ①
$-2x + y = 6$ ②
Multiply equation ① by 2 so the x-terms are opposites.
$2(x + 2y = -2)$
$2x + 4y = -4$ ③
Add equations ② and ③ to eliminate x.
$-2x + y = 6$ ②
$\underline{+(2x + 4y = -4)}$ ③
$y + 4y = 6 - 4$
$5y = 2$
$y = \dfrac{2}{5}$
Multiply equation ② by 2 so the y-terms are equal.
$2(-2x + y = 6)$
$-4x + 2y = 12$ ④
Subtract equation ④ from equation ① to eliminate y.
$x + 2y = -2$ ①
$\underline{-(-4x + 2y = 12)}$ ④
$x + 4x = -2 - 12$
$5x = -14$
$x = -\dfrac{14}{5}$

Verify the solution.
In each equation, substitute: $x = -\dfrac{14}{5}$ and $y = \dfrac{2}{5}$

$x + 2y = -2$ ①

L.S. $= x + 2y$
$= -\dfrac{14}{5} + 2\left(\dfrac{2}{5}\right)$
$= -\dfrac{14}{5} + \dfrac{4}{5}$

$-2x + y = 6$ ②

L.S. $= -2x + y$
$= -2\left(-\dfrac{14}{5}\right) + \dfrac{2}{5}$
$= \dfrac{28}{5} + \dfrac{2}{5}$

$= -\frac{10}{5}$ $\quad\quad$ $= \frac{30}{5}$

$= -2$ $\quad\quad$ $= 6$

= R.S. $\quad\quad$ = R.S.

For each equation, the left side is equal to the right side; so the solution is:

$x = -\frac{14}{5}$ and $y = \frac{2}{5}$

5. Write a linear system to model the situation.
Let s represent the number of times Trish bought soup.
Let c represent the number of times Trish bought a main course.
Trish bought 160 food items.
So, one equation is: $s + c = 160$
The amount in dollars that Trish spent on soup is: $1.75s$
The amount in dollars that Trish spent on a main course is: $4.75c$
Trish spent a total of \$490.
So, another equation is: $1.75s + 4.75c = 490$
A linear system is:
$s + c = 160$ ①
$1.75s + 4.75c = 490$ ②
Multiply equation ① by 1.75.
$1.75(s + c = 160)$
$1.75s + 1.75c = 280$ ③
Subtract equation ③ from equation ② to eliminate s.

$$\begin{aligned} 1.75s + 4.75c &= 490 & ② \\ \underline{-(1.75s + 1.75c = 280)} & & ③ \\ 4.75c - 1.75c &= 490 - 280 & ③ \\ 3c &= 210 \\ c &= 70 \end{aligned}$$

Substitute $c = 70$ in equation ①.

$$\begin{aligned} s + c &= 160 & ① \\ s + 70 &= 160 \\ s &= 90 \end{aligned}$$

Trish bought soup 90 times and a main course 70 times.

Verify the solution.
The total number of meals Trish bought is: 90 + 70 = 160; this agrees with the given information.
The cost of the food Trish bought is: 90(\$1.75) + 70(\$4.75) = \$157.50 + \$332.50, or \$490.00; this agrees with the given information.
So, the solution is correct.

6. Write a linear system to model the situation.
Let l millilitres represent the larger volume.
Let s millilitres represent the smaller volume.
The difference in volumes is 1000 mL.
So, one equation is: $l - s = 1000$
The amount of acid, in millilitres, in the larger volume is: $0.055l$
The amount of acid, in millilitres, in the smaller volume is: $0.045s$
The total amount of acid is 100 mL.

So, another equation is: $0.055l + 0.045s = 100$
A linear system is:
$l - s = 1000$ ①
$0.055l + 0.045s = 100$ ②
Solve the linear system.
Multiply equation ① by 0.045.
$0.045(l - s = 1000)$
$0.045l - 0.045s = 45$ ③
Add equations ② and ③ to eliminate s.
$0.055l + 0.045s = 100$ ②
$\underline{+(0.045l - 0.045s = 45)}$ ③
$0.055l + 0.045l = 100 + 45$
$0.1l = 145$
$l = 1450$
Substitute $l = 1450$ in equation ①.
$l - s = 1000$ ①
$1450 - s = 1000$
$-s = 1000 - 1450$
$s = 450$
The larger volume of acid solution was 1450 mL and the smaller volume of the solution was 450 mL.

Verify the solution.
The difference in volumes is: 1450 mL – 450 mL = 1000 mL; this agrees with the given information.
The total amount of acid is: 0.055(1450) + 0.045(450 mL) = 79.75 mL + 20.25 mL, or 100 mL; this agrees with the given information.
So, the solution is correct.

7. Write a linear system to model the situation.
The sum of the angles in a triangle is 180°.
So, one equation is: $x + y + 60 = 180$, or $x + y = 120$
The sum of the angles in a quadrilateral is 360°.
So, another equation is: $2x + y + 95 + 90 = 360$, or $2x + y = 175$
A linear system is:
$x + y = 120$ ①
$2x + y = 175$ ②
The y-coefficients are the same, so subtract equation ① from equation ② to eliminate y.
$2x + y = 175$ ②
$\underline{-(x + y = 120)}$ ①
$2x - x = 175 - 120$
$x = 55$
Substitute $x = 55$ in equation ①.
$x + y = 120$ ①
$55 + y = 120$
$y = 65$
In the diagram, $x = 55$ and $y = 65$

Verify the solution.
The sum of the angles in the triangle is: 55° + 65° + 60° = 180°; this agrees with the given information.

The sum of the angles in the quadrilateral is: $2(55°) + 65° + 95° + 90° = 360°$; this agrees with the given information.
So, the solution is correct.

Lesson 7.6 Properties of Systems of Linear Equations Exercises (pages 448-449)

A

4. a) Write each equation in slope-intercept form to identify the slope.

i) $-x + y = 5$
$y = x + 5$
The coefficient of x is 1, so the slope is 1.

ii) $-x - y = 10$
$-y = x + 10$
$y = -x - 10$
The coefficient of x is –1, so the slope is –1.

iii) $-2x + 2y = 10$
$2y = 2x + 10$
$y = x + 5$
The coefficient of x is 1, so the slope is 1.

iv) $x + y = 5$
$y = -x + 5$
The coefficient of x is –1, so the slope is –1.

b) Parallel lines have the same slope. These lines are parallel:
$-x + y = 5$ and $-2x + 2y = 10$
$-x - y = 10$ and $x + y = 5$

c) Lines that are not parallel intersect. These lines intersect:
$-x + y = 5$ and $-x - y = 10$
$-x + y = 5$ and $x + y = 5$
$-x - y = 10$ and $-2x + 2y = 10$
$-2x + 2y = 10$ and $x + y = 5$

5. a) Two lines form a linear system with exactly one solution when they intersect in exactly one point. These are: lines A and C; and lines B and C

b) Two lines form a linear system with no solution when they are parallel. These are lines A and B.

6. Write each equation in slope-intercept form to identify the slope and y-intercept.
$4x + 2y = 20$
$2y = -4x + 20$ Divide each side by 2.
$y = -2x + 10$ The slope is –2 and the y-intercept is 10.

$x - 3y = 12$
$-3y = -x + 12$ Divide each side by –3.
$y = \frac{1}{3}x - 4$ The slope is $\frac{1}{3}$ and the y-intercept is –4.

$5x - 15y = -60$
$-15y = -5x - 60$ Divide each side by –15.
$y = \frac{1}{3}x + 4$ The slope is $\frac{1}{3}$ and the y-intercept is 4.

$2x + y = 10$
$y = -2x + 10$ The slope is –2 and the y-intercept is 10.

$6x + 3y = 5$
$3y = -6x + 5$ Divide each side by 3.
$y = -2x + \frac{5}{3}$ The slope is –2 and the y-intercept is $\frac{5}{3}$.

$2x - 6y = 24$
$-6y = -2x + 24$ Divide each side by –6.
$y = \frac{1}{3}x - 4$ The slope is $\frac{1}{3}$ and the y-intercept is –4.

a) A linear system with no solution has equations that represent parallel lines; that is, their slopes are equal but their y-intercepts are different. Here are these linear systems:
$4x + 2y = 20$ and $6x + 3y = 5$
$2x + y = 10$ and $6x + 3y = 5$
$x - 3y = 12$ and $5x - 15y = -60$
$5x - 15y = -60$ and $2x - 6y = 24$

b) A linear system with exactly one solution has equations that do not represent parallel lines or coincident lines; that is, their slopes are different. Here are these linear systems:
$4x + 2y = 20$ and $x - 3y = 12$
$4x + 2y = 20$ and $5x - 15y = -60$
$4x + 2y = 20$ and $2x - 6y = 24$
$2x + y = 10$ and $x - 3y = 12$
$2x + y = 10$ and $5x - 15y = -60$
$2x + y = 10$ and $2x - 6y = 24$
$6x + 3y = 5$ and $x - 3y = 12$
$6x + 3y = 5$ and $5x - 15y = -60$
$6x + 3y = 5$ and $2x - 6y = 24$

c) A linear system with an infinite number of solutions has equations with lines that are coincident; that is, their slopes are equal and their y-intercepts are equal. Here are these linear systems:
$4x + 2y = 20$ and $2x + y = 10$
$x - 3y = 12$ and $2x - 6y = 24$

B

7. Write each equation in slope-intercept form to identify the slope and y-intercept.

a) First equation: $x + 2y = 6$
$2y = -x + 6$ Divide each side by 2.

$y = -\frac{1}{2}x + 3$ The slope is $-\frac{1}{2}$ and the y-intercept is 3.

Second equation: $x + y = -2$

$y = -x - 2$ The slope is –1 and the y-intercept is –2.

The slopes are different, so the lines intersect at one point, and there is exactly one solution.

b) First equation: $3x + 5y = 9$

$5y = -3x + 9$ Divide each side by 5.

$y = -\frac{3}{5}x + \frac{9}{5}$ The slope is $-\frac{3}{5}$ and the y-intercept is $\frac{9}{5}$.

Second equation: $6x + 10y = 18$

$10y = -6x + 18$ Divide each side by 10.

$y = -\frac{6}{10}x + \frac{18}{10}$ Simplify.

$y = -\frac{3}{5}x + \frac{9}{5}$ The slope is $-\frac{3}{5}$ and the y-intercept is $\frac{9}{5}$.

The slopes are equal and the y-intercepts are equal, so the lines are coincident and the linear system has infinite solutions.

c) First equation: $2x - 5y = 30$

$-5y = -2x + 30$ Divide each side by –5.

$y = \frac{2}{5}x - 6$ The slope is $\frac{2}{5}$ and the y-intercept is –6.

Second equation: $4x - 10y = 15$

$-10y = -4x + 15$ Divide each side by –10.

$y = \frac{4}{10}x + \frac{15}{-10}$ Simplify.

$y = \frac{2}{5}x - \frac{3}{2}$ The slope is $\frac{2}{5}$ and the y-intercept is $-\frac{3}{2}$.

The slopes are equal but the y-intercepts are different, so the lines are parallel and there is no solution.

d) First equation: $\frac{x}{2} + \frac{y}{3} = \frac{1}{2}$

$\frac{y}{3} = -\frac{x}{2} + \frac{1}{2}$ Multiply each side by 3.

$y = -\frac{3}{2}x + \frac{3}{2}$ The slope is $-\frac{3}{2}$ and the y-intercept is $\frac{3}{2}$.

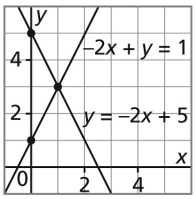
Second equation: $\frac{x}{2} + \frac{y}{3} = \frac{1}{4}$

$\frac{y}{3} = -\frac{x}{2} + \frac{1}{4}$ Multiply each side by 3.

$y = -\frac{3}{2}x + \frac{3}{4}$ The slope is $-\frac{3}{2}$ and the y-intercept is $\frac{3}{4}$.

The slopes are equal but the y-intercepts are different, so the lines are parallel and there is no solution.

8. a) One equation of a linear system is: $-2x + y = 1$
Solve for y.
$-2x + y = 1$
$y = 2x + 1$
This line has slope 2 and y-intercept 1.
Sketch a graph of this line in the first quadrant.
For the second line, choose a y-intercept such as 5.
Choose a slope that is different from the slope of the first line, such as –2.
Draw the second line on the grid.



Use the slope-intercept form to write the equation of the second line as: $y = -2x + 5$
A linear system is:
$-2x + y = 1$
$y = -2x + 5$

b) One equation of a linear system is: $-2x + y = 1$
From part a, it can be written as: $y = 2x + 1$
This line has slope 2 and y-intercept 1.
The second line does not intersect this line, so it has the same slope but different y-intercept.
Let the y-intercept be –3; the slope is 2.
Use the slope-intercept form to write the equation of the second line as: $y = 2x - 3$

A linear system is:
$-2x + y = 1$
$y = 2x - 3$

c) One equation of a linear system is: $-2x + y = 1$
To determine a coincident line, multiply the equation by a constant, such as 3.
$3(-2x + y = 1)$
$-6x + 3y = 3$
A linear system is:
$-2x + y = 1$
$-6x + 3y = 3$

9. a) Line A has slope –0.5 and y-intercept 4.
Line B has slope –0.5 and y-intercept 2.
Lines A and B have the same slope but different y-intercepts, so the lines are parallel and the linear system has no solution.

b) Line A has slope –0.5 and y-intercept 4.
Line C has slope 0.5 and y-intercept 4.

Lines A and C have different slopes, so the lines intersect at exactly one point and the linear system has exactly one solution.

c) Line B has slope –0.5 and y-intercept 2.
Line C has slope 0.5 and y-intercept 4.
Lines B and C have different slopes, so the lines intersect at exactly one point and the linear system has exactly one solution.

10. When the slopes of two lines are different, the lines intersect in exactly one point, so the linear system has exactly one solution.

11. When two lines have the same slope, they could be parallel or coincident.
I need to know the y-intercepts. If the y-intercepts are different, the lines are parallel and the linear system has not solution. If the y-intercepts are equal, then the lines are coincident and the linear system has infinite solutions.

12. One equation is: $3x - 4y = 12$
Write this equation in slope-intercept form.
$3x - 4y = 12$
$-4y = -3x + 12$ Divide each side by –4.
$y = \frac{3}{4}x - 3$ The slope is $\frac{3}{4}$ and the y-intercept is –3.
A linear system with exactly one solution will have a second equation with a different slope, such as 3; and may have a different y-intercept, such as 5.
The second equation could be: $y = 3x + 5$

A linear system with no solution will have a second equation with the same slope of $\frac{3}{4}$, and different y-intercept, such as 4.
The second equation could be: $y = \frac{3}{4}x + 4$

A linear system with infinite solutions will have a second equation that has the same slope and the same y-intercept. Multiply the first equation by a constant, such as 2.
$2(3x - 4y = 12)$
$6x - 8y = 24$
The second equation could be: $6x - 8y = 24$

13. Write a linear system that is modelled by the balance scales.
Let the mass of the large container be represented by x kilograms.
Let the mass of the small container be represented by y kilograms.
For the first balance scales:
$2x + y = 3$ ①
For the second balance scales:
$6x + 3y = 9$ Simplify; divide each term by 3.
$2x + y = 3$ ②
The two equations are equivalent, so their graphs are coincident and the linear system has infinite solutions.
So, the problem has infinite solutions.

14. Write a linear system to model the situation.
Let n represent the number of nickels.
Let d represent the number of dimes.
The total number of coins is 300.
So, one equation is: $n + d = 300$
Write this equation in slope-intercept form.
$d = -n + 300$ ①
The value of the coins is \$23.25.
So, another equation is: $0.05n + 0.10d = 23.25$
Write this equation in slope-intercept form.
$0.10d = -0.05n + 23.25$ Divide each side by 0.10.
$d = -0.5n + 232.5$ ②
The slopes of the graphs of the lines are different, so the linear system has one solution.
So, the problem has one solution.

15. Write a linear system to model the situation.
Let s dollars represent the amount of money in the savings account.
Let c dollars represent the amount of money in the chequing account.
The total amount of money is \$85.
So, one equation is: $s + c = 85$
Write this equation in slope-intercept form.

$c = -s + 85$ ①
The amount of money in each account is doubled and the total is then \$170.
So, another equation is: $2s + 2c = 170$
Write this equation in slope-intercept form.

$2s + 2c = 170$
$2c = -2s + 170$ Divide each side by 2.
$c = -s + 85$ ②
The equations are equivalent, so the linear system has infinite solutions.
However, the number of solutions to the problem is not infinite, because the possible sums of money are not infinite. There are many solutions to the problem.

16. Write a linear system to model the situation.
Let a represent the number of people on Saturday.
Let b represent the number of people on Sunday.
The total number of people is 568.
So, one equation is: $a + b = 568$
Write this equation in slope-intercept form.
$b = -a + 568$ ①
There were 44 more people on Sunday than on Saturday.
So, another equation is: $b = a + 44$ ②
The slopes of the graphs of the lines are different, so the linear system has one solution.
So, the problem has one solution.

17. Write a linear system to model the situation.
Let a represent the number of adults who visited the museum.
Let s represent the number of students who visited the museum.
The total number of people is 75.
So, one equation is: $a + s = 75$

Write this equation in slope-intercept form.
$s = -a + 75$ ①
An adult's ticket cost \$5 and a student's ticket cost \$3. The total cost was \$275.
So, another equation is: $5a + 3s = 275$
Write this equation in slope-intercept form.
$3s = -5a + 275$ Divide each side by 3.
$s = -\frac{5}{3}a + \frac{275}{3}$ ②
The slopes of the graphs of the lines are different, so the linear system has one solution.
So, the problem has one solution.

18. For two lines to have 0 points of intersection, the lines are parallel, which means their slopes are equal and their y-intercepts are different.
For two lines to have 1 point of intersection, the lines are not parallel, which means that their slopes are different and their y-intercepts may or may not be equal. If the y-intercepts are equal, this point is the point of intersection of the lines.
For two lines to have infinite points of intersection, the lines are coincident, which means their slopes are equal and their y-intercepts are equal.

19. a) For a linear system to have infinite solutions, the graphs are coincident.
Write any equation, such as: $3x + 4y = 5$
Multiply the equation by a constant, such as –2.
$-2(3x + 4y = 5)$
$-6x - 8y = -10$
A linear system is:
$3x + 4y = 5$ ①
$-6x - 8y = -10$ ②

b) To solve this system by elimination, I would multiply equation ① by –2 to make the x-coefficients the same. But when I multiply, I make the y-coefficients the same and the constants the same. When I then subtract to eliminate the x-coefficients, I also eliminate the y-coefficients, and the constants, and I am left with $0 = 0$.

20. a) For a linear system with no solution, the graphs are parallel. The lines have the same slope.
Write any equation, such as: $y = 4x - 7$
Write another equation with the same slope but different y-intercept, such as: $y = 4x + 8$
A linear system is:
$y = 4x - 7$ ①
$y = 4x + 8$ ②

b) To solve this system by elimination, I subtract equation ② from equation ① to eliminate y. But, when I subtract, I also eliminate x, and I am left with $0 = -15$, which is not a true statement.

21. If the graphs intersect at a point that is not on the grid, then I cannot determine the coordinates of the point of intersection. But if I know or can determine the equations, I can calculate the coordinates.
The graphs may look parallel and I might think there is no solution, but if I know or can determine the equations, then I know if the lines are parallel.

Sometimes the slopes of the graphs are very close in value, and I cannot identify the coordinates of the point of intersection from the graph. But if I know or can determine the equations, I can calculate the coordinates.

C

22. a) i) $2x + 3y = 4$ ①
$4x + 6y = 8$ ②
Write each equation in slope-intercept form.
For equation ①:
$2x + 3y = 4$
$3y = -2x + 4$
$y = -\frac{2}{3}x + \frac{4}{3}$ The slope is $-\frac{2}{3}$ and the y-intercept is $\frac{4}{3}$.
For equation ②:
$4x + 6y = 8$
$6y = -4x + 8$
$y = -\frac{2}{3}x + \frac{4}{3}$ The slope is $-\frac{2}{3}$ and the y-intercept is $\frac{4}{3}$.
Since the slopes are equal and the y-intercepts are equal, the lines are coincident and the linear system has infinite solutions.

ii) $2x + 3y = 4$ ①
$4x + 6y = 7$ ②
Write each equation in slope-intercept form.
From part a) i), equation ① is:
$y = -\frac{2}{3}x + \frac{4}{3}$ The slope is $-\frac{2}{3}$ and the y-intercept is $\frac{4}{3}$.
For equation ②:
$4x + 6y = 7$
$6y = -4x + 7$
$y = -\frac{2}{3}x + \frac{7}{6}$ The slope is $-\frac{2}{3}$ and the y-intercept is $\frac{7}{6}$.
Since the slopes are equal and the y-intercepts are different, the lines are parallel and the linear system has no solution.

iii) $2x + 3y = 4$ ①
$4x + 5y = 8$ ②
Write each equation in slope-intercept form.
From part a) i), equation ① is:
$y = -\frac{2}{3}x + \frac{4}{3}$ The slope is $-\frac{2}{3}$ and the y-intercept is $\frac{4}{3}$.
For equation ②:
$4x + 5y = 8$
$5y = -4x + 8$
$y = -\frac{4}{5}x + \frac{8}{5}$ The slope is $-\frac{4}{5}$ and the y-intercept is $\frac{8}{5}$.
Since the slopes are different, the lines intersect and the linear system has one solution.

b) **i)** $2x + 3y = 4$ ①
$4x + 6y = 8$ ②
Each coefficient and constant term in equation ② is 2 times the corresponding coefficient and constant term in equation ①.
So, to determine if a linear system has infinite solutions, I check to see if the two equations are equivalent; that is, if, when I divide each pair of coefficients and constant terms, I get the same quotient; for example, for the linear system above,
$\frac{4}{2} = 2$; $\frac{6}{3} = 2$, and $\frac{8}{4} = 2$

ii) $2x + 3y = 4$ ①
$4x + 6y = 7$ ②
Each coefficient in equation ② is 2 times the corresponding coefficient in equation ①.
So, to determine if a linear system has no solution, I check to see if, when I divide each pair of coefficients, I get the same quotient; for example, for the linear system above, $\frac{4}{2} = 2$ and $\frac{6}{3} = 2$; there is no relation ship between the pair of constant terms.

iii) $2x + 3y = 4$ ①
$4x + 5y = 8$ ②
There is no relationship between the pairs of coefficients and the pair of constant terms; for example, for the linear system above, $\frac{4}{2} \neq \frac{5}{3} \neq \frac{8}{4}$. So, if no relationship exists, then the linear system has one solution.

23. Write each equation in slope-intercept form.

$$Ax + By = C \quad ①$$
$$By = -Ax + C$$
$$y = -\frac{A}{B}x + \frac{C}{B} \quad ③ \qquad \text{This line has slope } -\frac{A}{B}.$$
$$Dx + Ey = F \quad ②$$
$$Ey = -Dx + F$$
$$y = -\frac{D}{E}x + \frac{F}{E} \quad ④ \qquad \text{This line has slope } -\frac{D}{E}.$$

$AE - DB = 0$ can be written as:

$AE = DB$ Divide each side by E.

$A = \frac{DB}{E}$ Divide each side by B.

$\frac{A}{B} = \frac{D}{E}$

Since $\frac{A}{B} = \frac{D}{E}$, then $-\frac{A}{B} = -\frac{D}{E}$ and the slopes of the graphs of the equations are equal.
This means that the lines are parallel and the linear system does not have exactly one solution.

24. a) Write each equation in slope-intercept form.
For equation ①:

$\frac{1}{2}x + \frac{5}{3}y = 2$

$\frac{5}{3}y = -\frac{1}{2}x + 2$ Multiply each side by $\frac{3}{5}$.

$y = -\frac{3}{10}x + \frac{6}{5}$

For equation ②:

$kx + \frac{5}{2}y = 3$

$\frac{5}{2}y = -kx + 3$ Multiply each side by $\frac{2}{5}$.

$y = \frac{-2k}{5}x + \frac{6}{5}$

i) For the linear system to have exactly one solution, the slopes must be different.
That is,

$\frac{-2k}{5} \neq -\frac{3}{10}$ Multiply each side by 10.

$-4k \neq -3$ Divide each side by –4.

$k \neq \frac{3}{4}$

For exactly one solution, k can be any real number except $\frac{3}{4}$.

ii) Since the y-intercepts are equal, for the linear system to have infinite solutions, the slopes must also be equal.
That is,

$\frac{-2k}{5} = -\frac{3}{10}$ Multiply each side by 10.

$-4k = -3$ Divide each side by –4.

$k = \frac{3}{4}$

For infinite solutions, $k = \frac{3}{4}$

b) For the system to have no solution, the lines must be parallel; that is, their slopes are equal and their y-intercepts are different. But the lines have the same y-intercept, so they cannot be parallel.

Review (pages 452-454)

7.1

1. a) Let s represent the number of times that teams from Saskatchewan have won the BRIT.
Let o represent the number of times that teams from outside Saskatchewan have won the BRIT.
There have been 41 tournaments.
So, one equation is: $s + o = 41$
Teams from outside Saskatchewan have won the BRIT 17 more times than teams from Saskatchewan.
So, another equation is: $o - s = 17$
A linear system is:
$s + o = 41$
$o - s = 17$

b) Solution A is incorrect because it states that teams from Saskatchewan have won the BRIT more times than team from outside Saskatchewan; this disagrees with the given information.
Solution B is correct.
The difference in numbers of times (outside the province – inside the province) is:
$29 - 12 = 17$; this agrees with the given information.
The total number of tournaments is: $12 + 29 = 41$; this agrees with the given information.

2. a) Let s represent the number of small driveways cleared.
Let l represent the number of large driveways cleared.
The cost to clear a small driveway is \$15 and the cost to clear a large driveway is \$25.
The total cost is \$475.
So, one equation is: $15s + 25l = 475$
25 driveways were cleared.
So, another equation is: $s + l = 25$
A linear system is:
$15s + 25l = 475$
$s + l = 25$

b) If solution A is correct, then:
The total number of driveways cleared is: $10 + 15 = 25$; this agrees with the given information.
The total cost is: 10(\$15) + 15(\$25) = \$150 + \$375, or \$525; this disagrees with the given information, so solution A is not correct.
If solution B is correct, then:
The total number of driveways cleared is: $15 + 10 = 25$; this agrees with the given information.
The total cost is: 15(\$15) + 10(\$25) = \$225 + \$250, or \$475; this agrees with the given information, so solution B is correct.

3. At a theatre, a ticket costs more than a box of popcorn, so let \$9.95 be the cost of a ticket and let \$5.50 be the cost of a box of popcorn.
Then t represents the number of tickets and p represents the number of boxes of popcorn purchased.
The first equation represents the total cost of popcorn and tickets.
The second equation represents the difference in the number of tickets purchased and the number of boxes of popcorn purchased.

A problem could be:
A group of people went to a movie theatre and spent \$76.20 on tickets and popcorn.
A ticket costs \$9.95 and a box of popcorn costs \$5.50.
There were 3 more tickets purchased than boxes of popcorn.
How many tickets were purchased? How many boxes of popcorn were purchased?

7.2

4. a) To determine the equation of each line, use the form of a linear equation that involves the coordinates of two points on the line:

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$$

For the line that goes up to the right, the coordinates of two points are: (8, 5) and (–2, –1)
Substitute: $y_1=5$, $x_1=8$, $y_2=-1$, and $x_2=-2$

$$\frac{y-5}{x-8}=\frac{-1-5}{-2-8}$$

$$\frac{y-5}{x-8}=\frac{-6}{-10}$$

$$\frac{y-5}{x-8}=\frac{3}{5}$$ Multiply each side by $(x-8)$.

$$y-5=\frac{3}{5}(x-8)$$ Multiply each side by 5.

$5(y-5)=3(x-8)$
$5y-25=3x-24$
$-1=3x-5y$
$3x-5y=-1$ ①

For the line that goes down to the right, the coordinates of two points are: (2, 5) and (4, –1)
Substitute: $y_1=5$, $x_1=2$, $y_2=-1$, and $x_2=4$

$$\frac{y-5}{x-2}=\frac{-1-5}{4-2}$$

$$\frac{y-5}{x-2}=\frac{-6}{2}$$

$$\frac{y-5}{x-2}=-3$$ Multiply each side by $(x-2)$.

$y-5=-3(x-2)$ Remove brackets.
$y-5=-3x+6$
$3x+y=6+5$
$3x+y=11$ ②

A linear system is:
$3x-5y=-1$ ①
$3x+y=11$ ②

b) From the graph, the solution appears to be: $x=3$ and $y=2$

Verify the solution.
Substitute the coordinates of the solution in each equation.
$3x-5y=-1$ ① $3x+y=11$ ②

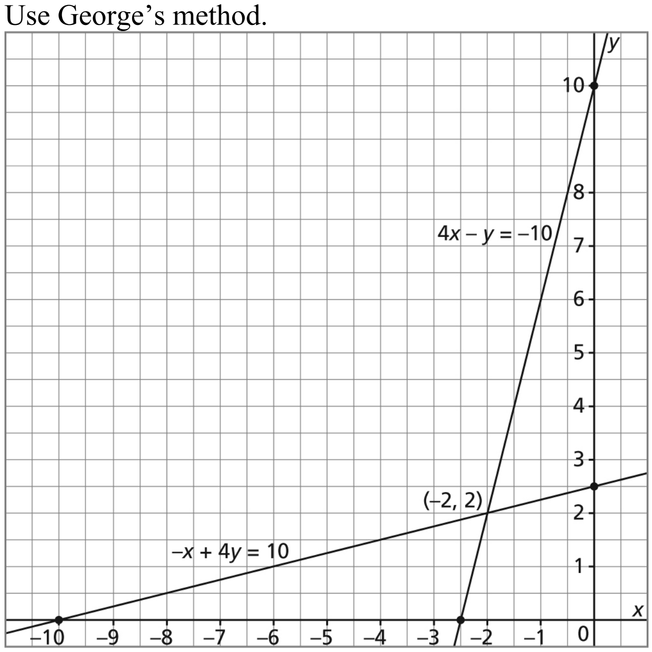
L.S. = $3x - 5y$
$= 3(3) - 5(2)$
$= 9 - 10$
$= -1$
= R.S.

L.S. = $3x + y$
$= 3(3) + 2$
$= 9 + 2$
$= 11$
= R.S.

For each equation, the left side is equal to the right side, so the solution is exact.

5. **a)** George plotted two points on each line, so he will draw a line through each pair of points, then identify the coordinates of the point where the lines intersect.
Sunita has each equation in slope-point form. For each line, she will mark a point at the y-intercept, then use the rise and run indicated by the slope to mark another point. Sunita will then draw a line through the points, and identify the coordinates of the point where the lines intersect.

b) Use George's method.



From the graph, the solution appears to be: $x = -2$ and $y = 2$

Verify the solution.
Substitute the coordinates of the solution in each equation.

$-x + 4y = 10$ ①
L.S. = $-x + 4y$
$= -(-2) + 4(2)$
$=2 + 8$
$= 10$
= R.S.

$4x - y = -10$ ②
L.S. = $4x - y$
$= 4(-2) - 2$
$= -8 - 2$
$= -10$
= R.S.

For each equation, the left side is equal to the right side, so the solution is exact.

6. $x - y = 15$
$2x + y = 6$

To graph $x - y = 15$:
I determine the intercepts as two points through which to draw the line.

When $x = 0$, $y = -15$
When $y = 0$, $x = 15$
I mark points at (0, –15) and (15, 0), then draw a line through them.

To graph $2x + y = 6$:
I determine the intercepts as two points through which to draw the line.
When $x = 0$, $y = 6$
When $y = 0$, $x = 3$
I mark points at (0, 6) and (3, 0), then draw a line through them.
I choose a scale on the grid so that the point of intersection is shown.
I identify the coordinates of this point, then verify the solution by substituting these coordinates in each equation to make sure the left side is equal to the right side.

7. **a)** $4x - 2y = 1$ ①
$3x - 4y = 16$ ②
To graph $4x - 2y = 1$:
Determine the intercepts as two points through which to draw the line.

When $x = 0$:
$$4(0) - 2y = 1$$
$$-2y = 1$$
$$y = -\frac{1}{2}$$

When $y = 0$:
$$4x - 2(0) = 1$$
$$4x = 1$$
$$x = \frac{1}{4}$$

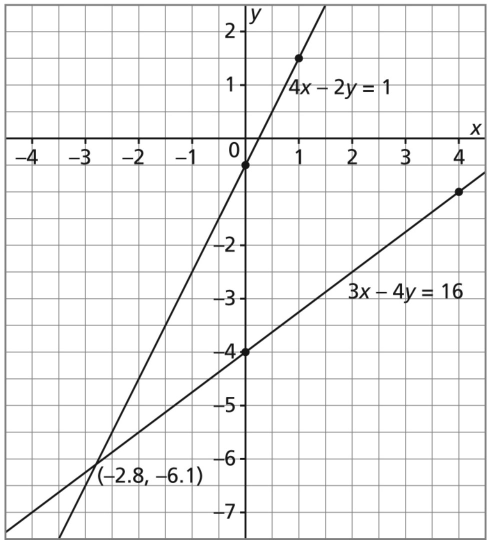
It may be difficult to plot a point at $\left(\frac{1}{4}, 0\right)$, so determine the coordinates of another point.
Substitute $x = 1$:
$$4(1) - 2y = 1$$
$$-2y = -3$$
$$y = \frac{3}{2}$$
On a grid, mark points at $\left(0, -\frac{1}{2}\right)$ and $\left(1, \frac{3}{2}\right)$, then draw a line through them.

To graph $3x - 4y = 16$:
The x-intercept will not be a whole number, so choose a value of x that is a multiple of 4, since the y-coefficient and the constant are multiples of 4.

Substitute $x = 4$:
$$3(4) - 4y = 16$$
$$-4y = 16 - 12$$
$$-4y = 4$$
$$y = -1$$

Substitute $x = 0$:
$$3(0) - 4y = 16$$
$$-4y = 16$$
$$y = -4$$

On a grid, mark points at (4, –1) and (0, –4), then draw a line through them.



From the graph, the point of intersection appears to be: (–2.8, –6.1)

b) Verify the solution. In each equation, substitute: $x = -2.8$ and $y = -6.1$

$4x - 2y = 1$ ①	$3x - 4y = 16$ ②
L.S. $= 4x - 2y$	L.S. $= 3x - 4y$
$= 4(-2.8) - 2(-6.1)$	$= 3(-2.8) - 4(-6.1)$
$= -11.2 + 12.2$	$= -8.4 + 24.4$
$= 1$	$= 16$
= R.S.	= R.S.

In each equation, the left side is equal to the right side, so the solution is exact.

7.3

8. a) Let c milligrams represent the mass of sodium in a bowl of cereal.
Let b milligrams represent the mass of sodium in a slice of bacon.
Owen’s breakfast of 2 bowls of cereal and 4 slices of bacon contained 940 mg of sodium.
So, one equation is: $2c + 4b = 940$
Natalie’s breakfast of 1 bowl of cereal and 3 slices of bacon contained 620 mg of sodium.
So, another equation is: $c + 3b = 620$
A linear system is:
$2c + 4b = 940$ ①
$c + 3b = 620$ ②

b) Each line represents one of the equations in the linear system.
Write each equation in slope-intercept form.
$2c + 4b = 940$ ①
$4b = -2c + 940$
$b = -\frac{1}{2}c + 235$ The b-intercept is 235.

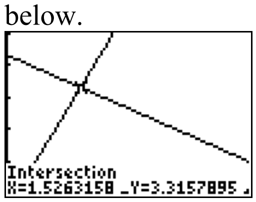
$c + 3b = 620$ ②
$3b = -c + 620$
$b = -\frac{1}{3}c + \frac{620}{3}$ The b-intercept is $206.\overline{6}$.

Equation ① has the greater *b*-intercept, so it is represented by the pink line.
Equation ② has the lesser *b*-intercept, so it is represented by the blue line.

c) From the graph, the solution is: (170, 150)
This means that 1 bowl of cereal contains 170 mg of sodium and 1 slice of bacon contains 150 mg.

Verify the solution.
The sodium in Owen's breakfast is: 2(170 mg) + 4(150 mg) = 940 mg; this agrees with the given information.
The sodium in Natalie's breakfast is: 170 mg + 3(150 mg) = 620 mg; this agrees with the given information.
So, the solution is exact.

9. a) $2x + 3y = 13$ ①
$5x - 2y = 1$ ②
Write each equation in $y = mx + b$ form.
For equation ①:
$2x + 3y = 13$
$3y = -2x + 13$ Divide each side by 3.
$y = -\frac{2}{3}x + \frac{13}{3}$
For equation ②:
$5x - 2y = 1$
$-2y = -5x + 1$ Divide each side by –2.
$y = \frac{5}{2}x - \frac{1}{2}$
On a TI-83 graphing calculator, press Y=, then next to Y1 = input the expression (–2/3)X+13/3. Move the cursor down to Y2= and input the expression (5/2)X–1/2. Press GRAPH. To see the point of intersection, set the WINDOW to Xmin = 0, Xmax = 5, Ymin = 0, and Ymax = 5. To show the coordinates of the point of intersection, press 2nd TRACE for CALC, then selected 5:intersect. Press ENTER 3 times to get the screen below.



From the calculator screen, the solution is: $x \doteq 1.526$ and $y \doteq 3.316$

Verify the solution.
Substitute $x = 1.526$ and $y = 3.316$ into each equation.

$2x + 3y = 13$ ①
L.S. $= 2x + 3y$
$= 2(1.526) + 3(3.316)$
$= 13$
$=$ R.S.

$5x - 2y = 1$ ②
L.S. $= 5x - 2y$ R.S. = 1
$= 5(1.526) - 2(3.316)$
$= 0.998$

The left side is equal to the right side in equation ①, but the left side is only approximately equal to the right side in equation ②, so the solution is approximate.

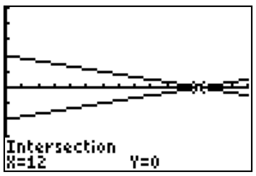
b) $y = \frac{1}{6}x - 2$ ①

$y = -\frac{1}{6}x + 2$ ②

Each equation is written in $y = mx + b$ form.

On a TI-83 graphing calculator, press $\boxed{\text{Y=}}$, then next to Y1 = input the expression (1/6)X–2. Move the cursor down to Y2= and input the expression (–1/6)X+2.

Press $\boxed{\text{GRAPH}}$. To see the point of intersection, set the $\boxed{\text{WINDOW}}$ to Xmin = 0, Xmax = 15, Ymin = –5, and Ymax = 5. To show the coordinates of the point of intersection, press $\boxed{\text{2nd}}\boxed{\text{TRACE}}$ for CALC, then selected 5:intersect. Press $\boxed{\text{ENTER}}$ 3 times to get the screen below.



From the calculator screen, the solution is: $x = 12$ and $y = 0$

Verify the solution.
Substitute $x = 12$ and $y = 0$ into each equation.

$y = \frac{1}{6}x - 2$ ①

L.S. $= y$ $= 0$

R.S. $= \frac{1}{6}x - 2$
$= \frac{1}{6}(12) - 2$
$= 2 - 2$
$= 0$

$y = -\frac{1}{6}x + 2$ ②

L.S. $= y$ $= 0$

R.S. $= -\frac{1}{6}x + 2$
$= -\frac{1}{6}(12) + 2$
$= -2 + 2$
$= 0$

In each equation, the left side is equal to the right side, so the solution is correct.

c) $4x - 5y = 20$ ①
$8x + 5y = 19$ ②

Write each equation in $y = mx + b$ form.

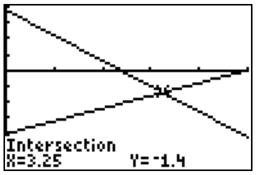
For equation ①:
$4x - 5y = 20$
$-5y = -4x + 20$ Divide each side by –5.
$y = \frac{4}{5}x - 4$

For equation ②:
$8x + 5y = 19$
$5y = -8x + 19$ Divide each side by 5.
$y = -\frac{8}{5}x + \frac{19}{5}$

On a TI-83 graphing calculator, press $\boxed{\text{Y=}}$, then next to Y1 = input the expression (4/5)X–4. Move the cursor down to Y2= and input the expression (–8/5)X+19/5.

Press $\boxed{\text{GRAPH}}$. To see the point of intersection, set the $\boxed{\text{WINDOW}}$ to Xmin = 0, Xmax = 5, Ymin = –6 , and Ymax = 4. To show the coordinates of the point of intersection, press

2nd TRACE for CALC, then selected 5:intersect. Press ENTER 3 times to get the screen below.



From the calculator screen, the solution is: $x = 3.25$ and $y = -1.4$

Verify the solution.
Substitute $x = 3.25$ and $y = -1.4$ into each equation.

$4x - 5y = 20$ ①
L.S. $= 4x - 5y$
$= 4(3.25) - 5(-1.4)$
$= 20$
$=$ R.S.

$8x + 5y = 19$ ②
L.S. $= 8x + 5y$
$= 8(3.25) + 5(-1.4)$
$= 19$
$=$ R.S.

In each equation, the left side is equal to the right side, so the solution is correct.

d) $\frac{x}{2} + \frac{3y}{4} = -\frac{25}{16}$ ①
$-2x + 4y = 20$ ②
Write each equation in $y = mx + b$ form.
For equation ①:

$$\frac{x}{2} + \frac{3y}{4} = -\frac{25}{16}$$

$$\frac{3y}{4} = -\frac{x}{2} - \frac{25}{16}$$ Multiply each side by $\frac{4}{3}$.

$$\frac{4}{3}\left(\frac{3y}{4}\right) = \frac{4}{3}\left(-\frac{x}{2}\right) - \frac{4}{3}\left(\frac{25}{16}\right)$$

$$y = -\frac{2}{3}x - \frac{25}{12}$$

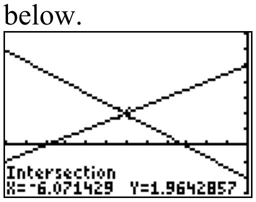
For equation ②:

$$-2x + 4y = 20$$

$$4y = 2x + 20$$ Divide each side by 4.

$$y = \frac{1}{2}x + 5$$

On a TI-83 graphing calculator, press Y=, then next to Y1 = input the expression (–2/3)X–25/12. Move the cursor down to Y2= and input the expression (1/2)X+5.
Press GRAPH. To see the point of intersection, set the WINDOW to Xmin = –12 , Xmax = 0, Ymin = –3, and Ymax = 7. To show the coordinates of the point of intersection, press 2nd TRACE for CALC, then selected 5:intersect. Press ENTER 3 times to get the screen below.



From the calculator screen, the solution is: $x \doteq -6.071$ and $y \doteq 1.964$

Verify the solution.
Substitute $x = -6.071$ and $y = 1.964$ into each equation.

$\frac{x}{2} + \frac{3y}{4} = -\frac{25}{16}$ ① $-2x + 4y = 20$ ②

$$\begin{aligned} \text{L.S.} &= \frac{x}{2} + \frac{3y}{4} \\ &= \frac{-6.071}{2} + \frac{3(1.964)}{4} \\ &= -1.5625 \\ &= \text{R.S.} \end{aligned}$$

$$\begin{aligned} \text{L.S.} &= -2x + 4y \\ &= -2(-6.071) + 4(1.964) \\ &= 19.998 \end{aligned}$$

$\text{R.S.} = 20$

The left side is equal to the right side in equation ①, but the left side is only approximately equal to the right side in equation ②, so the solution is approximate.

7.4

10. a) $x + y = -5$ ①
$x + 3y = -15$ ②
Solve equation ① for x.

$$\begin{aligned} x + y &= -5 \quad ① \\ x &= -5 - y \end{aligned}$$

Substitute $x = -5 - y$ in equation ②.

$$\begin{aligned} x + 3y &= -15 \quad ② \\ -5 - y + 3y &= -15 \\ 2y &= -10 \\ y &= -5 \end{aligned}$$

Substitute $y = -5$ in equation ①.

$$\begin{aligned} x + y &= -5 \quad ① \\ x + (-5) &= -5 \\ x &= 0 \end{aligned}$$

Verify the solution.
In each equation, substitute: $x = 0$ and $y = -5$

$x + y = -5$ ①

$$\begin{aligned} \text{L.S.} &= x + y \\ &= 0 + (-5) \\ &= -5 \\ &= \text{R.S.} \end{aligned}$$

$x + 3y = -15$ ②

$$\begin{aligned} \text{L.S.} &= x + 3y \\ &= 0 + 3(-5) \\ &= -15 \\ &= \text{R.S.} \end{aligned}$$

For each equation, the left side is equal to the right side, so the solution is:
$x = 0$ and $y = -5$

b) $7x + y = 10$ ①
$3x - 2y = -3$ ②
Solve equation ① for y.

$$\begin{aligned} 7x + y &= 10 \quad ① \\ y &= -7x + 10 \end{aligned}$$

Substitute $y = -7x + 10$ in equation ②.

$$\begin{aligned} 3x - 2y &= -3 \quad ② \\ 3x - 2(-7x + 10) &= -3 \\ 3x + 14x - 20 &= -3 \\ 17x &= 17 \end{aligned}$$

$x = 1$

Substitute $x = 1$ in equation ①.

$7x + y = 10$ ①
$7(1) + y = 10$
$7 + y = 10$
$y = 3$

Verify the solution.
In each equation, substitute: $x = 1$ and $y = 3$

$7x + y = 10$ ①
L.S. $= 7x + y$
$= 7(1) + 3$
$= 10$
= R.S.

$3x - 2y = -3$ ②
L.S. $= 3x - 2y$
$= 3(1) - 2(3)$
$= 3 - 6$
$= -3$
= R.S.

For each equation, the left side is equal to the right side, so the solution is: $x = 1$ and $y = 3$

c) $\frac{1}{2}x + 3y = \frac{5}{6}$ ①

$\frac{1}{3}x - 5y = \frac{16}{9}$ ②

Write an equivalent system with integer coefficients.
For equation ①, the common denominator is the lowest common multiple of 2 and 6, which is 6:

$\frac{1}{2}x + 3y = \frac{5}{6}$ Multiply each term by 6.

$6\left(\frac{1}{2}x\right) + 6(3y) = 6\left(\frac{5}{6}\right)$ Simplify.

$3x + 18y = 5$ ③

For equation ②, the common denominator is the lowest common multiple of 3 and 9, which is 9:

$\frac{1}{3}x - 5y = \frac{16}{9}$ Multiply each term by 9.

$9\left(\frac{1}{3}x\right) - 9(5y) = 9\left(\frac{16}{9}\right)$ Simplify.

$3x - 45y = 16$ ④

Solve equation ④ for $3x$.

$3x - 45y = 16$ ④
$3x = 16 + 45y$

Substitute for $3x$ in equation ③.

$3x + 18y = 5$ ③
$(16 + 45y) + 18y = 5$
$16 + 45y + 18y = 5$
$63y = -11$
$y = -\frac{11}{63}$

To determine the value of x, solve equation ③ for $9y$.

$3x + 18y = 5$ ③

$$18y = 5 - 3x$$
$$9y = \frac{5}{2} - \frac{3}{2}x$$

Substitute for $9y = \frac{5}{2} - \frac{3}{2}x$ in equation ④.

$$3x - 45y = 16 \quad ④$$
$$3x - 5\left(\frac{5}{2} - \frac{3}{2}x\right) = 16$$
$$3x - \frac{25}{2} + \frac{15}{2}x = 16$$
$$3x + \frac{15}{2}x = 16 + \frac{25}{2}$$
$$\frac{6}{2}x + \frac{15}{2}x = \frac{32}{2} + \frac{25}{2}$$
$$\frac{21}{2}x = \frac{57}{2}$$
$$21x = 57$$
$$x = \frac{57}{21}, \text{ or } \frac{19}{7}$$

Verify the solution.

In each original equation, substitute: $x = \frac{19}{7}$ and $y = -\frac{11}{63}$

$$\frac{1}{2}x + 3y = \frac{5}{6} \quad ①$$

$$\begin{aligned} \text{L.S.} &= \frac{1}{2}x + 3y \\ &= \frac{1}{2}\left(\frac{19}{7}\right) + 3\left(-\frac{11}{63}\right) \\ &= \frac{1}{2}\left(\frac{19}{7}\right) + 3\left(-\frac{11}{63}\right) \\ &= \frac{19}{14} - \frac{11}{21} \\ &= \frac{57}{42} - \frac{22}{42} \\ &= \frac{35}{42}, \text{ or } \frac{5}{6} \\ &= \text{R.S.} \end{aligned}$$

$$\frac{1}{3}x - 5y = \frac{16}{9} \quad ②$$

$$\begin{aligned} \text{L.S.} &= \frac{1}{3}x - 5y \\ &= \frac{1}{3}\left(\frac{19}{7}\right) - 5\left(-\frac{11}{63}\right) \\ &= \frac{1}{3}\left(\frac{19}{7}\right) - 5\left(-\frac{11}{63}\right) \\ &= \frac{19}{21} + \frac{55}{63} \\ &= \frac{57}{63} + \frac{55}{63} \\ &= \frac{112}{63}, \text{ or } \frac{16}{9} \\ &= \text{R.S.} \end{aligned}$$

For each equation, the left side is equal to the right side, so the solution is:
$x = \frac{19}{7}$ and $y = -\frac{11}{63}$

d) $0.6x - 0.2y = -0.2$ ①
$-0.03x - 0.07y = 0.17$ ②
Consider the x-coefficients: $0.6 = 0.03 \times 20$

Solve equation ② for $0.03x$.
$-0.03x - 0.07y = 0.17$ ②
$-0.03x = 0.17 + 0.07y$
$0.03x = -0.17 - 0.07y$
Write equation ① as:
$20(0.03x) - 0.2y = -0.2$
Substitute $0.03x = -0.17 - 0.07y$
$20(-0.17 - 0.07y) - 0.2y = -0.2$
$-3.4 - 1.4y - 0.2y = -0.2$
$-1.6y = 3.2$
$y = -2$
Substitute $y = -2$ in equation ①.
$0.6x - 0.2y = -0.2$ ①
$0.6x - 0.2(-2) = -0.2$
$0.6x + 0.4 = -0.2$
$0.6x = -0.6$
$x = -1$

Verify the solution.
In each equation, substitute: $x = -1$ and $y = -2$

$0.6x - 0.2y = -0.2$ ①
L.S. $= 0.6x - 0.2y$
$= 0.6(-1) - 0.2(-2)$
$= -0.6 + 0.4$
$= -0.2$
$=$ R.S.

$-0.03x - 0.07y = 0.17$ ②
L.S. $= -0.03x - 0.07y$
$= -0.03(-1) - 0.07(-2)$
$= 0.03 + 0.14$
$= 0.17$
$=$ R.S.

For each equation, the left side is equal to the right side, so the solution is:
$x = -1$ and $y = -2$

11. a) Laura multiplied equation ① by 4 because that is the lowest common denominator of the fractions in that equation, and this multiplication will remove the fractions from the equation.
Laura multiplied equation ② by 6 because that is the lowest common denominator of the fractions in that equation.

b) The new linear system will have the same solution as the original equation because multiplying the equations in a linear system by a constant does not change their graphs, so the point of intersection of the graphs in both linear systems will be the same.

c) $-\frac{3}{2}x - \frac{1}{4}y = -\frac{1}{2}$ ①

$\frac{1}{3}x + \frac{5}{6}y = \frac{19}{3}$ ②

Write an equivalent system with integer coefficients.
From part a, multiply equation ① by 4.

$4\left(-\frac{3}{2}x\right) - 4\left(\frac{1}{4}y\right) = 4\left(-\frac{1}{2}\right)$ Simplify.

$-6x - y = -2$ Multiply each term by -1.
$6x + y = 2$ ③

From part a, multiply equation ② by 6.

$6\left(\frac{1}{3}x\right)+6\left(\frac{5}{6}y\right)=6\left(\frac{19}{3}\right)$ Simplify.

$2x+5y=38$ ④

Solve equation ③ for y.

$6x+y=2$ ③

$y=-6x+2$

Substitute for $y=-6x+2$ in equation ④.

$2x+5y=38$ ④

$2x+5(-6x+2)=38$

$2x-30x+10=38$

$-28x=28$

$x=-1$

Substitute $x=-1$ in equation ③.

$6x+y=2$ ③

$6(-1)+y=2$

$-6+y=2$

$y=8$

Verify the solution.

In each original equation, substitute: $x=-1$ and $y=8$

$-\frac{3}{2}x-\frac{1}{4}y=-\frac{1}{2}$ ①

L.S. $=-\frac{3}{2}x-\frac{1}{4}y$

$=-\frac{3}{2}(-1)-\frac{1}{4}(8)$

$=\frac{3}{2}-\frac{4}{2}$

$=-\frac{1}{2}$

= R.S.

$\frac{1}{3}x+\frac{5}{6}y=\frac{19}{3}$ ②

L.S. $=\frac{1}{3}x+\frac{5}{6}y$

$=\frac{1}{3}(-1)+\frac{5}{6}(8)$

$=-\frac{1}{3}+\frac{20}{3}$

$=\frac{19}{3}$

= R.S.

For each equation, the left side is equal to the right side, so the solution is:
$x=-1$ and $y=8$

12. a) Let q represent the number of quarter cup measures.
Let t represent the number of two-thirds cup measures.

$5\frac{3}{4}$ cups were measured using both cups.

So, one equation is: $\frac{1}{4}q+\frac{2}{3}t=5\frac{3}{4}$

One more quarter cup measure was used than two-thirds cup measure.
So, another equation is: $q-t=1$

A linear system is:

$\frac{1}{4}q+\frac{2}{3}t=5\frac{3}{4}$ ①

$q-t=1$ ②

b) Solve the linear system.
Solve equation ② for q.

$q - t = 1$ ②
$q = t + 1$

Substitute $q = t + 1$ in equation ①.

$\frac{1}{4}q + \frac{2}{3}t = 5\frac{3}{4}$ ①

$\frac{1}{4}(t + 1) + \frac{2}{3}t = 5\frac{3}{4}$

$\frac{1}{4}t + \frac{1}{4} + \frac{2}{3}t = 5\frac{3}{4}$

$\frac{1}{4}t + \frac{2}{3}t = 5\frac{3}{4} - \frac{1}{4}$ Write the fractions on the left side with a common denominator.

$\frac{3}{12}t + \frac{8}{12}t = 5\frac{1}{2}$ Write the mixed number as an improper fraction.

$\frac{11}{12}t = \frac{11}{2}$ Multiply each side by $\frac{12}{11}$.

$t = \frac{12}{2}$, or 6

Substitute $t = 6$ in equation ②.

$q - t = 1$ ②
$q - 6 = 1$
$q = 7$

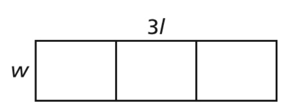
7 quarter cup measures were used and 6 two-thirds cup measured were used.

Verify the solution.

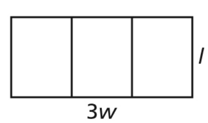
7 quarter cups and 6 two-thirds cups measure: $7\left(\frac{1}{4}\right) + 6\left(\frac{2}{3}\right) = 1\frac{3}{4} + 4$, or $5\frac{3}{4}$; this agrees with the given information.
The difference in numbers of cups used is: $7 - 6 = 1$; this agrees with the given information. So the solution is correct.

13. a) Let l feet represent the length of a table.
Let w feet represent the width of a table.

First 3 tables placed end to end:



First 3 tables placed side by side:



b) From the first diagram above, when the tables are placed end to end, the total length is $30l$ feet and the width is w feet. The perimeter is 306 ft.
So, one equation is:

$30l + 30l + w + w = 306$ Simplify.
$60l + 2w = 306$

From the second diagram above, when the tables are placed side by side, the total length is $30w$ feet and the width is l feet. The perimeter is 190 ft.

So, one equation is:

$30w + 30w + l + l = 190$ Simplify.
$60w + 2l = 190$

A linear system is:

$60l + 2w = 306$ ①
$2l + 60w = 190$ ②

c) To solve the linear system:

Solve equation ② for $2l$.

$2l + 60w = 190$ ②
$2l = -60w + 190$

Substitute $2l = -60w + 190$ into equation ①.

$60l + 2w = 306$ ①
$30(-60w + 190) + 2w = 306$
$-1800w + 5700 + 2w = 306$
$-1798w = 306 - 5700$
$-1798w = -5394$
$w = 3$

Substitute $w = 3$ in equation ②.

$2l + 60w = 190$ ②
$2l + 60(3) = 190$
$2l + 180 = 190$
$2l = 10$
$l = 5$

Each table is 5 ft. long and 3 ft. wide.

Verify the solution.

When the tables are placed end to end:
The length is: 30(5 ft.) = 150 ft.
The width is: 3 ft.
So, the perimeter is: 2(150 ft.) + 2(3 ft.) = 306 ft.; this agrees with the given information.
When the tables are placed side by side:
The length is: 30(3 ft.) = 90 ft.
The width is: 5 ft.
So, the perimeter is: 2(90 ft.) + 2(5 ft.) = 190 ft.; this agrees with the given information.
So, the solution is correct.

14. Write a linear system to model the problem.
Let t represent the number of triangles.
Let s represent the number of squares.
The total number of shapes is 150.
So, one equation is: $t + s = 150$
40% of the triangles were blue; this is: $0.4t$
60% of the squares were blue; this is: $0.6s$
83 shapes were blue.
So, another equation is: $0.4t + 0.6s = 83$

A linear system is:
$t + s = 150$ ①
$0.4t + 0.6s = 83$ ②
Solve equation ① for t.
$t + s = 150$ ①
$t = -s + 150$
Substitute $t = -s + 150$ in equation ②.
$0.4t + 0.6s = 83$ ②
$0.4(-s + 150) + 0.6s = 83$
$-0.4s + 60 + 0.6s = 83$
$0.2s = 23$ Divide each side by 0.2.
$s = 115$
Substitute $s = 115$ in equation ①.
$t + s = 150$ ①
$t + 115 = 150$
$t = 35$
In the design, there were 35 triangles and 115 squares.

Verify the solution.
The total number of shapes was: 115 + 35 = 150; this agrees with the given information.
40% of the triangles and 60% of the squares are: 0.4(35) + 0.6(115) = 83; this agrees with the given information.
So, the solution is correct.

7.5

15. a) $-3x - y = 5$ ①
$2x + y = -5$ ②
Since the y-terms are opposites, add the equations to eliminate y.
$-3x - y = 5$ ①
$\underline{+(2x + y = -5)}$ ②
$-3x + 2x = 5 - 5$
$-x = 0$
$x = 0$
Substitute $x = 0$ into equation ②.
$2x + y = -5$ ②
$2(0) + y = -5$
$y = -5$

Verify the solution.
In each equation, substitute: $x = 0$ and $y = -5$

$-3x - y = 5$ ①
L.S. $= -3x - y$
$= -3(0) - (-5)$
$= 5$
= R.S.

$2x + y = -5$ ②
L.S. $= 2x + y$
$= 2(0) + (-5)$
$= -5$
= R.S.

Since the left side is equal to the right side for each equation, the solution is:
$x = 0$ and $y = -5$

b) $2x - 4y = 13$ ①
$4x - 5y = 8$ ②
Multiply equation ① by 2.

$2(2x - 4y = 13)$
$4x - 8y = 26$ ③
Subtract equation ③ from equation ② to eliminate x.
$4x - 5y = 8$ ②
$\underline{-(4x - 8y = 26)}$ ③
$-5y + 8y = 8 - 26$
$3y = -18$
$y = -6$
Substitute $y = -6$ into equation ②.
$4x - 5y = 8$ ②
$4x - 5(-6) = 8$
$4x + 30 = 8$
$4x = -22$
$x = \frac{-22}{4}$, or $-\frac{11}{2}$

Verify the solution.

In each original equation, substitute: $x = -\frac{11}{2}$ and $y = -6$

$2x - 4y = 13$ ①
L.S. $= 2x - 4y$
$= 2\left(-\frac{11}{2}\right) - 4(-6)$
$= -11 + 24$
$= 13$
$=$ R.S.

$4x - 5y = 8$ ②
L.S. $= 4x - 5y$
$= 4\left(-\frac{11}{2}\right) - 5(-6)$
$= -22 + 30$
$= 8$
$=$ R.S.

Since the left side is equal to the right side for each equation, the solution is:
$x = -\frac{11}{2}$ and $y = -6$

16. a) $3x - 4y = 8.5$ ①
$4x + 2y = 9.5$ ②
Since the y-coefficient in equation ① is a multiple of the y-coefficient in equation ②, I would multiply equation ② by 2 so the coefficients have the same numerical value.

b) Since the y-coefficients would then be opposites, I would add the new equation to equation ① to eliminate y.

c) $3x - 4y = 8.5$ ①
$4x + 2y = 9.5$ ②
Multiply equation ② by 2.
$2(4x + 2y = 9.5)$
$8x + 4y = 19$ ③
Add equations① and ③ to eliminate y.
$3x - 4y = 8.5$ ①
$\underline{+(8x + 4y = 19)}$ ③
$3x + 8x = 8.5 + 19$
$11x = 27.5$
$x = 2.5$

Substitute $x = 2.5$ into equation ②.

$$\begin{aligned} 4x + 2y &= 9.5 \quad ② \\ 4(2.5) + 2y &= 9.5 \\ 10 + 2y &= 9.5 \\ 2y &= -0.5 \\ y &= -0.25 \end{aligned}$$

Verify the solution.
In each original equation, substitute: $x = 2.5$ and $y = -0.25$

$3x - 4y = 8.5$ ①

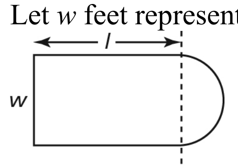
$$\begin{aligned} \text{L.S.} &= 3x - 4y \\ &= 3(2.5) - 4(-0.25) \\ &= 7.5 + 1 \\ &= 8.5 \\ &= \text{R.S.} \end{aligned}$$

$4x + 2y = 9.5$ ②

$$\begin{aligned} \text{L.S.} &= 4x + 2y \\ &= 4(2.5) + 2(-0.25) \\ &= 10 - 0.5 \\ &= 9.5 \\ &= \text{R.S.} \end{aligned}$$

Since the left side is equal to the right side for each equation, the solution is:
$x = 2.5$ and $y = -0.25$

17. a) Let l feet represent the length of the rectangular part.
Let w feet represent the width of the rectangular part.



The length of the rectangular part is 7 ft. longer than the width.
So, one equation is: $l = 7 + w$

The perimeter is approximately $68\frac{5}{6}$ ft.

The perimeter is also: $2l + w +$ length of semicircle
The diameter of the semicircle is the width, w feet.

The length of the semicircle is: $\frac{1}{2}\pi \times \text{diameter} = \frac{1}{2}\pi w$

Another equation is: $2l + w + \frac{1}{2}\pi w = 68\frac{5}{6}$

A linear system is:

$l = 7 + w$ ①

$2l + w + \frac{1}{2}\pi w = 68\frac{5}{6}$ ②

b) Since equation ① is solved for l, use substitution to solve the linear system.
Substitute $l = 7 + w$ in equation ②.

$$\begin{aligned} 2l + w + \frac{1}{2}\pi w &= 68\frac{5}{6} \quad ② \\ 2(7 + w) + w + \frac{1}{2}\pi w &= 68\frac{5}{6} \\ 14 + 2w + w + \frac{1}{2}\pi w &= 68\frac{5}{6} \end{aligned}$$

Write the mixed number as an improper fraction.

$14 + 3w + \frac{1}{2}\pi w = \frac{413}{6}$ Remove w as a common factor.

$w(3 + \frac{1}{2}\pi) = \frac{413}{6} - 14$

$w(3 + \frac{1}{2}\pi) = \frac{413}{6} - \frac{84}{6}$

$w(3 + \frac{1}{2}\pi) = \frac{329}{6}$ Divide each side by $(3 + \frac{1}{2}\pi)$.

$w = \frac{329}{6} \div (3 + \frac{1}{2}\pi)$

$w = 11.9964\ldots$

$w \doteq 12$

Substitute $w = 12$ in equation ①.

$l = 7 + w$ ①

$l = 7 + 12$

$l = 19$

The length of the rectangular part is approximately 19 ft. and the width is approximately 12 ft.

Verify the solution.

The difference between the length and width is approximately: 19 ft. – 12 ft. = 7ft.; this agrees with the given information.

The perimeter, in feet, is: $2(19) + 12 + \frac{1}{2}\pi(12) = 50 + 6\pi$, or approximately 68.85; this agrees approximately with the given information.

So, the lengths are correct to the nearest foot.

7.6

18. a) For a linear system with infinite solutions, one equation is multiplied by a constant to get the other equation. For example,

$5x - 2y = 12$ Multiply this equation by 3.

$15x - 6y = 36$

For a linear system with no solution, the x-coefficients can be the same, the y-coefficients can be the same, and the constant terms are different. For example,

$5x - 2y = 12$

$5x - 2y = 7$

b) When I graph the equations of the linear system with infinite solutions, the graphs coincide.

When I graph the equations of the linear system with no solution, the lines are parallel.

c) When I write each equation in slope-intercept form:

The equations of the linear system with infinite solutions are the same.

The equations of the linear system with no solution have the same x-coefficients, which are the slopes of the graphs; but different constant terms, which are the y-intercepts of the graphs.

19. Let g and o represent the numbers on the shirts.

Write a linear system for each set of clues.

a) Clue 1:
The difference between the numbers is 33.
So, one equation is: $g - o = 33$
The difference between triple the numbers is 99.
So, another equation is: $3g - 3o = 99$
A linear system is:
$g - o = 33$
$3g - 3o = 99$
These equations are equivalent. There are infinite solutions to the linear system, and there are many possible solutions to the problem. There is not sufficient information to determine one solution.
Clue 2:
The sum of the numbers is 57.
So, one equation is: $g + o = 57$
When you divide each number by 3, then add the quotients, the sum is 20.
So, another equation is: $\frac{g}{3} + \frac{o}{3} = 20$
Multiply each side by 3.
$g + o = 60$
A linear system is:
$g + o = 57$
$g + o = 60$
This linear system has no solution.
There must be an error in the clues because the sum of the numbers cannot be 57 and 60.
So, there is not enough information to determine the numbers.
Clue 3:
The sum of the numbers is 57.
So, one equation is: $g + o = 57$
The difference of the numbers is 33.
So, another equation is: $g - o = 33$
A linear system is:
$g + o = 57$
$g - o = 33$

b) Solve the linear system in Clue 3 in part a.
$g + o = 57$ ①
$g - o = 33$ ②
Add the equations to eliminate o.
$g + o = 57$ ①
$\underline{+(g - o = 33)}$ ②
$2g = 90$
$g = 45$
Substitute $g = 45$ in equation ①.
$g + o = 57$ ①
$45 + o = 57$
$o = 12$
The numbers on the shirts are 45 and 12.

Verify the solution.

The sum of the numbers is: 45 + 12 = 57; this agrees with the information in Clue 3.
The difference of the numbers is: 45 – 12 = 33; this agrees with the information in Clue 3.
So, the solution is correct.

20. a) $-x + 5y = 8$ ①
$2x - 10y = 7$ ②
If I multiply the x-coefficient in equation ① by –2, I get the x-coefficient in equation ②.
If I multiply the y-coefficient in equation ① by –2, I get the y-coefficient in equation ②.
When two equations have their coefficients related in this way, the slopes of the graphs of the lines are equal, the lines are parallel, and the linear system has no solution.

b) $-\frac{3}{2}x + \frac{1}{4}y = -\frac{1}{4}$ ①
$\frac{3}{4}x - \frac{y}{8} = \frac{1}{8}$ ②
Multiply each equation by the lowest common denominator of its fractions to write the equation with integer coefficients.
Multiply equation ① by 4.
$-6x + y = -1$ ③
Multiply equation ② by 8.
$6x - y = 1$ ④
If I multiply equation ① by –1, I get equation ②. So, the equations are equivalent, their graphs coincide and the linear system has infinite solutions.

c) $0.5x + y = 0.3$
$-x + 2y = 0.6$
There is no relationship between corresponding coefficients, so the equations do not represent graphs that are parallel or coincident, and the linear system has one solution.

d) $2x - y = -5$ ①
$6x - 3y = 15$ ②
If I multiply the x-coefficient in equation ① by 3, I get the x-coefficient in equation ②.
If I multiply the y-coefficient in equation ① by 3, I get the y-coefficient in equation ②.
So, the slopes of the graphs of the lines are equal, the lines are parallel, and the linear system has no solution.

21. a) In question 20c, the linear system is:
$0.5x + y = 0.3$ ①
$-x + 2y = 0.6$ ②
Write each equation in slope-intercept form.
For equation ①:
$0.5x + y = 0.3$
$y = -0.5x + 0.3$ The slope of the graph is –0.5.
For equation ②:
$-x + 2y = 0.6$
$2y = x + 0.6$ Divide each side by 2.
$y = 0.5x + 0.3$ The slope of the graph is 0.5.
Since the slopes are different, the graphs intersect at one point, and there is only 1 solution.

So, when two lines in a linear system have different slopes, the linear system has only 1 solution.

b) The slopes of the graphs are not sufficient information to distinguish between a linear system that has no solution and a linear system that has infinite solutions.
In question 20a, the linear system is:
$-x + 5y = 8$ ①
$2x - 10y = 7$ ②
Write each equation in slope-intercept form.
For equation ①:
$-x + 5y = 8$
$5y = x + 8$ Divide each side by 5.
$y = \frac{1}{5}x + \frac{8}{5}$ The slope of the graph is $\frac{1}{5}$.
For equation ②:
$2x - 10y = 7$
$-10y = -2x + 7$ Divide each side by -10.
$y = \frac{1}{5}x - \frac{7}{10}$ The slope of the graph is $\frac{1}{5}$.
The graphs have the same slope so I have to look at their y-intercepts to determine whether the graphs are parallel (then the linear system would have no solution), or coincident (then the linear system would have infinite solutions). In the linear system above, the y-intercepts are different, so the lines are parallel and the linear system has no solution.

Practice Test **(page 455)**

1. $3x - 2y = 4.5$ ①

$-x + \frac{y}{2} = -1.25$ ②

A. If I multiply equation ① by 3, it becomes:
$3(3x - 2y = 4.5)$
$9x - 6y = 13.5$ ③
If I add equation ③ to equation ②, I get:
$9x - 6y = 13.5$ ③
$+(-x + \frac{y}{2} = -1.25)$ ②
This will not eliminate x because the x-coefficients are not opposites.
So, A is not the correct answer.

B. Solve each equation for y to determine the slope of its graph.
For equation ①:
$3x - 2y = 4.5$
$-2y = -3x + 4.5$ — Divide each side by –2.
$y = 1.5x - 2.25$ — The slope of the line is 1.5.
For equation ②:
$-x + \frac{y}{2} = -1.25$
$\frac{y}{2} = x - 1.25$ — Multiply each side by 2.
$y = 2x - 2.5$ — The slope of the line is 2.
Since the slopes are different, the lines intersect at one point and there is one solution.
So, B is the correct answer.

C. The new system will be equivalent if the equation $4x - 2y = -5$ is equivalent to
$-x + \frac{y}{2} = -1.25$ ②
Multiply ② by –4 so the x-coefficient is equal to the x-coefficient of $4x - 2y = -5$:
$-4(-x + \frac{y}{2} = -1.25)$
$4x - 2y = 5$ ③
Since the constant terms in equations ③ and $4x - 2y = -5$ are different, the new system will not have the same solution as the original system. So, C is not the correct answer.

D. Verify the solution.
Substitute $x = 1$ and $y = -0.75$ in both equations.

$3x - 2y = 4.5$ ①

L.S. $= 3x - 2y$

$-x + \frac{y}{2} = -1.25$ ②

L.S. $= -x + \frac{y}{2}$

$= 3(1) - 2(-0.75)$
$= 3 + 1.5$
$= 4.5$
$=$ R.S.

$= -1 + \dfrac{(-0.75)}{2}$
$= -1 - 0.375$
$= -1.375$
$\neq$ R.S.

Since the left side is not equal to the right side in both equations, D is not the correct answer.

2. In A, the slopes of the lines are different so the linear system has exactly one solution. So, A is the correct answer.
In B, the equations are equivalent and the linear system has infinite solutions. So, B is not the correct answer.
In C, the graphs have the same slope, and different y-intercepts. The linear system has no solution. So, C is not the correct answer.
In D, only the left sides of the equations are equivalent. The linear system has no solution. So, D is not the correct answer.

3. The linear system in question 2, A is:
$y = 3x - 2$
$y = -4x + 5$
The slopes of the graphs are 3 and –4. The graphs have different slopes so they intersect in exactly one point, and the linear system has exactly one solution.

The linear system in question 2, B is:
$4x - 2y = -0.2$ ①
$-x + 0.5y = 0.05$ ②
Multiply equation ② by –4.
$-4(-x + 0.5y = 0.05)$
$4x - 2y = -0.2$ ③
Equations ① and ③ are the same, so equations ① and ② are equivalent. This means that their graphs coincide, so they intersect at an infinite number of points, and the linear system has infinite solutions.

The linear system in question 2, C is:
$y = 3x - 2$
$y = 3x + 2$
The slopes of both graphs is 3. The graphs have different y-intercepts, so the graphs are parallel. A linear system has no solution when its graphs are parallel because the lines do not intersect.

The linear system in question 2, D is:
$\frac{1}{3}x + \frac{1}{2}y = \frac{1}{6}$ ①
$\frac{1}{6}x + \frac{1}{4}y = \frac{1}{6}$ ②

Multiply equation ① by 6.
$\frac{1}{3}x + \frac{1}{2}y = \frac{1}{6}$ ①

Multiply equation ② by 12.
$\frac{1}{6}x + \frac{1}{4}y = \frac{1}{6}$ ②

$$6\left(\frac{1}{3}\right)x + 6\left(\frac{1}{2}\right)y = 6\left(\frac{1}{6}\right) \qquad 12\left(\frac{1}{6}\right)x + 12\left(\frac{1}{4}\right)y = 12\left(\frac{1}{6}\right)$$

$2x + 3y = 1$ ③ $\qquad$ $2x + 3y = 1$ ④

Equations ③ and ④ have the same x-coefficients and y-coefficients, but different constant terms. So, the linear system has no solution.

4. **a)** A linear system is:

$1.75s + 2.50a = 15.50$ ①

$s - a = 4$ ②

Since the situation involves adults and students and their fares on a transit system, then s could represent the number of students who travel and a could represent the number of adults who travel.

Since the coefficients of s and a in equation ① are decimals, then these coefficients could represent the fares for a students and an adult.
Equation ① represents the total cost for s students and a adults.
Equation ② represents the difference in the numbers of students and adults who travel.
A problem could be:
A group of students and adults travel for a total cost of \$15.50.
A student ticket costs \$1.75 and an adult ticket costs \$2.50.
There are 4 more students than adults.
How many students travelled? How many adults travelled?

b) $1.75s + 2.50a = 15.50$ ①

$s - a = 4$ ②

To solve the linear system:
Solve equation ② for s.

$s - a = 4$ ②

$s = a + 4$

Substitute $s = a + 4$ in equation ①.

$1.75s + 2.50a = 15.50$ ①

$1.75(a + 4) + 2.50a = 15.50$

$1.75a + 7 + 2.50a = 15.50$ Collect like terms, then solve for a.

$4.25a = 8.50$ Collect like terms, then solve for a.

$a = 2$

Substitute $a = 2$ in equation ②.

$s - a = 4$ ②

$s - 2 = 4$

$s = 6$

Six students and 2 adults travelled.

Verify the solution.
The difference in numbers of students and adults is: $6 - 2 = 4$; this agrees with the given information.
The cost for 6 students and 2 adults is: 6(\$1.75) + 2(\$2.50) = \$15.50; this agrees with the given information.
So, the solution is correct.

5. a) i) $-3x - 4y = -2$ ①
$x + 2y = 3$ ②
Solve equation ② for x.
$x + 2y = 3$ ②
$x = -2y + 3$
Substitute $x = -2y + 3$ in equation ①.
$-3x - 4y = -2$ ①
$-3(-2y + 3) - 4y = -2$ Simplify, then solve for y.
$6y - 9 - 4y = -2$
$2y = 7$
$y = \frac{7}{2}$
Substitute $y = \frac{7}{2}$ in equation ②.
$x + 2y = 3$ ②
$x + 2\left(\frac{7}{2}\right) = 3$
$x + 7 = 3$
$x = -4$

Verify the solution.
In each equation, substitute: $x = -4$ and $y = \frac{7}{2}$

$-3x - 4y = -2$ ①
L.S. $= -3x - 4y$
$= -3(-4) - 4\left(\frac{7}{2}\right)$
$= 12 - 14$
$= -2$
$=$ R.S.

$x + 2y = 3$ ②
L.S. $= x + 2y$
$= -4 + 2\left(\frac{7}{2}\right)$
$= -4 + 7$
$= 3$
$=$ R.S.

In each equation, the left side is equal to the right side, so the solution is:
$x = -4$ and $y = \frac{7}{2}$

ii) $-0.5x + 0.2y = -1$ ①
$0.3x - 0.6y = -1.8$ ②
Multiply equation ① by 3.
$3(-0.5x + 0.2y = -1)$
$-1.5x + 0.6y = -3$ ③
Add equations ② and ③ to eliminate y.
$0.3x - 0.6y = -1.8$ ②
$\underline{+(-1.5x + 0.6y = -3)}$ ③
$0.3x - 1.5x = -1.8 - 3$
$-1.2x = -4.8$
$x = 4$
Substitute $x = 4$ in equation ①.
$-0.5x + 0.2y = -1$ ①
$-0.5(4) + 0.2y = -1$ Simplify, then solve for y.
$-2 + 0.2y = -1$

$$0.2y = 1 \qquad \text{Divide each side by 0.2.}$$
$$y = 5$$

Verify the solution.
In each equation, substitute: $x = 4$ and $y = 5$

$-0.5x + 0.2y = -1$ ①
L.S. $= -0.5x + 0.2y$
$= -0.5(4) + 0.2(5)$
$= -2 + 1$
$= -1$
$=$ R.S.

$0.3x - 0.6y = -1.8$ ②
L.S. $= 0.3x - 0.6y$
$= 0.3(4) - 0.6(5)$
$= 1.2 - 3$
$= -1.8$
$=$ R.S.

In each equation, the left side is equal to the right side, so the solution is:
$x = 4$ and $y = 5$

iii) $x - \frac{1}{3}y = \frac{4}{3}$ ①

$\frac{5}{6}x + \frac{1}{2}y = \frac{3}{2}$ ②

Multiply each equation by a common denominator to produce a linear system with integer coefficients.
Multiply equation ① by 3.

$$3x - 3\left(\frac{1}{3}y\right) = 3\left(\frac{4}{3}\right)$$
$$3x - y = 4 \quad ③$$

Multiply equation ② by 6.

$$6\left(\frac{5}{6}x\right) + 6\left(\frac{1}{2}y\right) = 6\left(\frac{3}{2}\right)$$
$$5x + 3y = 9 \quad ④$$

Solve equation ③ for y.

$$3x - y = 4 \quad ③$$
$$-y = -3x + 4$$
$$y = 3x - 4$$

Substitute $y = 3x - 4$ in equation ④.

$$5x + 3y = 9 \quad ④$$
$$5x + 3(3x - 4) = 9 \qquad \text{Simplify, then solve for } x.$$
$$5x + 9x - 12 = 9$$
$$14x = 21$$
$$x = \frac{21}{14}, \text{ or } \frac{3}{2}$$

Substitute $x = \frac{3}{2}$ in equation ③.

$$3x - y = 4 \quad ③$$
$$3\left(\frac{3}{2}\right) - y = 4$$
$$\frac{9}{2} - y = 4$$

$$-y = \frac{8}{2} - \frac{9}{2}$$
$$-y = -\frac{1}{2}$$
$$y = \frac{1}{2}$$

Verify the solution.

In each equation, substitute: $x = \frac{3}{2}$ and $y = \frac{1}{2}$

$x - \frac{1}{3}y = \frac{4}{3}$ ①

$$\begin{aligned}\text{L.S.} &= x - \frac{1}{3}y \\ &= \frac{3}{2} - \frac{1}{3}\left(\frac{1}{2}\right) \\ &= \frac{3}{2} - \frac{1}{6} \\ &= \frac{9}{6} - \frac{1}{6} \\ &= \frac{8}{6} \\ &= \frac{4}{3} \\ &= \text{R.S.}\end{aligned}$$

$\frac{5}{6}x + \frac{1}{2}y = \frac{3}{2}$ ②

$$\begin{aligned}\text{L.S.} &= \frac{5}{6}x + \frac{1}{2}y \\ &= \frac{5}{6}\left(\frac{3}{2}\right) + \frac{1}{2}\left(\frac{1}{2}\right) \\ &= \frac{15}{12} + \frac{1}{4} \\ &= \frac{15}{12} + \frac{3}{12} \\ &= \frac{18}{12} \\ &= \frac{3}{2} \\ &= \text{R.S.}\end{aligned}$$

In each equation, the left side is equal to the right side, so the solution is:

$x = \frac{3}{2}$ and $y = \frac{1}{2}$

b) For the linear system in part a) ii), the solution is: $x = 4$ and $y = 5$
This means that if the equations below are graphed on the same grid, they will intersect at the point with coordinates (4, 5); that is, the solution of a linear system is the coordinates of the point of intersection of the graphs of the system.
$-0.5x + 0.2y = -1$
$0.3x - 0.6y = -1.8$

6. a) Let s represent the number of squares used in the design.
Let t represent the number of triangles used in the design.
The design contains 90 shapes.
So, one equation is: $s + t = 90$
The area of a square is 25 cm^2.
So, the area of s squares is: $25s$ square centimetres
The area of a triangle is 12.5 cm^2.
So, the area of t triangles is: $12.5t$ square centimetres
The total area of the design is 1500 cm^2.
So, another equation is: $25s + 12.5t = 1500$

A linear system is:
$s + t = 90$ ①
$25s + 12.5t = 1500$ ②

b) Solve the linear system to solve the problem.
Solve equation ① for s.
$s + t = 90$ ①
$s = -t + 90$
Substitute $s = -t + 90$ in equation ②.

$$25s + 12.5t = 1500 \quad ②$$
$$25(-t + 90) + 12.5t = 1500$$
$$-25t + 2250 + 12.5t = 1500 \quad \text{Simplify, then solve for } t.$$
$$-12.5t = -750$$
$$t = \frac{-750}{-12.5}$$
$$t = 60$$

Substitute $t = 60$ in equation ①.

$$s + t = 90 \quad ①$$
$$s + 60 = 90$$
$$s = 30$$

Thirty squares and 60 triangles were used.

Verify the solution.
The total number of shapes used is: $30 + 60 = 90$; this agrees with the given information.
The total area is: $30(25 \text{ cm}^2) + 60(12.5 \text{ cm}^2) = 1500 \text{ cm}^2$; this agrees with the given information.
So, the solution is correct.